Supplementary Material for "Considerations for Fitting Dynamic Bayesian Networks with Latent Variables: A Monte Carlo Study"

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## Results of the Simulation Study

Tables S1 - S10 contain the results of the parameter recovery simulation study for the raw bias, relative bias, and efficiency in the estimation of the transition probability and measurement model parameters as well as classification accuracy values for all models save for the L-CDM model. Table S11 presents the relative bias values for the L-DCM measurement model parameters. Additional tables, such as results for other parameters (i.e., the prior probability of mastery parameter) and/or for other indices (i.e., root mean squared error [RMSE]) are available upon request of the corresponding author (rreichenberg@unl.edu).

Table S1.
Raw bias in the estimation of the transition probability across manipulated design facets.

|  |  | $M Q=$ Low |  |  |  | $M Q=$ Med |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |  |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | .166 | .129 | .102 | .008 | .001 | .002 |  |
|  | $J=1, T=10$ | .047 | .025 | .015 | .000 | .001 | .003 |  |
|  | $J=3, T=5$ | .041 | .023 | .015 | .001 | .000 | .001 |  |
|  | $J=3, T=10$ | .005 | .005 | .007 | .001 | .001 | .001 |  |
|  | $J=5, T=5$ | .014 | .008 | .005 | .000 | .001 | .001 |  |
|  | $J=5, T=10$ | .004 | .005 | .005 | .002 | .000 | .001 |  |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | .061 | .045 | .037 | .000 | .001 | -.001 |  |
|  | $J=1, T=10$ | .018 | .007 | -.003 | -.003 | -.001 | .000 |  |
|  | $J=3, T=5$ | .024 | .011 | .004 | .002 | .001 | .001 |  |
|  | $J=3, T=10$ | .000 | .003 | -.003 | .002 | .001 | .002 |  |
|  | $J=5, T=5$ | .011 | .004 | .003 | .002 | .001 | .001 |  |
|  | $J=5, T=10$ | -.003 | .001 | .002 | .001 | .001 | .001 |  |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | .158 | .118 | .080 | -.002 | .000 | .002 |  |
|  | $J=1, T=10$ | .061 | .028 | .018 | .000 | .002 | .003 |  |
|  | $J=3, T=5$ | .027 | .015 | .009 | -.001 | .000 | .001 |  |
|  | $J=3, T=10$ | .004 | .005 | .008 | .002 | .001 | .000 |  |
|  | $J=5, T=5$ | .005 | .004 | .005 | .002 | .001 | .001 |  |
|  | $J=5, T=10$ | .003 | .005 | .007 | .003 | .000 | .000 |  |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | .047 | .049 | .029 | -.011 | -.008 | -.002 |  |
|  | $J=1, T=10$ | .021 | .005 | -.007 | -.010 | -.004 | .000 |  |
|  | $J=3, T=5$ | .014 | .007 | -.001 | -.001 | .001 | .002 |  |
|  | $J=3, T=10$ | .003 | -.008 | -.006 | -.001 | -.001 | .001 |  |
|  | $J=5, T=5$ | -.002 | -.002 | .003 | .001 | .000 | .001 |  |
|  | $J=5, T=10$ | -.004 | .000 | .005 | .002 | .001 | .001 |  |

Table S2.
Relative bias in the estimation of the transition probability across manipulated design facets.

