Online Supplement

Not Restricted to Selection Research:
Accounting for Indirect Range Restriction in Organizational Research

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Appendix A
Four Equivalent Alternative Formulations of the UVIRR Correction and Attenuation Formulas

The BVIRR correction equation given in the text is,

$$\rho_{TP_a} = \frac{\rho_{XY_i} u_X u_Y + \lambda\sqrt{|1-u_X^2||1-u_Y^2|}}{\sqrt{\rho_{XX_a}\rho_{YY_a}}}, \tag{A1a}$$

which can be re-expressed using true-score *u* ratios for one or both variables as:

$$\rho_{TP_a} = \frac{\rho_{XY_i} u_T u_P}{\sqrt{\rho_{XX_i}\rho_{YY_i}}} + \lambda\sqrt{|1-u_T^2||1-u_P^2|}, \tag{A1b}$$

$$\rho_{TP_a} = \left(\frac{\rho_{XY_i}}{\sqrt{\rho_{YY_i}}} u_X u_P + \lambda\sqrt{|1-u_X^2||1-u_P^2|}\right) / \sqrt{\rho_{XX_a}}, \tag{A1c}$$

and

$$\rho_{TP_a} = \left(\frac{\rho_{XY_i}}{\sqrt{\rho_{XX_i}}} u_T u_Y + \lambda\sqrt{|1-u_T^2||1-u_Y^2|}\right) / \sqrt{\rho_{YY_a}}, \tag{A1d}$$

where ρ_{XY_i} is the observed correlation between *X* and *Y*, ρ_{TP_a} is the corrected correlation between *T* and *P* (i.e., the constructs represented by *X* and *Y*, respectively), u_X and u_Y are the ratios of the observed standard deviations of *X* and *Y* to the population standard deviations of *X* and *Y*, respectively, u_T and u_P are the true-score *u* ratios of *X* and *Y*, respectively, ρ_{XX_a} and ρ_{YY_a} are applicant-group reliabilities for *X* and *Y*, respectively, and ρ_{XX_i} and ρ_{YY_i} are incumbent-group reliabilities for *X* and *Y*, respectively. All four of the above equations yield identical results, but we recommend Equation A1a because it is generally the easiest version to use.

If one has access to reliability estimates from applicant samples (i.e., ρ_{XX_a} or ρ_{YY_a}), those values can be converted to incumbent reliabilities using the formulas

$$\rho_{XX_i} = 1 - \frac{1-\rho_{XX_a}}{u_X^2} \tag{A2}$$

and

$$\rho_{YY_i} = 1 - \frac{1-\rho_{YY_a}}{u_Y^2}, \tag{A3}$$

(Schmidt & Hunter, 2015, p. 127). The true-score *u* ratios required in Equations A1b, A1c, and A1d can be computed as

$$u_T = \sqrt{\frac{u_X^2 - (1-\rho_{XX_a})}{\rho_{XX_a}}} = \sqrt{\frac{\rho_{XX_i} u_X^2}{1+\rho_{XX_i} u_X^2 - u_X^2}} \tag{A4}$$

for the true-score *u* ratio of *X* and

$$u_P = \sqrt{\frac{u_Y^2 - (1-\rho_{YY_a})}{\rho_{YY_a}}} = \sqrt{\frac{\rho_{YY_i} u_Y^2}{1+\rho_{YY_i} u_Y^2 - u_Y^2}}, \tag{A5}$$

for the true-score *u* ratio of *Y* (Le et al., 2016, p. 983; Schmidt & Hunter, 2015, p. 127).

The attenuation formulas that correspond to A1a, A1b, A1c, and A1d are, respectively:

$$\rho_{XY_i} = \frac{\rho_{TP_a}\sqrt{\rho_{XX_a}\rho_{YY_a}} - \lambda\sqrt{|1-u_X^2||1-u_Y^2|}}{u_X u_Y}, \tag{A6a}$$

$$\rho_{XY_i} = \sqrt{\rho_{XX_i}\rho_{YY_i}}\left(\frac{\rho_{TP_a} - \lambda\sqrt{|1-u_T^2||1-u_P^2|}}{u_T u_P}\right), \tag{A6b}$$

$$\rho_{XY_i} = \sqrt{\rho_{YY_i}}\left(\frac{\rho_{TP_a}\sqrt{\rho_{XX_a}} - \lambda\sqrt{|1-u_X^2||1-u_P^2|}}{u_X u_P}\right), \tag{A6c}$$

and

$$\rho_{XY_i} = \sqrt{\rho_{XX_i}}\left(\frac{\rho_{TP_a}\sqrt{\rho_{YY_a}} - \lambda\sqrt{|1-u_T^2||1-u_Y^2|}}{u_T u_Y}\right). \tag{A6d}$$

The equations for computing the λ coefficients for Equations A1a, A1b, A1c, and A1d (and A1a, A6b, A6c, and A6d, respectively) are, respectively:

$$\lambda = \operatorname{sign}\left[\rho_{ST_a}\rho_{SP_a}(1-u_X)(1-u_Y)\right]\frac{\operatorname{sign}(1-u_X)\min\left(u_X,\frac{1}{u_X}\right) + \operatorname{sign}(1-u_Y)\min\left(u_Y,\frac{1}{u_Y}\right)}{\min\left(u_X,\frac{1}{u_X}\right) + \min\left(u_Y,\frac{1}{u_Y}\right)}, \tag{A7a}$$

$$\lambda = \operatorname{sign}\left[\rho_{ST_a}\rho_{SP_a}(1-u_T)(1-u_P)\right]\frac{\operatorname{sign}(1-u_T)\min\left(u_T,\frac{1}{u_T}\right) + \operatorname{sign}(1-u_P)\min\left(u_P,\frac{1}{u_P}\right)}{\min\left(u_T,\frac{1}{u_T}\right) + \min\left(u_P,\frac{1}{u_P}\right)}, \tag{A7b}$$

$$\lambda = \operatorname{sign}\left[\rho_{ST_a}\rho_{SP_a}(1-u_X)(1-u_P)\right]\frac{\operatorname{sign}(1-u_X)\min\left(u_X,\frac{1}{u_X}\right) + \operatorname{sign}(1-u_P)\min\left(u_P,\frac{1}{u_P}\right)}{\min\left(u_X,\frac{1}{u_X}\right) + \min\left(u_Y,\frac{1}{u_P}\right)}, \tag{A7c}$$

and

$$\lambda = \operatorname{sign}\left[\rho_{ST_a}\rho_{SP_a}(1-u_T)(1-u_Y)\right]\frac{\operatorname{sign}(1-u_T)\min\left(u_T,\frac{1}{u_T}\right) + \operatorname{sign}(1-u_Y)\min\left(u_Y,\frac{1}{u_Y}\right)}{\min\left(u_T,\frac{1}{u_T}\right) + \min\left(u_Y,\frac{1}{u_Y}\right)}. \tag{A7d}$$

