**APPENDIX**

**1. Annotated Bibliography of Select Bayesian Analysis Books**

1. Bolstad, W.M. 2017. *Introduction to Bayesian Statistics*. Wiley: Hoboken, NJ.

* An introduction to Bayesian statistics suitable for both beginners and intermediate levels
* The topics in the book are covered in more depth compared to Kruschke (2015), but less depth compared to Gill (2015)
* Uses Minitab macros and R for examples and exercises

1. Gelman, A., Carlin, J.B., Stern, H.S., Dunson, D.B., Vehtari, A., & Rubin, D.B. 2013. *Bayesian data analysis (3rd ed).* Chapman & Hall/CRC Press: Boca Raton, Fl.
   * Requires general understanding of mathematical statistics
   * Covers wide variety of regression models
   * Provides introduction to Bayes and Bayesian statistics before going into more advanced computation
   * Includes some intro guidance on programming with R and Stan in the appendices
2. Gill J. 2015. *Bayesian Methods: A social and behavioral sciences approach*. CRC Press: Boca Raton, FL.

* Introduction and application to Bayesian approaches
* Illustrates how to use Bayesian methods in practice and thus focuses on how to conduct Bayesian analysis
* Uses R and BUGS for examples and exercises

1. Hahn, E.D. 2014. *Bayesian Methods for Management and Business: Pragmatic Solutions for Real Problems.* Wiley: Hoboken, NJ.

* Explains how Bayesian statistics can provide insights into critical issues facing management and business
* Utilizes real-world examples drawn from various management disciplines
* Uses WinBUGS and R for examples and exercises

1. Kaplan D. 2014. *Bayesian statistics for the social sciences*. The Guilford Press: New York, NY.

* An intermediate-level overview of Bayesian statistics
* Examples utilize large education and social science databases
* Divided into 3 parts: basics of Bayesian inference; Baysian testing, model building and reqression analysis; and extended modeling
* Touches on multilevel modeling and modeling for continuous and categorical latent variables
* Uses R software packages available on CRAN for examples and exercises

1. Kruschke JK. 2014. *Doing bayesian data analysis: A tutorial with R, JAGS, and Stan (2nd ed.)*. Elsevier: Amsterdam.

* An easy and accessible guide on conducting Bayesian analysis
* Highly recommended for beginners
* Covers nested/hierarchicial models
* Uses R, JAGS, and Stan for examples and exercies

1. Lancaster, T. 2004. *An Introduction to Modern Bayesian Econometrics*. Blackwell: Malden, MA.

* Geared toward econometricians and requires a firm understanding of basic statistics
* Covers variety of econometric linear and non-linear models using cross-sectional, time series, and nested (e.g., panel) data
* Provides frequentist to Bayesian “Conversion” reference manual
* Uses S programming language and Bugs software for examples and exercises

1. Lee, M.D. and Wagenmakers, E-J. 2014. *Bayesian Cognitive Modeling: A Practical Course.* Cambridge University Press: Cambridge.

* An accessible guide of Bayesian modeling
* Covers basics of parameter estimation and model comparison
* Set in context of cognitive sciences – including signal detection theory, psychophysics, and decision making
* Does not require advance statistical knowledge
* Uses WinBUGS, JAGS, Matlab and R for examples and exercises

1. McElreath R. 2016. *Statistical rethinking: A Bayesian course with examples in R and Stan*. CRC Press: Boca Raton, FL.

* Performs step-by-step calculations so that readers are able to make informed choices and interpretations in their own modeling
* Requires coding and (at minimum) basic statistical knowledge
* Covers linear and multilevel modeling
* 2nd edition contains additional material on a number of topics including prior distributions, categorical predictors, cross-validation, and instrumental variables
* Provides optional sections with more detailed mathematics explanations
* Uses R and Stan code for examples and exercises

1. McGrayne S. 2011. *The theory that would not die: How Bayes’ rule cracked the enigma code, hunted down Russian submarines & mereged triumphant from two centuries of controversies*. Yale University Press: New Haven, CT.

* A historical account of the emergence and development of Bayesian statistics
* Describes the battles between Bayesians and frequentists

1. Stone J. 2013. *Bayes’ rule: A tutorial introduction to Bayesian analysis*. Sebtel Press.

* Targeted and well-suited for beginners
* Does not require prior mathematical/statistical knowledge
* Utilizes graphics and illustrations to visually explain Bayes’ rule
* Uses MatLab, Python, and R for examples and exercises