|  |  | $M Q=$ Low |  |  |  | $M Q=$ Med |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |  |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | $83.00 \%$ | $64.50 \%$ | $51.00 \%$ | $4.00 \%$ | $0.50 \%$ | $1.00 \%$ |  |
|  | $J=1, T=10$ | $23.50 \%$ | $12.50 \%$ | $7.50 \%$ | $0.00 \%$ | $0.50 \%$ | $1.50 \%$ |  |
|  | $J=3, T=5$ | $20.50 \%$ | $11.50 \%$ | $7.50 \%$ | $0.50 \%$ | $0.00 \%$ | $0.50 \%$ |  |
|  | $J=3, T=10$ | $2.50 \%$ | $2.50 \%$ | $3.50 \%$ | $0.50 \%$ | $0.50 \%$ | $0.50 \%$ |  |
|  | $J=5, T=5$ | $7.00 \%$ | $4.00 \%$ | $2.50 \%$ | $0.00 \%$ | $0.50 \%$ | $0.50 \%$ |  |
|  | $J=5, T=10$ | $2.00 \%$ | $2.50 \%$ | $2.50 \%$ | $1.00 \%$ | $0.00 \%$ | $0.50 \%$ |  |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | $15.25 \%$ | $11.25 \%$ | $9.25 \%$ | $0.00 \%$ | $0.25 \%$ | $-0.25 \%$ |  |
|  | $J=1, T=10$ | $4.50 \%$ | $1.75 \%$ | $-0.75 \%$ | $-0.75 \%$ | $-0.25 \%$ | $0.00 \%$ |  |
|  | $J=3, T=5$ | $6.00 \%$ | $2.75 \%$ | $1.00 \%$ | $0.50 \%$ | $0.25 \%$ | $0.25 \%$ |  |
|  | $J=3, T=10$ | $0.00 \%$ | $0.75 \%$ | $-0.75 \%$ | $0.50 \%$ | $0.25 \%$ | $0.50 \%$ |  |
|  | $J=5, T=5$ | $2.75 \%$ | $1.00 \%$ | $0.75 \%$ | $0.50 \%$ | $0.25 \%$ | $0.25 \%$ |  |
|  | $J=5, T=10$ | $-0.75 \%$ | $0.25 \%$ | $0.50 \%$ | $0.25 \%$ | $0.25 \%$ | $0.25 \%$ |  |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | $79.00 \%$ | $59.00 \%$ | $40.00 \%$ | $-1.00 \%$ | $0.00 \%$ | $1.00 \%$ |  |
|  | $J=1, T=10$ | $30.50 \%$ | $14.00 \%$ | $9.00 \%$ | $0.00 \%$ | $1.00 \%$ | $1.50 \%$ |  |
|  | $J=3, T=5$ | $13.50 \%$ | $7.50 \%$ | $4.50 \%$ | $-0.50 \%$ | $0.00 \%$ | $0.50 \%$ |  |
|  | $J=3, T=10$ | $2.00 \%$ | $2.50 \%$ | $4.00 \%$ | $1.00 \%$ | $0.50 \%$ | $0.00 \%$ |  |
|  | $J=5, T=5$ | $2.50 \%$ | $2.00 \%$ | $2.50 \%$ | $1.00 \%$ | $0.50 \%$ | $0.50 \%$ |  |
|  | $J=5, T=10$ | $1.50 \%$ | $2.50 \%$ | $3.50 \%$ | $1.50 \%$ | $0.00 \%$ | $0.00 \%$ |  |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | $11.75 \%$ | $12.25 \%$ | $7.25 \%$ | $-2.75 \%$ | $-2.00 \%$ | $-0.50 \%$ |  |
|  | $J=1, T=10$ | $5.25 \%$ | $1.25 \%$ | $-1.75 \%$ | $-2.50 \%$ | $-1.00 \%$ | $0.00 \%$ |  |
|  | $J=3, T=5$ | $3.50 \%$ | $1.75 \%$ | $-0.25 \%$ | $-0.25 \%$ | $0.25 \%$ | $0.50 \%$ |  |
|  | $J=3, T=10$ | $0.75 \%$ | $-2.00 \%$ | $-1.50 \%$ | $-0.25 \%$ | $-0.25 \%$ | $0.25 \%$ |  |
|  | $J=5, T=5$ | $-0.50 \%$ | $-0.50 \%$ | $0.75 \%$ | $0.25 \%$ | $0.00 \%$ | $0.25 \%$ |  |
|  | $J=5, T=10$ | $-1.00 \%$ | $0.00 \%$ | $1.25 \%$ | $0.50 \%$ | $0.25 \%$ | $0.25 \%$ |  |

Table S3.
Efficiency in the estimation of the transition probability across manipulated design facets.