The equations for computing the V coefficients associated with Equations A1a, A1b, A1c, and A1d are, respectively:

$$V = \frac{u_X u_Y}{\sqrt{\rho_{XX_a}\rho_{YY_a}}}, \tag{A7a}$$

$$V = \frac{u_T u_P}{\sqrt{\rho_{XX_i}\rho_{YY_i}}}, \tag{A7b}$$

$$V = \frac{u_T u_Y}{\sqrt{\rho_{XX_i}\rho_{YY_a}}}, \tag{A7c}$$

and

$$V = \frac{u_X u_P}{\sqrt{\rho_{XX_a}\rho_{YY_i}}}. \tag{A7d}$$

Appendix B
Conceptual Derivation of the λ Formula

A Two-Stage Correction Procedure

In exploring methods for using the BVIRR procedure when *u* ratios fall on either side of 1, we devised a method by which the BVIRR correction could be used to make a mixed-variation correction in a two-stage procedure. Regardless of how large or small the *u* ratios in a correction are, the choice of some arbitrarily large population standard deviation to use in making a correction can turn any application of the BVIRR procedure into a correction for range restriction; similarly, the choice of an arbitrarily small population standard deviation can turn any application of BVIRR into a correction for range enhancement. When applied in succession, corrections involving large and small hypothetical population standard deviations can successfully assuage issues related to mismatched *u* ratios without breaking any of the "rules" we have outlined for how to use the BVIRR correction. We describe this procedure below to illustrate the intuition of the approach and then, in the subsequent section, we present a shortcut that we have derived to compute a λ value between −1 and +1 that obviates the need for a two-stage correction. We emphasize that scenarios in which one must utilize a λ that is less than 1 in absolute value are not typical and that this procedure for handling mixed patterns of *u* ratios is our solution to a problem that otherwise appears intractable; it is a bivariate approximation of a correction that should ideally be computed using the multivariate range-correction procedure and is therefore only a convenience for use in settings where such a correction is infeasible.

To account for range variation with *u* ratios that fall on either side of 1, first correct the observed correlation toward arbitrarily large hypothetical population standard deviations. We recommend defining a hypothetical population standard deviation by adding some infinitesimally small value (e.g., 10^{-10}) to each of one's *u* ratios; if the resulting *u* ratio is smaller than 1, set the hypothetical *u* ratio to 1 plus the chosen small value (e.g., $1 + 10^{-10}$). We denote these new hypothetical values for u_X and u_Y as u'_X and u'_Y. Next, compute another set of hypothetical *u* ratios by dividing each observed *u* ratio by its *u*-prime counterpart, such that $u''_X = u_X \,/\, u'_X$ and $u''_Y = u_Y \,/\, u'_Y$. The *u*-prime values are all at least trivially larger than 1 and the *u*-double-prime values are all at least trivially smaller than 1, which means that λ values can be chosen unambiguously for the u'_X and u'_Y set and for the u''_X and u''_Y set.

With hypothetical *u* ratios in hand, correct r_{XY_i} for range restriction by using u''_X and u''_Y in Equation 1 with λ set to 1 and without making a correction for measurement error. Then pass the result of that operation through Equation 1 yet again to correct for range enhancement, this time using u'_X and u'_Y, setting λ to −1, and correcting the result for measurement error using applicant reliabilities. This sequence of operations accounts for the mixed pattern of range restriction and range enhancement without having to select a λ value different from 1 in absolute value. The two-stage procedure makes full use of all range-variation information and results in a corrected correlation that gives greater weight to the type of range variation (i.e., restriction or enhancement) that is most impactful in the given data. Next, we present a procedure for obtaining the same results as the two-stage correction that is much simpler to use and that allows one to choose an appropriate λ value to use in Equations 1 and 2.

A General Procedure for Computing a λ Coefficient

We suggest a general formula for estimating λ that (a) reduces to the +1/−1 rules described above when both *u* ratios fall on the same side of 1, (b) converges with the two-stage correction procedure described in the previous section, and (c) makes the BVIRR correction tolerant of *u* ratios that fall on either side of 1 for substantive reasons or simply due to sampling error. Our generalized λ formula was intended to "tilt" the BVIRR correction toward being either a correction for range restriction or a correction for range enhancement, according to which type of range variation has the larger effect.

Our procedure for computing a λ coefficient is,

$$\lambda = \text{sign}\left[\rho_{ST_a}\rho_{SP_a}(1-u_X)(1-u_Y)\right]\frac{\text{sign}(1-u_X)\min\left(u_X,\frac{1}{u_X}\right)+\text{sign}(1-u_Y)\min\left(u_Y,\frac{1}{u_Y}\right)}{\min\left(u_X,\frac{1}{u_X}\right)+\min\left(u_Y,\frac{1}{u_Y}\right)}. \quad \text{(B1)}$$

The result of Equation B1 ranges from −1 to +1, with λ values smaller than 1 in absolute value indicating the extent to which the BVIRR correction must make a compromise between increasing the magnitude of an observed correlation and decreasing the magnitude of the correlation. In deriving our λ formula, we began with the assumption that the magnitude of the effect of a variable's range variation on a correlation is proportional to the extent to which the variable's *u* ratio deviates from 1. That is, we assumed that the impacts of two variables' range variation of their correlation can be compared by computing $\min\left(u_X, {}^{1}/_{u_X}\right)$ and $\min\left(u_Y, {}^{1}/_{u_Y}\right)$, where the largest proportional difference between u_X and u_Y from 1 signals which *u* ratio has the strongest effect on the correlation between *X* and *Y*. Next, because our goal was to determine which *u* ratio in a set should play a more prominent role in a correction, we added $\min\left(u_X, {}^{1}/_{u_X}\right)$ and $\min\left(u_Y, {}^{1}/_{u_Y}\right)$ together, giving each value the sign of the difference between 1 and the corresponding *u* ratio before computing the sum (this process determines the emphasis given to a range-restriction correction versus a range-enhancement correction). We then divided the previously described result by the sum of $\min\left(u_X, {}^{1}/_{u_X}\right)$ and $\min\left(u_Y, {}^{1}/_{u_Y}\right)$ to standardize to result in terms of the total relative effect of both variables' range variation. Finally, we determined the final sign of the λ coefficient by multiplying the previously computed result by the sign of the product of ρ_{ST_a}, ρ_{SP_a}, $(1-u_X)$, and $(1-u_Y)$.