**Appendix 2. Bayesian Analysis Code for R and Stata**

|  |  |  |
| --- | --- | --- |
|  | **R** | **Stata** |
| Library | library(dplyr)  library(fBasics)  library(coda)  library(brms)  library(bayesplot)  library(ggplot2)  library(psych)  library(sjPlot)  library(lme4)  library(stats4)  library(ggeffects)  library(sjmisc) |  |
| Model 1: Frequentist Multilevel Model | m1 <- lmer(y ~ 1 + x1between + x1within + x2between + x2within + x3 + factor(*time*) + (1 | *lev2*), data=a)  summary(m1, digits=3) | xtset panelvar timevar [, *tsoptions*]  xthybrid y x1within x1between x2within x2between x3, clusterid(*varname*) |
| Model 2: Frequentist Multilevel Model w/ Interaction | m2 <- lmer(y ~ 1 + x1between + x1within + x2between + x2within + x3 + x1within:x3 + factor(*time*) + (1 | *lev2*), data=a)  summary(m2, digits=3)  Gender:ROAw | xtset panelvar timevar [, *tsoptions*]  xthybrid y x1within x1between x2within x2between x3 c.x1within#i.x3, clusterid(*varname*) |
| Model 3: Bayesian Default Multilevel Model | m3 <- brm(data = a,  y ~ 1 + x1between + x1within + x2between + x2within + x3 + factor(*time*) + (1 | *lev2*), warmup =#, iter = #, chains = 4)  summary(m3, digits=3)  postm3 <- posterior\_samples (m3, add\_chain = T) | bayes, prior(*priorspec*) mcmcsize(*#*) nchains(#): mixed y x1between x1 within x2between x2within x3 || *lev2*:,  bayesstats ess  bayesstats summary |
| Model 4: Bayesian Multilevel Model w/ Interaction | m4 <- brm(data = a, family= gaussian(link = "identity"),  y ~ 1 + x1between + x1within + x2between + x2within + x3 + x1within:x3 + factor(*time*) + (1 | *lev2*), warmup =#, iter = #, chains = 4)  summary(m4, digits=3)  postm4 <- posterior\_samples (m4, add\_chain = T) | bayesmh, prior(*priorspec*) mcmcsize(*#*) nchains(#): mixed y x1between x1 within x2between x2within x3 c.x1within#i.x3 || *lev2*:  bayesstats ess  bayesstats summary |
| Model 5: Bayesian Multilevel Model w/ Gaussian likelihood & Prior for Gender | m5 <- brm(data = a, family= gaussian(link = "identity"), y ~ 1 + x1between + x1within + x2between + x2within + x3 + factor(*time*) + (1 | *lev2*), warmup =#, iter = #,  prior = c(set\_prior("normal(0, 1)", class = "b", coef = "x3")), chains = 4)  summary(m5, digits=3)  postm5 <- posterior\_samples (m5, add\_chain = T) | bayesmh mcmcsize(*#*) nchains(#): mixed y x1between x1 within x2between x2within x3 || *lev2*:  prior({x3}, normal(0, 1))  bayesstats ess  bayesstats summary |
| Model 6: Bayesian Multilevel Model w/ Gamma likelihood & Raw Compensation as the dependent variable | m6 <- brm(data = a, family= Gamma(link="log"),  y2 ~ 1 + x1between + x1within + x2between + x2within + x3 + factor(*time*) + (1 | *lev2*), warmup =#, iter = #, chains = 4)  summary(m6, digits=3)  postm6 <- posterior\_samples (m6, add\_chain = T) | Bayes mcmcsize(*#*) nchains(#): mixed y2 x1between x1 within x2between x2within x3 || *lev2*:, family(gamma) link(log)  bayesstats ess  bayesstats summary |
| Interaction Plot for frequentist model | plot\_model(m2, type = “int”) | margins x3, at(x1within = (values for x1within))  marginsplot, ci |
| Interaction Plot for Bayesian model | plot(conditional\_effects(m4, effects = “x1within:x3”)) | findit marginscontplot  (install package)  mcp x1within x3, ci |
| Distribution of simulated ys by randomly selecting *#* samples from the posterior distribution | pp\_check(m3, nsamples = #) | bayesstat ppvalues  bayespredict pmean, mean rseed(#)  bayesreps yrep\*, nreps(#) rseed(#)  list lnoutput yrep1 yrep2 yrep3 pmean in 1/10  twoway histogram lnoutput || histogram yrep1, color(*color*) ||  histogram yrep2, color(*color*) || histogram yrep3, color(*color*)  legend(off) title(Observed vs replicated data) |
| Trace plots for model | postm3T <- subset(postm3, select=c("x1", "x2", "x3", "x4", "x5", "sigma", "chain"))  mcmc\_trace(postm3T, pars = c("x1", "x2", "x3", "x4", "x5", "sigma")) | bayesgraph trace *spec* [*spec* . . .] [, *multiopts*]  bayesgraph diagnostics \_all |
| Kernel density plot | mcmc\_areas(postm3, pars = c("x1"), prob = 0.95, point\_est=c("median")) | bayesgraph kdensity *spec* [*spec* . . .] [, *multiopts*]  bayesgraph diagnostics \_all |
| Correlation Matrix | postm3CM <- subset(m3, select=c(("x1", "x2", "x3", "x4", "x5", "sigma")  panel.cor <- function(x, y, cex.cor,...)  {  usr <- par("usr"); on.exit(par(usr))  par(usr = c(0, 1, 0, 1))  r <- round(cor(x, y), digits =3)  txt <- paste0(r)  if(missing(cex.cor)) cex.cor <- 0.8/strwidth(txt)  text(0.5, 0.5, txt)  }  panel.hist <- function(x, ...)  {  usr <- par("usr"); on.exit(par(usr))  par(usr = c(usr[1:2], 0, 1.5) )  h <- hist(x, plot = FALSE)  breaks <- h$breaks; nB <- length(breaks)  y <- h$counts; y <- y/max(y)  rect(breaks[-nB], 0, breaks[-1], y, col.bar="#b0c4de",...)  }  pairs(postm3CM) | graph matrix x1 x2 x3 x4 x5 sigma |
| Caterpillar Plot | Please refer to:  https://mc-stan.org/users/documentation/case-studies/tutorial\_rstanarm.html | Please refer to:  Laura Bellows & Paul T. von Hippel, 2017. "CATERPILLAR: Stata module to generate confidence intervals, Bonferroni-corrected confidence intervals, and null distribution," Statistical Software Components S458360, Boston College Department of Economics, revised 02 Feb 2020. |