|  |  | $M Q=$ Low |  |  | $M Q=$ Med |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | .192 | .146 | .089 | .045 | .031 | .020 |
|  | $J=1, T=10$ | .113 | .069 | .040 | .024 | .017 | .011 |
|  | $J=3, T=5$ | .096 | .061 | .037 | .024 | .016 | .011 |
|  | $J=3, T=10$ | .043 | .029 | .019 | .017 | .011 | .007 |
|  | $J=5, T=5$ | .059 | .039 | .023 | .020 | .015 | .010 |
|  | $J=5, T=10$ | .030 | .020 | .013 | .016 | .011 | .007 |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | .181 | .136 | .087 | .060 | .046 | .029 |
|  | $J=1, T=10$ | .141 | .104 | .064 | .043 | .031 | .021 |
|  | $J=3, T=5$ | .113 | .075 | .047 | .033 | .024 | .015 |
|  | $J=3, T=10$ | .076 | .052 | .034 | .029 | .020 | .013 |
|  | $J=5, T=5$ | .074 | .055 | .034 | .028 | .021 | .013 |
|  | $J=5, T=10$ | .055 | .037 | .023 | .026 | .020 | .012 |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | .185 | .152 | .100 | .048 | .033 | .022 |
|  | $J=1, T=10$ | .141 | .088 | .047 | .029 | .020 | .013 |
|  | $J=3, T=5$ | .115 | .062 | .039 | .028 | .020 | .012 |
|  | $J=3, T=10$ | .051 | .035 | .022 | .020 | .013 | .009 |
|  | $J=5, T=5$ | .060 | .043 | .026 | .024 | .018 | .011 |
|  | $J=5, T=10$ | .033 | .024 | .016 | .019 | .014 | .008 |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | .188 | .153 | .104 | .066 | .049 | .035 |
|  | $J=1, T=10$ | .160 | .119 | .079 | .052 | .037 | .023 |
|  | $J=3, T=5$ | .126 | .091 | .055 | .038 | .027 | .018 |
|  | $J=3, T=10$ | .100 | .063 | .039 | .033 | .023 | .014 |
|  | $J=5, T=5$ | .090 | .062 | .041 | .035 | .024 | .015 |
|  | $J=5, T=10$ | .069 | .046 | .029 | .030 | .021 | .014 |

Table S4.
Raw bias in the estimation of the conditional probability of a correct response for a Master.

|  |  | $M Q=$ Low |  |  |  | $M Q=$ Med |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |  |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | -.027 | -.028 | -.033 | -.008 | -.002 | -.001 |  |
|  | $J=1, T=10$ | -.003 | -.004 | .000 | -.001 | .001 | .002 |  |
|  | $J=3, T=5$ | -.011 | -.009 | -.005 | -.003 | -.002 | .000 |  |
|  | $J=3, T=10$ | .000 | .000 | .003 | .000 | .000 | .000 |  |
|  | $J=5, T=5$ | -.004 | -.003 | .000 | -.004 | -.002 | .000 |  |
|  | $J=5, T=10$ | .000 | .001 | .002 | -.001 | -.001 | .000 |  |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | .001 | -.002 | -.004 | .001 | .000 | .002 |  |
|  | $J=1, T=10$ | .001 | .002 | .003 | .002 | .002 | .003 |  |
|  | $J=3, T=5$ | -.002 | -.001 | .002 | -.002 | .000 | .000 |  |
|  | $J=3, T=10$ | .002 | .001 | .004 | -.001 | .000 | .000 |  |
|  | $J=5, T=5$ | .000 | .000 | .002 | -.002 | -.001 | .000 |  |
|  | $J=5, T=10$ | .001 | .002 | .002 | -.001 | .000 | .000 |  |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | -.014 | -.015 | -.017 | .000 | .001 | .001 |  |
|  | $J=1, T=10$ | -.001 | .000 | .001 | .001 | .002 | .002 |  |
|  | $J=3, T=5$ | -.002 | -.003 | -.001 | -.002 | -.001 | .000 |  |
|  | $J=3, T=10$ | .002 | .001 | .003 | -.001 | .000 | .000 |  |
|  | $J=5, T=5$ | .000 | .000 | .001 | -.002 | .000 | .000 |  |
|  | $J=5, T=10$ | .000 | .002 | .002 | -.001 | -.001 | .000 |  |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | .006 | .001 | .000 | .005 | .004 | .003 |  |
|  | $J=1, T=10$ | .003 | .003 | .004 | .002 | .003 | .003 |  |
|  | $J=3, T=5$ | .001 | .001 | .003 | .000 | -.001 | .001 |  |
|  | $J=3, T=10$ | .002 | .003 | .005 | .000 | .001 | .000 |  |
|  | $J=5, T=5$ | .003 | .001 | .002 | -.001 | .000 | .000 |  |
|  | $J=5, T=10$ | .002 | .003 | .003 | -.001 | .000 | .000 |  |

Table S5.
Relative bias in the estimation of the conditional probability of a correct response for a Master.