Appendix C
Standard Errors of *u* Ratios and Reliability Indices

Assuming one has observed (not estimated) values for $q_{X_a} = \sqrt{\rho_{XX_a}}$ and $q_{Y_a} = \sqrt{\rho_{YY_a}}$, the individual sampling error variance formulas of correlations and all artifacts are as follows:

$$SE^2_{\rho_{XY_i}} = \frac{\left(1 - \rho^2_{XY_i}\right)^2}{N_i - 1} \tag{C1a}$$

$$SE^2_{q_{X_a}} = \frac{\left(1 - q^2_{X_a}\right)^2}{N_a - 1} \tag{C1b}$$

$$SE^2_{q_{Y_a}} = \frac{\left(1 - q^2_{Y_a}\right)^2}{N_a - 1} \tag{C1c}$$

$$SE^2_{u_X} \approx .5\, u^2_X \left[\frac{1}{N_i - 1} + \frac{1}{N_a - 1}\right] \tag{C1d}$$

$$SE^2_{u_Y} \approx .5\, u^2_Y \left[\frac{1}{N_i - 1} + \frac{1}{N_a - 1}\right] \tag{C1e}$$

Where N_i is the restricted-group sample size and N_a is the sample size for the unrestricted referent sample. If one is computing an artifact-distribution meta-analysis with BVIRR, we strongly recommend using the above estimates of artifacts' standard errors to residualize one's artifact distributions and remove the artifact variance that is likely attributable to sampling error. Simply use the mean value for the artifact and the mean sample size of studies that provide the artifact to estimate the sampling variance, subtract that error variance from the observed variance of artifacts, and use the residualized artifact variances to compute the Taylor series approximation of $SD_{\rho_{TP_a}}$, as described in Appendix E.

These standard errors can also be computed for individual studies using the mean artifact values and the sample-specific sample sizes as inputs to the error-variance equations; the resulting error variances can be used as inputs to the Taylor series approximation of $SE_{\rho_{TP_a}}$, described in Appendix D. Use of the mean artifact value (rather than study-specific estimates) in estimating the sampling error allows for more stable sampling error estimates, just as using the mean correlation produces better estimates of sampling error in meta-analyses of correlations.

If q_{X_a} or q_{Y_a} are not observed values, but rather are estimated from q_{X_i} or q_{Y_i} and u_X or u_Y, the following procedure can be used to estimate the sampling error of the applicant reliability estimates.

If q_{X_a} or q_{Y_a} are computed from q_{X_i} or q_{Y_i}, respectively, then the following standard errors should be used:

$$SE^2_{q_{X_i}} = \frac{\left(1 - q^2_{X_i}\right)^2}{N_i - 1} \tag{C2a}$$

$$SE^2_{q_{Y_i}} = \frac{\left(1 - q^2_{Y_i}\right)^2}{N_i - 1} \tag{C2b}$$

$$\begin{aligned} SE^2_{q_{X_a}} &\approx \left[\frac{u_X\left(1 - q^2_{X_i}\right)}{\sqrt{1 - u^2_X\left(1 - q^2_{X_i}\right)}}\right]^2 SE^2_{u_X} + \left[\frac{u_X q_{X_i}}{\sqrt{1 - u^2_X\left(1 - q^2_{X_i}\right)}}\right]^2 SE^2_{q_{X_i}} \\ &= .5u^4_X\left[\frac{\left(1 - q^2_{X_i}\right)^2}{1 - u^2_X\left(1 - q^2_{X_i}\right)}\right]\left[\frac{1}{N_i - 1} + \frac{1}{N_a - 1}\right] + \frac{u^2_X q^2_{X_i}\left(1 - q^2_{X_i}\right)^2}{\left[1 - u^2_X\left(1 - q^2_{X_i}\right)\right](N_i - 1)} \end{aligned} \tag{C2c}$$

$$\begin{aligned} SE^2_{q_{Y_a}} &\approx \left[\frac{u_Y\left(1 - q^2_{Y_i}\right)}{\sqrt{1 - u^2_Y\left(1 - q^2_{Y_i}\right)}}\right]^2 SE^2_{u_Y} + \left[\frac{u_Y q_{Y_i}}{\sqrt{1 - u^2_Y\left(1 - q^2_{Y_i}\right)}}\right]^2 SE^2_{q_{Y_i}} \\ &= .5\, u^4_Y\left[\frac{\left(1 - q^2_{Y_i}\right)^2}{1 - u^2_Y\left(1 - q^2_{Y_i}\right)}\right]\left[\frac{1}{N_i - 1} + \frac{1}{N_a - 1}\right] + \frac{u^2_Y q^2_{Y_i}\left(1 - q^2_{Y_i}\right)^2}{\left[1 - u^2_Y\left(1 - q^2_{Y_i}\right)\right](N_i - 1)} \end{aligned} \tag{C2d}$$

Note that q_{X_i} and u_X are likely to be correlated. If this this the case, then the equations for $SE^2_{q_{X_{a\,j}}}$ in Equation C2c will underestimate the sampling error. In that case, a better approximation would account for this correlation:

$$SE^2_{q_{X_a}} \approx \left[\frac{u_X(1-q^2_{X_i})}{\sqrt{1-u^2_X(1-q^2_{X_i})}}\right]^2 SE^2_{u_X} + \left[\frac{u_X q_{X_i}}{\sqrt{1-u^2_X(1-q^2_{X_i})}}\right]^2 SE^2_{q_{X_i}} \tag{C2e}$$

$$+ 2\left[\frac{u_X(1-q^2_{X_i})}{\sqrt{1-u^2_X(1-q^2_{X_i})}}\right]\left[\frac{u_X q_{X_i}}{\sqrt{1-u^2_X(1-q^2_{X_i})}}\right] SE_{u_X} SE_{q_{X_i}} \rho_{u_X q_{X_i}}$$

$$= .5\, u^4_X \left[\frac{(1-q^2_{X_i})^2}{1-u^2_X(1-q^2_{X_i})}\right]\left[\frac{1}{N_i-1}+\frac{1}{N_a-1}\right] + \frac{u^2_X q^2_{X_i}(1-q^2_{X_i})^2}{[1-u^2_X(1-q^2_{X_i})](N_i-1)}$$

$$+ 2\left[\frac{u^2_X q_{X_i}(1-q^2_{X_i})^2}{1-u^2_X(1-q^2_{X_i})}\right]\sqrt{\frac{\frac{1}{N_i-1}+\frac{1}{N_a-1}}{N_i+N_a-1}}\,\rho_{u_X q_{X_i}}$$

Where $\rho_{u_X q_{X_i}}$ is a value that must be derived analytical based on the type of reliability estimate one is using. Equation C2e also applies for $SE^2_{q_{Y_a}}$ by using u_Y instead of u_X and using q_{Y_i} instead of q_{X_i}.

Appendix D
Taylor Series Approximation of Sampling Variance ρ_{TP_a} Estimates Computed Using the Bivariate Indirect Range Restriction (BVIRR) Correction

Beginning with the disattenuation formula for BVIRR range restriction given in the text:

$$\rho_{TP_a} = \left[\rho_{XY_i} u_X u_Y + \lambda\sqrt{|1-u_X^2||1-u_Y^2|}\right] / \left(q_{X_a} q_{Y_a}\right) \tag{D1}$$

Per the principles of propagation of uncertainty and assuming $q_{X_a}, q_{Y_a}, u_X, u_Y$, and ρ_{TP_a}, are independent, we can approximate:

$$SE^2_{\rho_{TP_a}} \approx \beta_1^2 SE^2_{q_{X_a}} + \beta_2^2 SE^2_{q_{Y_a}} + \beta_3^2 SE^2_{u_X} + \beta_4^2 SE^2_{u_Y} + \beta_5^2 SE^2_{\rho_{XY_i}} \tag{D2}$$

where, $\beta_1 \ldots \beta_5$ are the first order partial derivatives of ρ_{TP_a} with respect to $q_{X_a}, q_{Y_a}, u_X, u_Y$, and ρ_{XY_i}, respectively.

These partial derivatives are as follows:

$$\beta_1 = \frac{\partial \rho_{TP_a}}{\partial q_{X_a}} = -\frac{\rho_{XY_i} u_X u_Y + \lambda\sqrt{|1-u_X^2||1-u_Y^2|}}{q_{X_a}^2 q_{Y_a}} \tag{D3a}$$

$$\beta_2 = \frac{\partial \rho_{TP_a}}{\partial q_{X_a}} = -\frac{\rho_{XY_i} u_X u_Y + \lambda\sqrt{|1-u_X^2||1-u_Y^2|}}{q_{X_a} q_{Y_a}^2} \tag{D3b}$$

$$\beta_3 = \frac{\partial \rho_{TP_a}}{\partial u_X} = \left[\rho_{XY_i} u_Y - \frac{\lambda u_X (1-u_X^2)\sqrt{|1-u_Y^2|}}{|1-u_X^2|^{1.5}}\right] / \left(q_{X_a} q_{Y_a}\right) \tag{D3c}$$

$$\beta_4 = \frac{\partial \rho_{TP_a}}{\partial u_Y} = \left[\rho_{XY_i} u_X - \frac{\lambda u_Y (1-u_Y^2)\sqrt{|1-u_X^2|}}{|1-u_Y^2|^{1.5}}\right] / \left(q_{X_a} q_{Y_a}\right) \tag{D3d}$$

$$\beta_5 = \frac{\partial \rho_{TP_a}}{\partial \rho_{XY_i}} = \frac{u_X u_Y}{q_{X_a} q_{Y_a}} \tag{D3e}$$

For equations to estimate the sampling-error variances of artifacts to use in D2, see Appendix C.

Appendix E

Taylor Series Approximation of the Random-Effects Variance of ρ_{TP_a} Using the Bivariate Indirect Range Restriction (BVIRR) Correction

Under BVIRR range restriction, the attenuation formula yielding ρ_{XY_i} (solved from Equation 1 in the manuscript) is:

$$\rho_{XY_i} = \left(\rho_{TP_a} q_{X_a} q_{Y_a} - \lambda\sqrt{|1-u_X^2||1-u_Y^2|}\right)/(u_X u_Y) \quad \text{(E1)}$$

where $q_{X_a} = \sqrt{\rho_{XX_a}}$ and $q_{Y_a} = \sqrt{\rho_{YY_a}}$ to simplify notation, which can be linearly approximated to:

$$\rho_{XY_i} \approx \bar{\rho}_{XY_i} + b_1 d_{q_{X_a}} + b_2 d_{q_{Y_a}} + b_3 d_{u_X} + b_4 d_{u_Y} + b_5 d_{\rho_{TP_a}} \quad \text{(E2)}$$

where *d* indicates deviation scores for each variable.

Via a first-order Taylor Series Approximation (delta method):

$$var_{\rho_{XY_i}} \approx b_1^2 var_{q_{X_a}} + b_2^2 var_{q_{Y_a}} + b_3^2 var_{u_X} + b_4^2 var_{u_Y} + b_5^2 var_{\rho_{TP_a}} \quad \text{(E3)}$$

where, b_1 ... b_5 are the first order partial derivatives of ρ_{XY_i} with respect to $q_{X_a}, q_{Y_a}, u_X, u_Y$, and ρ_{TP_a}, respectively.

These partial derivatives are as follows:

$$b_1 = \frac{\partial \rho_{XY_i}}{\partial q_{X_a}} = \frac{\rho_{TP_a} q_{Y_a}}{u_X u_Y} \quad \text{(E4)}$$

$$b_2 = \frac{\partial \rho_{xy_i}}{\partial q_{Y_a}} = \frac{\rho_{TP_a} q_{X_a}}{u_X u_Y}$$

$$b_3 = \frac{\partial \rho_{XY_i}}{\partial u_X} = \frac{\lambda(1-u_X^2)\sqrt{|1-u_Y^2|}}{u_y|1-u_X^2|^{1.5}} - \frac{\rho_{TP_a} q_{X_a} q_{Y_a} - \lambda\sqrt{|1-u_X^2||1-u_Y^2|}}{u_X^2 u_Y}$$

$$b_4 = \frac{\partial \rho_{XY_i}}{\partial u_Y} = \frac{\lambda(1-u_Y^2)\sqrt{|1-u_X^2|}}{u_X|1-u_Y^2|^{1.5}} - \frac{\rho_{TP_a} q_{X_a} q_{Y_a} - \lambda\sqrt{|1-u_X^2||1-u_Y^2|}}{u_X u_Y^2}$$

$$b_5 = \frac{\partial \rho_{XY_i}}{\partial \rho_{TP_a}} = \frac{q_{X_a} q_{Y_a}}{u_X u_Y}$$

Note that if one or both of the mean *u* ratios equals 1, b_3 and/or b_4 will be undefined; in such a scenario, re-compute the partial derivative after adding a small value (e.g., .01) to the *u* ratio associated with the derivative to obtain a proper result.