|  |  | $M Q=$ Low |  |  | $M Q=\mathrm{Med}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | -4.50\% | -4.67\% | -5.50\% | -1.07\% | -0.27\% | -0.13\% |
|  | $J=1, T=10$ | -0.50\% | -0.67\% | 0.00\% | -0.13\% | 0.13\% | 0.27\% |
|  | $J=3, T=5$ | -1.83\% | -1.50\% | -0.83\% | -0.40\% | -0.27\% | 0.00\% |
|  | $J=3, T=10$ | 0.00\% | 0.00\% | 0.50\% | 0.00\% | 0.00\% | 0.00\% |
|  | $J=5, T=5$ | -0.67\% | -0.50\% | 0.00\% | -0.53\% | -0.27\% | 0.00\% |
|  | $J=5, T=10$ | 0.00\% | 0.17\% | 0.33\% | -0.13\% | -0.13\% | 0.00\% |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | 0.17\% | -0.33\% | -0.67\% | 0.13\% | 0.00\% | 0.27\% |
|  | $J=1, T=10$ | 0.17\% | 0.33\% | 0.50\% | 0.27\% | 0.27\% | 0.40\% |
|  | $J=3, T=5$ | -0.33\% | -0.17\% | 0.33\% | -0.27\% | 0.00\% | 0.00\% |
|  | $J=3, T=10$ | 0.33\% | 0.17\% | 0.67\% | -0.13\% | 0.00\% | 0.00\% |
|  | $J=5, T=5$ | 0.00\% | 0.00\% | 0.33\% | -0.27\% | -0.13\% | 0.00\% |
|  | $J=5, T=10$ | 0.17\% | 0.33\% | 0.33\% | -0.13\% | 0.00\% | 0.00\% |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | -2.33\% | -2.50\% | -2.83\% | 0.00\% | 0.13\% | 0.13\% |
|  | $J=1, T=10$ | -0.17\% | 0.00\% | 0.17\% | 0.13\% | 0.27\% | 0.27\% |
|  | $J=3, T=5$ | -0.33\% | -0.50\% | -0.17\% | -0.27\% | -0.13\% | 0.00\% |
|  | $J=3, T=10$ | 0.33\% | 0.17\% | 0.50\% | -0.13\% | 0.00\% | 0.00\% |
|  | $J=5, T=5$ | 0.00\% | 0.00\% | 0.17\% | -0.27\% | 0.00\% | 0.00\% |
|  | $J=5, T=10$ | 0.00\% | 0.33\% | 0.33\% | -0.13\% | -0.13\% | 0.00\% |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | 1.00\% | 0.17\% | 0.00\% | 0.67\% | 0.53\% | 0.40\% |
|  | $J=1, T=10$ | 0.50\% | 0.50\% | 0.67\% | 0.27\% | 0.40\% | 0.40\% |
|  | $J=3, T=5$ | 0.17\% | 0.17\% | 0.50\% | 0.00\% | -0.13\% | 0.13\% |
|  | $J=3, T=10$ | 0.33\% | 0.50\% | 0.83\% | 0.00\% | 0.13\% | 0.00\% |
|  | $J=5, T=5$ | 0.50\% | 0.17\% | 0.33\% | -0.13\% | 0.00\% | 0.00\% |
|  | $J=5, T=10$ | 0.33\% | 0.50\% | 0.50\% | -0.13\% | 0.00\% | 0.00\% |

Table S6.
Efficiency in the estimation of the conditional probability of a correct response for a Master.

|  |  | $M Q=$ Low |  |  |  | $M Q=$ Med |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |  |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | .062 | .053 | .029 | .043 | .034 | .022 |  |
|  | $J=1, T=10$ | .030 | .021 | .014 | .017 | .013 | .008 |  |
|  | $J=3, T=5$ | .046 | .034 | .021 | .023 | .017 | .011 |  |
|  | $J=3, T=10$ | .019 | .014 | .009 | .013 | .009 | .006 |  |
|  | $J=5, T=5$ | .037 | .026 | .016 | .022 | .015 | .010 |  |
|  | $J=5, T=10$ | .017 | .012 | .008 | .012 | .009 | .006 |  |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | .044 | .034 | .019 | .031 | .024 | .015 |  |
|  | $J=1, T=10$ | .020 | .014 | .009 | .014 | .011 | .007 |  |
|  | $J=3, T=5$ | .031 | .021 | .014 | .021 | .014 | .009 |  |
|  | $J=3, T=10$ | .015 | .011 | .007 | .011 | .008 | .005 |  |
|  | $J=5, T=5$ | .027 | .018 | .012 | .018 | .013 | .008 |  |
|  | $J=5, T=10$ | .014 | .010 | .006 | .011 | .008 | .005 |  |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | .053 | .046 | .028 | .034 | .026 | .018 |  |
|  | $J=1, T=10$ | .029 | .021 | .012 | .016 | .012 | .007 |  |
|  | $J=3, T=5$ | .039 | .027 | .018 | .021 | .015 | .009 |  |
|  | $J=3, T=10$ | .018 | .013 | .009 | .012 | .008 | .005 |  |
|  | $J=5, T=5$ | .029 | .022 | .014 | .018 | .013 | .008 |  |
|  | $J=5, T=10$ | .016 | .011 | .007 | .011 | .008 | .005 |  |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | .043 | .032 | .018 | .027 | .021 | .014 |  |
|  | $J=1, T=10$ | .019 | .014 | .009 | .014 | .010 | .006 |  |
|  | $J=3, T=5$ | .031 | .021 | .013 | .019 | .013 | .008 |  |
|  | $J=3, T=10$ | .015 | .011 | .007 | .011 | .008 | .005 |  |
|  | $J=5, T=5$ | .027 | .018 | .012 | .018 | .013 | .008 |  |
|  | $J=5, T=10$ | .014 | .010 | .006 | .011 | .008 | .005 |  |

Table S7.
Raw bias in the estimation of the conditional probability of a correct response for a Non-master.

|  |  | $M Q=$ Low |  |  |  | $M Q=$ Med |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |  |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | -.029 | -.037 | -.044 | -.004 | .000 | -.001 |  |
|  | $J=1, T=10$ | -.013 | -.012 | -.006 | .007 | .004 | -.001 |  |
|  | $J=3, T=5$ | -.012 | -.009 | -.006 | .002 | .001 | .000 |  |
|  | $J=3, T=10$ | .002 | .000 | .000 | .003 | .002 | .001 |  |
|  | $J=5, T=5$ | -.003 | -.002 | -.001 | .002 | .002 | .000 |  |
|  | $J=5, T=10$ | .001 | .000 | .000 | .004 | .002 | .000 |  |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | -.021 | -.024 | -.023 | .013 | .007 | .002 |  |
|  | $J=1, T=10$ | .002 | .001 | .005 | .030 | .015 | .007 |  |
|  | $J=3, T=5$ | -.003 | -.004 | .001 | .005 | .003 | .001 |  |
|  | $J=3, T=10$ | .009 | .007 | .011 | .009 | .005 | .002 |  |
|  | $J=5, T=5$ | .001 | .000 | .001 | .004 | .001 | .001 |  |
|  | $J=5, T=10$ | .009 | .005 | .005 | .008 | .004 | .002 |  |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | -.006 | -.015 | -.027 | .007 | .003 | -.001 |  |
|  | $J=1, T=10$ | .003 | -.002 | -.004 | .011 | .005 | .000 |  |
|  | $J=3, T=5$ | -.008 | -.005 | -.004 | .004 | .001 | .000 |  |
|  | $J=3, T=10$ | .006 | .003 | .001 | .005 | .003 | .001 |  |
|  | $J=5, T=5$ | .000 | -.003 | -.001 | .003 | .001 | .001 |  |
|  | $J=5, T=10$ | .004 | .001 | .001 | .006 | .003 | .001 |  |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | .005 | .001 | .000 | .031 | .016 | .007 |  |
|  | $J=1, T=10$ | .025 | .020 | .024 | .046 | .023 | .010 |  |
|  | $J=3, T=5$ | .006 | .005 | .005 | .009 | .004 | .001 |  |
|  | $J=3, T=10$ | .019 | .016 | .015 | .017 | .008 | .003 |  |
|  | $J=5, T=5$ | .006 | .004 | .003 | .006 | .003 | .000 |  |
|  | $J=5, T=10$ | .013 | .008 | .005 | .011 | .006 | .003 |  |

Table S8.
Relative bias in the estimation of the conditional probability of a correct response for a Non-master.