We can rearrange Equation E3 to yield an approximation formula for $var_{\rho_{TP_a}}$:

$$var_{\rho_{TP_a}} \approx \left[var_{\rho_{XY_i}} - \left(b_1^2 var_{q_{X_a}} + b_2^2 var_{q_{Y_a}} + b_3^2 var_{u_X} + b_4^2 var_{u_Y}\right)\right]/b_5^2 \quad \text{(E5)}$$

We can expand Equation E5 to account for the fact that $var_{\rho_{XY_i}}$ is a value to be estimated from $var_{r_{XY_i}} - var_e$ (that is, $var_{\rho_{XY_i}}$ is the variance of observed correlations after subtracting out the sampling error of observed correlations):

$$var_{\rho_{TP_a}} \approx \left[var_{r_{XY_i}} - var_e - \left(b_1^2 var_{q_{X_a}} + b_2^2 var_{q_{Y_a}} + b_3^2 var_{u_X} + b_4^2 var_{u_Y}\right)\right]/b_5^2 \quad \text{(E6)}$$

For the most accurate estimate of $var_{\rho_{TP_a}}$, we strongly recommend residualizing the variance of one's distributions of q_{X_a}, q_{Y_a}, u_X, and u_Y values by subtracting out the variance in artifacts that would be predicted by sampling error only. Failure to residualize the variance of artifacts will lead to biased estimates of $var_{\rho_{TP_a}}$ with the BVIRR correction. See Appendix C for procedures to estimate the sampling variance of artifacts.

For a complete set of meta-analytic variance estimates, one can estimate the variance in correlations predicted from artifacts (var_{art}), the total variance of correlations predicted from artifacts and sampling error (var_{pre}), and the residual variance of observed correlations (var_{res}) as

$$var_{art} = b_1^2 var_{q_{X_a}} + b_2^2 var_{q_{Y_a}} + b_3^2 var_{u_X} + b_4^2 var_{u_Y} \,, \tag{E7}$$

$$var_{pre} = var_{art} + var_e \,, \tag{E8}$$

and

$$var_{res} = var_{r_{XY_i}} - var_{pre} \,. \tag{E9}$$

Appendix F
Tabled Results of Simulation Study by Condition and Meta-Analytic Method

Table F1

Means and Standard Deviations of $\bar{\rho}$ Estimates from Simulations Using Competing Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_{ρ}	Meta-analytic method					
			IC_A	IC_V	IC_{TSA}	AD (100%)	AD (50%)	(AD 20%)
10	.1	.00	.028 (.046)	.096 (.043)	.089 (.048)	.116 (.047)	.108 (.065)	.097 (.092)
		.05	.025 (.049)	.096 (.048)	.089 (.053)	.120 (.053)	.113 (.071)	.097 (.105)
		.10	.022 (.057)	.101 (.061)	.099 (.065)	.123 (.065)	.117 (.082)	.107 (.106)
		.15	.017 (.069)	.098 (.074)	.098 (.080)	.119 (.076)	.108 (.090)	.100 (.116)
		.20	.012 (.077)	.097 (.091)	.102 (.096)	.119 (.092)	.109 (.103)	.099 (.130)
	.3	.00	.256 (.081)	.296 (.041)	.292 (.046)	.318 (.042)	.310 (.062)	.292 (.088)
		.05	.258 (.083)	.299 (.043)	.296 (.048)	.318 (.046)	.307 (.065)	.301 (.087)
		.10	.246 (.107)	.298 (.056)	.297 (.062)	.318 (.058)	.310 (.073)	.302 (.097)
		.15	.230 (.125)	.298 (.069)	.298 (.073)	.319 (.070)	.312 (.084)	.303 (.104)
		.20	.212 (.154)	.299 (.091)	.305 (.095)	.321 (.092)	.312 (.103)	.304 (.120)
	.5	.00	.490 (.044)	.500 (.031)	.499 (.035)	.520 (.032)	.513 (.049)	.498 (.072)
		.05	.490 (.052)	.499 (.037)	.500 (.042)	.520 (.038)	.511 (.056)	.504 (.076)
		.10	.491 (.077)	.499 (.053)	.500 (.057)	.518 (.054)	.510 (.065)	.503 (.085)
		.15	.492 (.098)	.500 (.067)	.502 (.072)	.520 (.066)	.511 (.078)	.501 (.092)
		.20	.468 (.137)	.492 (.081)	.494 (.088)	.511 (.082)	.505 (.089)	.497 (.105)
20	.1	.00	.019 (.029)	.096 (.031)	.088 (.035)	.123 (.032)	.118 (.046)	.108 (.073)
		.05	.018 (.031)	.097 (.036)	.091 (.040)	.123 (.037)	.119 (.050)	.110 (.079)
		.10	.011 (.035)	.097 (.043)	.092 (.047)	.123 (.043)	.116 (.056)	.109 (.080)
		.15	.006 (.040)	.098 (.052)	.097 (.058)	.125 (.053)	.119 (.063)	.111 (.084)
		.20	.006 (.046)	.103 (.068)	.108 (.072)	.129 (.068)	.125 (.076)	.110 (.094)
	.3	.00	.238 (.083)	.296 (.027)	.292 (.031)	.321 (.028)	.316 (.042)	.308 (.067)
		.05	.242 (.085)	.298 (.029)	.294 (.034)	.324 (.030)	.320 (.043)	.309 (.064)
		.10	.222 (.103)	.298 (.040)	.295 (.044)	.325 (.041)	.322 (.048)	.312 (.070)
		.15	.198 (.121)	.300 (.052)	.300 (.056)	.326 (.053)	.323 (.061)	.314 (.078)
		.20	.165 (.128)	.299 (.064)	.302 (.068)	.325 (.064)	.321 (.071)	.311 (.086)
	.5	.00	.487 (.044)	.499 (.023)	.498 (.026)	.524 (.023)	.519 (.036)	.508 (.056)
		.05	.485 (.054)	.499 (.026)	.498 (.030)	.525 (.026)	.521 (.036)	.511 (.058)
		.10	.482 (.077)	.498 (.037)	.498 (.041)	.523 (.036)	.517 (.044)	.509 (.061)
		.15	.477 (.100)	.497 (.048)	.498 (.053)	.521 (.048)	.516 (.054)	.506 (.070)
		.20	.453 (.140)	.496 (.061)	.500 (.067)	.519 (.061)	.515 (.066)	.502 (.078)

(Table continues)

Table F1 (Continued)