|  |  | $M Q=$ Low |  |  | $M Q=$ Med |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | $-7.25 \%$ | $-9.25 \%$ | $-11.00 \%$ | $-1.60 \%$ | $0.00 \%$ | $-0.40 \%$ |
|  | $J=1, T=10$ | $-3.25 \%$ | $-3.00 \%$ | $-1.50 \%$ | $2.80 \%$ | $1.60 \%$ | $-0.40 \%$ |
|  | $J=3, T=5$ | $-3.00 \%$ | $-2.25 \%$ | $-1.50 \%$ | $0.80 \%$ | $0.40 \%$ | $0.00 \%$ |
|  | $J=3, T=10$ | $0.50 \%$ | $0.00 \%$ | $0.00 \%$ | $1.20 \%$ | $0.80 \%$ | $0.40 \%$ |
|  | $J=5, T=5$ | $-0.75 \%$ | $-0.50 \%$ | $-0.25 \%$ | $0.80 \%$ | $0.80 \%$ | $0.00 \%$ |
|  | $J=5, T=10$ | $0.25 \%$ | $0.00 \%$ | $0.00 \%$ | $1.60 \%$ | $0.80 \%$ | $0.00 \%$ |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | $-5.25 \%$ | $-6.00 \%$ | $-5.75 \%$ | $5.20 \%$ | $2.80 \%$ | $0.80 \%$ |
|  | $J=1, T=10$ | $0.50 \%$ | $0.25 \%$ | $1.25 \%$ | $12.00 \%$ | $6.00 \%$ | $2.80 \%$ |
|  | $J=3, T=5$ | $-0.75 \%$ | $-1.00 \%$ | $0.25 \%$ | $2.00 \%$ | $1.20 \%$ | $0.40 \%$ |
|  | $J=3, T=10$ | $2.25 \%$ | $1.75 \%$ | $2.75 \%$ | $3.60 \%$ | $2.00 \%$ | $0.80 \%$ |
|  | $J=5, T=5$ | $0.25 \%$ | $0.00 \%$ | $0.25 \%$ | $1.60 \%$ | $0.40 \%$ | $0.40 \%$ |
|  | $J=5, T=10$ | $2.25 \%$ | $1.25 \%$ | $1.25 \%$ | $3.20 \%$ | $1.60 \%$ | $0.80 \%$ |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | $-1.50 \%$ | $-3.75 \%$ | $-6.75 \%$ | $2.80 \%$ | $1.20 \%$ | $-0.40 \%$ |
|  | $J=1, T=10$ | $0.75 \%$ | $-0.50 \%$ | $-1.00 \%$ | $4.40 \%$ | $2.00 \%$ | $0.00 \%$ |
|  | $J=3, T=5$ | $-2.00 \%$ | $-1.25 \%$ | $-1.00 \%$ | $1.60 \%$ | $0.40 \%$ | $0.00 \%$ |
|  | $J=3, T=10$ | $1.50 \%$ | $0.75 \%$ | $0.25 \%$ | $2.00 \%$ | $1.20 \%$ | $0.40 \%$ |
|  | $J=5, T=5$ | $0.00 \%$ | $-0.75 \%$ | $-0.25 \%$ | $1.20 \%$ | $0.40 \%$ | $0.40 \%$ |
|  | $J=5, T=10$ | $1.00 \%$ | $0.25 \%$ | $0.25 \%$ | $2.40 \%$ | $1.20 \%$ | $0.40 \%$ |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | $1.25 \%$ | $0.25 \%$ | $0.00 \%$ | $12.40 \%$ | $6.40 \%$ | $2.80 \%$ |
|  | $J=1, T=10$ | $6.25 \%$ | $5.00 \%$ | $6.00 \%$ | $18.40 \%$ | $9.20 \%$ | $4.00 \%$ |
|  | $J=3, T=5$ | $1.50 \%$ | $1.25 \%$ | $1.25 \%$ | $3.60 \%$ | $1.60 \%$ | $0.40 \%$ |
|  | $J=3, T=10$ | $4.75 \%$ | $4.00 \%$ | $3.75 \%$ | $6.80 \%$ | $3.20 \%$ | $1.20 \%$ |
|  | $J=5, T=5$ | $1.50 \%$ | $1.00 \%$ | $0.75 \%$ | $2.40 \%$ | $1.20 \%$ | $0.00 \%$ |
|  | $J=5, T=10$ | $3.25 \%$ | $2.00 \%$ | $1.25 \%$ | $4.40 \%$ | $2.40 \%$ | $1.20 \%$ |

Table S9.
Efficiency in the estimation of the conditional probability of a correct response for a Non-master.