Means and Standard Deviations of $\bar{\rho}$ Estimates from Simulations Using Competing Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_ρ	Meta-analytic method					
			IC_A	IC_V	IC_{TSA}	AD (100%)	AD (50%)	(AD 20%)
50	.1	.00	.009 (.016)	.097 (.020)	.088 (.023)	.126 (.020)	.123 (.028)	.116 (.048)
		.05	.008 (.016)	.096 (.022)	.088 (.025)	.125 (.023)	.123 (.030)	.117 (.047)
		.10	.004 (.015)	.097 (.027)	.091 (.030)	.126 (.028)	.124 (.035)	.116 (.052)
		.15	.001 (.016)	.097 (.033)	.096 (.036)	.126 (.034)	.124 (.039)	.121 (.053)
		.20	-.001 (.017)	.101 (.041)	.104 (.045)	.129 (.041)	.127 (.045)	.123 (.060)
	.3	.00	.224 (.081)	.297 (.017)	.292 (.020)	.325 (.016)	.324 (.023)	.318 (.040)
		.05	.214 (.090)	.298 (.020)	.294 (.022)	.326 (.019)	.324 (.025)	.318 (.043)
		.10	.182 (.101)	.297 (.026)	.294 (.028)	.326 (.026)	.325 (.031)	.317 (.045)
		.15	.144 (.106)	.300 (.033)	.299 (.036)	.328 (.032)	.326 (.036)	.320 (.051)
		.20	.107 (.098)	.298 (.041)	.300 (.044)	.325 (.041)	.324 (.045)	.317 (.056)
	.5	.00	.485 (.031)	.498 (.014)	.497 (.016)	.526 (.014)	.524 (.021)	.519 (.035)
		.05	.485 (.045)	.498 (.017)	.497 (.020)	.526 (.017)	.524 (.024)	.518 (.035)
		.10	.480 (.074)	.500 (.023)	.500 (.026)	.526 (.023)	.524 (.028)	.519 (.040)
		.15	.463 (.106)	.498 (.031)	.500 (.035)	.525 (.031)	.523 (.034)	.520 (.043)
		.20	.405 (.152)	.491 (.037)	.494 (.041)	.517 (.036)	.515 (.039)	.509 (.050)
100	.1	.00	.005 (.010)	.096 (.014)	.088 (.016)	.125 (.014)	.124 (.020)	.120 (.032)
		.05	.003 (.009)	.097 (.016)	.089 (.018)	.126 (.016)	.126 (.021)	.123 (.032)
		.10	.002 (.008)	.097 (.019)	.091 (.021)	.127 (.019)	.126 (.024)	.123 (.034)
		.15	.000 (.008)	.100 (.025)	.098 (.027)	.129 (.025)	.129 (.030)	.125 (.037)
		.20	-.001 (.009)	.101 (.030)	.104 (.032)	.130 (.030)	.129 (.032)	.126 (.043)
	.3	.00	.197 (.089)	.297 (.012)	.292 (.014)	.326 (.012)	.325 (.016)	.321 (.027)
		.05	.187 (.095)	.296 (.014)	.292 (.016)	.325 (.014)	.325 (.018)	.322 (.029)
		.10	.150 (.098)	.297 (.018)	.294 (.020)	.327 (.018)	.326 (.021)	.323 (.030)
		.15	.102 (.091)	.299 (.024)	.298 (.026)	.328 (.024)	.328 (.026)	.325 (.034)
		.20	.072 (.074)	.298 (.029)	.301 (.031)	.326 (.029)	.325 (.031)	.324 (.038)
	.5	.00	.482 (.045)	.499 (.010)	.498 (.012)	.527 (.010)	.526 (.014)	.524 (.023)
		.05	.479 (.057)	.499 (.012)	.498 (.014)	.527 (.012)	.526 (.016)	.523 (.023)
		.10	.470 (.084)	.498 (.017)	.498 (.019)	.526 (.017)	.526 (.020)	.524 (.027)
		.15	.436 (.126)	.497 (.022)	.498 (.025)	.525 (.022)	.524 (.024)	.522 (.030)
		.20	.362 (.166)	.492 (.028)	.496 (.030)	.519 (.027)	.518 (.029)	.515 (.036)

Note. Values not in parentheses are means of $\bar{\rho}$ estimates. Values not in parentheses are standard deviations of $\bar{\rho}$ estimates.

Table F2
Means and Standard Deviations of SD_ρ Estimates from Simulations in Which All Studies Report Artifact Information Using Competing Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_ρ	Meta-analytic method			
			IC_A	IC_V	IC_{TSA}	$AD_{TSA\ res}$
10	.1	.00	.032 (.039)	.099 (.046)	.055 (.055)	.032 (.043)
		.05	.042 (.044)	.110 (.047)	.068 (.057)	.043 (.051)
		.10	.065 (.056)	.139 (.050)	.102 (.064)	.076 (.063)
		.15	.091 (.069)	.169 (.058)	.138 (.073)	.116 (.072)
		.20	.121 (.087)	.212 (.066)	.185 (.079)	.164 (.079)
	.3	.00	.041 (.051)	.075 (.042)	.038 (.047)	.032 (.042)
		.05	.051 (.058)	.085 (.043)	.047 (.051)	.043 (.048)
		.10	.091 (.066)	.121 (.045)	.087 (.061)	.080 (.060)
		.15	.132 (.076)	.159 (.054)	.131 (.070)	.123 (.069)
		.20	.170 (.090)	.201 (.067)	.178 (.078)	.173 (.077)
	.5	.00	.027 (.042)	.054 (.039)	.021 (.037)	.026 (.036)
		.05	.043 (.052)	.071 (.041)	.036 (.046)	.043 (.044)
		.10	.081 (.062)	.107 (.043)	.074 (.059)	.082 (.055)
		.15	.131 (.070)	.149 (.051)	.124 (.066)	.129 (.061)
		.20	.177 (.082)	.190 (.062)	.170 (.076)	.168 (.071)
20	.1	.00	.031 (.030)	.103 (.031)	.058 (.045)	.028 (.037)
		.05	.038 (.032)	.113 (.030)	.070 (.048)	.044 (.043)
		.10	.056 (.043)	.141 (.034)	.110 (.047)	.082 (.051)
		.15	.076 (.055)	.176 (.039)	.152 (.049)	.133 (.052)
		.20	.102 (.069)	.217 (.048)	.197 (.055)	.180 (.055)
	.3	.00	.049 (.051)	.076 (.030)	.033 (.038)	.028 (.034)
		.05	.064 (.053)	.090 (.030)	.049 (.043)	.045 (.040)
		.10	.110 (.054)	.127 (.032)	.099 (.049)	.087 (.047)
		.15	.146 (.059)	.165 (.038)	.144 (.050)	.137 (.047)
		.20	.176 (.076)	.207 (.045)	.191 (.052)	.184 (.051)
	.5	.00	.027 (.038)	.058 (.028)	.020 (.030)	.025 (.030)
		.05	.050 (.050)	.077 (.027)	.037 (.038)	.044 (.036)
		.10	.096 (.053)	.115 (.030)	.086 (.047)	.091 (.042)
		.15	.145 (.057)	.154 (.036)	.134 (.048)	.138 (.042)
		.20	.195 (.066)	.196 (.043)	.182 (.053)	.180 (.048)

(Table Continues)