|  |  | $M Q=$ Low |  |  | $M Q=$ Med |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | .052 | .046 | .033 | .040 | .030 | .020 |
|  | $J=1, T=10$ | .039 | .033 | .024 | .029 | .021 | .014 |
|  | $J=3, T=5$ | .043 | .031 | .019 | .022 | .015 | .010 |
|  | $J=3, T=10$ | .029 | .022 | .014 | .018 | .013 | .008 |
|  | $J=5, T=5$ | .034 | .023 | .015 | .020 | .014 | .009 |
|  | $J=5, T=10$ | .026 | .019 | .012 | .018 | .012 | .008 |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | .050 | .042 | .031 | .051 | .038 | .027 |
|  | $J=1, T=10$ | .038 | .034 | .029 | .044 | .036 | .025 |
|  | $J=3, T=5$ | .046 | .034 | .022 | .028 | .020 | .012 |
|  | $J=3, T=10$ | .036 | .031 | .021 | .026 | .019 | .012 |
|  | $J=5, T=5$ | .038 | .029 | .018 | .025 | .017 | .011 |
|  | $J=5, T=10$ | .035 | .025 | .016 | .023 | .017 | .011 |
| $T P=$ Low, $I P=$ High | $J=1, T=5$ | .063 | .055 | .041 | .046 | .034 | .026 |
|  | $J=1, T=10$ | .047 | .042 | .032 | .035 | .026 | .016 |
|  | $J=3, T=5$ | .053 | .037 | .024 | .026 | .018 | .012 |
|  | $J=3, T=10$ | .038 | .027 | .018 | .021 | .015 | .009 |
|  | $J=5, T=5$ | .040 | .029 | .018 | .024 | .017 | .011 |
|  | $J=5, T=10$ | .031 | .022 | .014 | .020 | .014 | .009 |
| $T P=$ High, $I P=$ High | $J=1, T=5$ | .056 | .047 | .035 | .059 | .052 | .037 |
|  | $J=1, T=10$ | .041 | .040 | .037 | .054 | .043 | .030 |
|  | $J=3, T=5$ | .056 | .042 | .029 | .034 | .025 | .016 |
|  | $J=3, T=10$ | .050 | .037 | .028 | .031 | .023 | .015 |
|  | $J=5, T=5$ | .051 | .037 | .025 | .029 | .021 | .013 |
|  | $J=5, T=10$ | .041 | .032 | .021 | .027 | .020 | .012 |

Table S10.
Classification accuracy (validation), as a percentage across manipulated design facets.

|  |  | $M Q=$ Low |  |  | $M Q=\mathrm{Med}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $N=200$ | $N=400$ | $N=1000$ | $N=200$ | $N=400$ | $N=1000$ |
| $T P=$ Low, $I P=$ Low | $J=1, T=5$ | 66.77 | 66.67 | 67.35 | 80.94 | 81.42 | 81.84 |
|  | $J=1, T=10$ | 88.06 | 88.84 | 89.21 | 93.19 | 93.20 | 93.25 |
|  | $J=3, T=5$ | 70.76 | 72.23 | 73.47 | 91.19 | 91.42 | 91.46 |
|  | $J=3, T=10$ | 89.91 | 90.20 | 90.33 | 97.13 | 97.11 | 97.09 |
|  | $J=5, T=5$ | 76.17 | 77.26 | 77.93 | 95.08 | 95.18 | 95.12 |
|  | $J=5, T=10$ | 91.59 | 91.63 | 91.84 | 98.40 | 98.37 | 98.39 |
| $T P=$ High, $I P=$ Low | $J=1, T=5$ | 87.58 | 88.39 | 89.48 | 91.09 | 91.22 | 91.38 |
|  | $J=1, T=10$ | 99.00 | 99.18 | 99.20 | 99.22 | 99.27 | 99.26 |
|  | $J=3, T=5$ | 88.86 | 89.41 | 89.73 | 95.63 | 95.64 | 95.65 |
|  | $J=3, T=10$ | 99.15 | 99.19 | 99.19 | 99.64 | 99.63 | 99.65 |
|  | $J=5, T=5$ | 89.92 | 90.18 | 90.31 | 97.48 | 97.49 | 97.52 |
|  | $J=5, T=10$ | 99.17 | 99.20 | 99.21 | 99.80 | 99.81 | 99.80 |
| $T P=$ Low, $I P=\mathrm{High}$ | $J=1, T=5$ | 73.75 | 73.97 | 75.06 | 84.66 | 85.14 | 85.38 |
|  | $J=1, T=10$ | 90.93 | 91.30 | 91.84 | 94.59 | 94.81 | 94.91 |
|  | $J=3, T=5$ | 76.57 | 78.02 | 78.84 | 93.20 | 93.34 | 93.38 |
|  | $J=3, T=10$ | 92.19 | 92.48 | 92.65 | 97.82 | 97.83 | 97.83 |
|  | $J=5, T=5$ | 80.92 | 81.82 | 82.35 | 96.22 | 96.34 | 96.33 |
|  | $J=5, T=10$ | 93.47 | 93.68 | 93.73 | 98.83 | 98.81 | 98.79 |
| $T P=$ High,$I P=$ High | $J=1, T=5$ | 89.70 | 91.12 | 92.07 | 93.02 | 93.19 | 93.35 |
|  | $J=1, T=10$ | 99.20 | 99.37 | 99.39 | 99.43 | 99.44 | 99.45 |
|  | $J=3, T=5$ | 91.18 | 91.80 | 92.18 | 96.61 | 96.68 | 96.70 |
|  | $J=3, T=10$ | 99.33 | 99.37 | 99.40 | 99.73 | 99.72 | 99.73 |
|  | $J=5, T=5$ | 91.96 | 92.50 | 92.67 | 98.14 | 98.12 | 98.14 |
|  | $J=5, T=10$ | 99.39 | 99.41 | 99.41 | 99.85 | 99.86 | 99.86 |
| Note. $: \leq 70 \%, \quad: 7$ | 0\%-79.99\%, | : $80 \%$ | 89.99\%, N | ne: $\geq 90 \%$. |  |  |  |

Table S11.
Relative estimation bias by sample size and measurement quality for the L-DCM measurement model parameters.

|  | $M Q=$ Low |  |  | $M Q=$ Medium |  |  | $M Q=$ High |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $N=200$ | $N=400$ | $N=1,000$ | $N=200$ | $N=400$ | $N=1,000$ | $N=200$ | $N=400$ | $N=1,000$ |
| $P\left(X_{1}=1 \mid \theta_{1}=N M\right)$ | $2.00 \%$ | $-1.25 \%$ | $-0.50 \%$ | $-0.80 \%$ | $3.60 \%$ | $2.40 \%$ | $7.00 \%$ | $1.00 \%$ | $0.00 \%$ |
| $P\left(X_{1}=1 \mid \theta_{1}=M\right)$ | $4.83 \%$ | $5.33 \%$ | $3.50 \%$ | $1.47 \%$ | $1.33 \%$ | $0.13 \%$ | $0.78 \%$ | $0.33 \%$ | $0.56 \%$ |
| $P\left(X_{5}=1 \mid \theta_{2}=N M\right)$ | $-5.25 \%$ | $-5.25 \%$ | $-0.75 \%$ | $1.60 \%$ | $0.00 \%$ | $1.60 \%$ | $11.00 \%$ | $3.00 \%$ | $2.00 \%$ |
| $P\left(X_{5}=1 \mid \theta_{2}=M\right)$ | $3.67 \%$ | $1.33 \%$ | $2.17 \%$ | $-0.80 \%$ | $0.13 \%$ | $-0.13 \%$ | $-0.44 \%$ | $-0.44 \%$ | $0.00 \%$ |
| $P\left(X_{11}=1 \mid \theta_{3}=N M\right)$ | $-2.50 \%$ | $-4.75 \%$ | $-3.25 \%$ | $2.00 \%$ | $0.80 \%$ | $0.00 \%$ | $5.00 \%$ | $0.00 \%$ | $4.00 \%$ |
| $P\left(X_{11}=1 \mid \theta_{3}=M\right)$ | $4.83 \%$ | $2.83 \%$ | $2.67 \%$ | $0.80 \%$ | $2.40 \%$ | $-0.80 \%$ | $0.00 \%$ | $0.56 \%$ | $-0.11 \%$ |
| $P\left(X_{16}=1 \mid \theta_{4}=N M\right)$ | $-3.00 \%$ | $-4.25 \%$ | $2.00 \%$ | $-1.60 \%$ | $-2.40 \%$ | $-0.40 \%$ | $-3.00 \%$ | $7.00 \%$ | $0.00 \%$ |
| $P\left(X_{16}=1 \mid \theta_{4}=M\right)$ | $3.67 \%$ | $5.17 \%$ | $0.00 \%$ | $-0.13 \%$ | $-0.40 \%$ | $0.00 \%$ | $0.00 \%$ | $-0.56 \%$ | $-0.33 \%$ |