Table F2 (Continued)
Means and Standard Deviations of SD_ρ Estimates from Simulations in Which All Studies Report Artifact Information Using Competing Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_ρ	Meta-analytic method			
			IC_A	IC_V	IC_{TSA}	AD_{TSA_res}
50	.1	.00	.026 (.019)	.103 (.019)	.059 (.034)	.024 (.030)
		.05	.030 (.022)	.114 (.019)	.077 (.032)	.043 (.034)
		.10	.041 (.029)	.142 (.021)	.115 (.031)	.091 (.034)
		.15	.052 (.039)	.180 (.025)	.160 (.030)	.142 (.031)
		.20	.069 (.051)	.221 (.030)	.204 (.035)	.188 (.032)
	.3	.00	.062 (.049)	.080 (.017)	.033 (.031)	.026 (.028)
		.05	.081 (.047)	.094 (.018)	.054 (.033)	.046 (.032)
		.10	.121 (.043)	.128 (.020)	.104 (.030)	.095 (.030)
		.15	.148 (.056)	.170 (.024)	.154 (.030)	.146 (.029)
		.20	.159 (.075)	.211 (.028)	.199 (.033)	.192 (.031)
	.5	.00	.026 (.039)	.061 (.016)	.013 (.022)	.021 (.024)
		.05	.053 (.041)	.079 (.016)	.037 (.031)	.048 (.029)
		.10	.108 (.044)	.117 (.019)	.093 (.030)	.099 (.026)
		.15	.164 (.048)	.160 (.022)	.145 (.029)	.147 (.026)
		.20	.211 (.058)	.200 (.026)	.189 (.031)	.187 (.029)
100	.1	.00	.020 (.015)	.103 (.013)	.062 (.026)	.022 (.026)
		.05	.023 (.017)	.115 (.013)	.081 (.022)	.045 (.029)
		.10	.031 (.023)	.145 (.015)	.120 (.020)	.095 (.022)
		.15	.040 (.029)	.181 (.018)	.163 (.021)	.143 (.022)
		.20	.050 (.036)	.223 (.022)	.207 (.025)	.192 (.023)
	.3	.00	.078 (.046)	.080 (.012)	.033 (.026)	.022 (.024)
		.05	.092 (.042)	.095 (.012)	.057 (.026)	.048 (.026)
		.10	.123 (.043)	.128 (.014)	.106 (.020)	.099 (.020)
		.15	.133 (.062)	.170 (.017)	.155 (.020)	.147 (.020)
		.20	.138 (.074)	.214 (.021)	.201 (.024)	.195 (.022)
	.5	.00	.029 (.045)	.061 (.011)	.009 (.017)	.022 (.021)
		.05	.061 (.045)	.080 (.012)	.037 (.027)	.050 (.023)
		.10	.117 (.043)	.117 (.013)	.096 (.020)	.100 (.016)
		.15	.175 (.048)	.160 (.017)	.146 (.022)	.147 (.020)
		.20	.218 (.060)	.201 (.018)	.192 (.022)	.189 (.020)

Note. Values not in parentheses are means of SD_ρ estimates. Values not in parentheses are standard deviations of SD_ρ estimates.

Table F3

Means and Standard Deviations of SD_ρ Estimates from Simulations in Which 50% of Studies Report Artifact Information Using Competing Artifact-Distribution Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_ρ	Meta-analytic method			
			AD_{Int}	AD_{Int_res}	AD_{TSA}	AD_{TSA_res}
10	.1	.00	.017 (.035)	.040 (.046)	.013 (.033)	.034 (.045)
		.05	.028 (.045)	.052 (.053)	.023 (.042)	.047 (.052)
		.10	.058 (.063)	.086 (.063)	.047 (.062)	.078 (.065)
		.15	.103 (.076)	.130 (.068)	.085 (.079)	.120 (.073)
		.20	.152 (.084)	.174 (.075)	.130 (.093)	.165 (.082)
	.3	.00	.020 (.036)	.039 (.044)	.016 (.033)	.034 (.043)
		.05	.031 (.046)	.052 (.050)	.022 (.041)	.045 (.050)
		.10	.067 (.062)	.089 (.059)	.052 (.060)	.079 (.063)
		.15	.111 (.073)	.131 (.067)	.091 (.078)	.122 (.072)
		.20	.164 (.081)	.179 (.074)	.140 (.089)	.168 (.080)
	.5	.00	.017 (.033)	.033 (.039)	.013 (.029)	.028 (.037)
		.05	.032 (.042)	.051 (.046)	.024 (.038)	.044 (.045)
		.10	.070 (.057)	.088 (.054)	.056 (.058)	.080 (.057)
		.15	.121 (.064)	.136 (.059)	.099 (.072)	.126 (.065)
		.20	.163 (.073)	.175 (.067)	.143 (.084)	.167 (.073)
20	.1	.00	.009 (.024)	.036 (.040)	.007 (.023)	.032 (.040)
		.05	.018 (.033)	.051 (.045)	.011 (.028)	.045 (.045)
		.10	.051 (.051)	.089 (.049)	.036 (.049)	.080 (.053)
		.15	.105 (.063)	.138 (.051)	.083 (.069)	.129 (.058)
		.20	.159 (.064)	.183 (.053)	.139 (.074)	.177 (.059)
	.3	.00	.012 (.026)	.035 (.038)	.008 (.022)	.031 (.037)
		.05	.020 (.034)	.049 (.042)	.013 (.029)	.044 (.042)
		.10	.061 (.051)	.091 (.046)	.046 (.049)	.085 (.049)
		.15	.117 (.057)	.140 (.048)	.100 (.063)	.134 (.052)
		.20	.169 (.057)	.186 (.051)	.151 (.069)	.180 (.056)
	.5	.00	.009 (.022)	.028 (.032)	.006 (.018)	.025 (.031)
		.05	.023 (.034)	.047 (.038)	.016 (.030)	.042 (.038)
		.10	.070 (.048)	.093 (.042)	.057 (.049)	.087 (.045)
		.15	.124 (.049)	.141 (.042)	.109 (.057)	.135 (.047)
		.20	.170 (.052)	.182 (.048)	.155 (.065)	.177 (.054)

(Table continues)

Table F3 (Continued)

Means and Standard Deviations of SD_ρ Estimates from Simulations in Which 50% of Studies Report Artifact Information Using Competing Artifact-Distribution Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_ρ	Meta-analytic method			
			AD_{Int}	AD_{Int_res}	AD_{TSA}	AD_{TSA_res}
50	.1	.00	.002 (.011)	.029 (.032)	.001 (.008)	.027 (.032)
		.05	.008 (.020)	.046 (.036)	.003 (.013)	.044 (.036)
		.10	.045 (.043)	.092 (.037)	.032 (.041)	.089 (.039)
		.15	.110 (.044)	.143 (.031)	.096 (.049)	.140 (.033)
		.20	.167 (.040)	.189 (.034)	.157 (.046)	.187 (.035)
	.3	.00	.003 (.012)	.027 (.029)	.002 (.009)	.026 (.029)
		.05	.012 (.024)	.047 (.034)	.008 (.020)	.046 (.035)
		.10	.059 (.041)	.095 (.032)	.050 (.042)	.093 (.033)
		.15	.123 (.037)	.145 (.030)	.116 (.041)	.144 (.032)
		.20	.176 (.035)	.192 (.031)	.171 (.038)	.191 (.032)
	.5	.00	.003 (.012)	.022 (.025)	.002 (.009)	.022 (.025)
		.05	.015 (.025)	.046 (.030)	.014 (.024)	.046 (.031)
		.10	.073 (.036)	.098 (.027)	.068 (.038)	.098 (.027)
		.15	.131 (.032)	.146 (.027)	.128 (.035)	.146 (.028)
		.20	.176 (.032)	.187 (.030)	.172 (.035)	.186 (.030)
100	.1	.00	.000 (.004)	.024 (.028)	.000 (.002)	.024 (.028)
		.05	.003 (.010)	.045 (.031)	.001 (.007)	.045 (.031)
		.10	.041 (.036)	.094 (.026)	.031 (.035)	.093 (.027)
		.15	.111 (.033)	.143 (.024)	.104 (.036)	.142 (.025)
		.20	.171 (.027)	.192 (.023)	.166 (.028)	.192 (.024)
	.3	.00	.001 (.005)	.023 (.025)	.000 (.003)	.024 (.025)
		.05	.007 (.016)	.046 (.029)	.005 (.014)	.047 (.028)
		.10	.062 (.034)	.098 (.022)	.056 (.035)	.098 (.023)
		.15	.126 (.025)	.146 (.021)	.123 (.026)	.147 (.021)
		.20	.180 (.025)	.194 (.023)	.178 (.025)	.195 (.023)
	.5	.00	.001 (.006)	.019 (.022)	.001 (.007)	.022 (.023)
		.05	.011 (.020)	.045 (.026)	.012 (.020)	.049 (.025)
		.10	.074 (.025)	.098 (.018)	.075 (.025)	.100 (.017)
		.15	.131 (.023)	.146 (.021)	.132 (.023)	.147 (.020)
		.20	.178 (.022)	.188 (.021)	.177 (.022)	.189 (.020)

Note. Values not in parentheses are means of SD_ρ estimates. Values not in parentheses are standard deviations of SD_ρ estimates.

Table F4

Means and Standard Deviations of SD_{ρ} Estimates from Simulations in Which 20% of Studies Report Artifact Information Using Competing Artifact-Distribution Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_{ρ}	Meta-analytic method			
			AD_{Int}	AD_{Int_res}	AD_{TSA}	AD_{TSA_res}
10	.1	.00	.035 (.046)	.046 (.048)	.032 (.046)	.045 (.049)
		.05	.046 (.053)	.058 (.054)	.041 (.054)	.056 (.055)
		.10	.077 (.066)	.089 (.064)	.066 (.068)	.084 (.067)
		.15	.117 (.075)	.128 (.072)	.102 (.083)	.121 (.077)
		.20	.164 (.083)	.174 (.078)	.145 (.092)	.166 (.084)
	.3	.00	.037 (.046)	.045 (.048)	.031 (.046)	.043 (.049)
		.05	.044 (.051)	.055 (.052)	.037 (.049)	.052 (.052)
		.10	.080 (.064)	.090 (.061)	.067 (.065)	.085 (.063)
		.15	.119 (.074)	.129 (.070)	.103 (.080)	.123 (.074)
		.20	.171 (.081)	.178 (.078)	.146 (.096)	.165 (.089)
	.5	.00	.029 (.039)	.037 (.041)	.021 (.035)	.032 (.040)
		.05	.044 (.047)	.054 (.047)	.034 (.045)	.048 (.048)
		.10	.080 (.058)	.090 (.056)	.065 (.061)	.081 (.059)
		.15	.128 (.066)	.135 (.063)	.107 (.075)	.125 (.069)
		.20	.167 (.075)	.174 (.071)	.145 (.087)	.163 (.080)
20	.1	.00	.018 (.033)	.043 (.042)	.017 (.034)	.039 (.042)
		.05	.030 (.042)	.057 (.047)	.026 (.042)	.052 (.048)
		.10	.062 (.055)	.092 (.052)	.050 (.058)	.083 (.057)
		.15	.114 (.063)	.139 (.053)	.092 (.072)	.128 (.062)
		.20	.166 (.067)	.186 (.058)	.139 (.085)	.172 (.071)
	.3	.00	.019 (.033)	.039 (.040)	.016 (.032)	.036 (.040)
		.05	.034 (.041)	.058 (.043)	.026 (.040)	.052 (.045)
		.10	.071 (.053)	.094 (.048)	.055 (.056)	.085 (.054)
		.15	.121 (.059)	.140 (.050)	.097 (.070)	.129 (.060)
		.20	.171 (.063)	.186 (.056)	.147 (.080)	.175 (.067)
	.5	.00	.017 (.029)	.033 (.033)	.012 (.024)	.028 (.033)
		.05	.031 (.038)	.050 (.039)	.023 (.036)	.045 (.040)
		.10	.077 (.049)	.096 (.044)	.062 (.053)	.085 (.051)
		.15	.127 (.052)	.141 (.046)	.106 (.064)	.130 (.056)
		.20	.171 (.055)	.182 (.051)	.147 (.073)	.172 (.061)

(Table continues)

Table F4 (Continued)
Means and Standard Deviations of SD_ρ Estimates from Simulations in Which 20% of Studies Report Artifact Information Using Competing Artifact-Distribution Meta-Analytic Methods with the Correction for Bivariate Indirect Range Restriction

k	$\bar{\rho}$	SD_ρ	Meta-analytic method			
			AD_{Int}	AD_{Int_res}	AD_{TSA}	AD_{TSA_res}
50	.1	.00	.007 (.019)	.035 (.036)	.005 (.018)	.031 (.036)
		.05	.014 (.027)	.052 (.039)	.008 (.023)	.045 (.040)
		.10	.053 (.046)	.095 (.039)	.035 (.045)	.085 (.047)
		.15	.110 (.050)	.143 (.036)	.087 (.060)	.134 (.045)
		.20	.167 (.043)	.189 (.035)	.143 (.060)	.181 (.045)
	.3	.00	.008 (.019)	.033 (.032)	.005 (.017)	.029 (.032)
		.05	.020 (.029)	.052 (.036)	.011 (.024)	.047 (.038)
		.10	.066 (.043)	.098 (.035)	.049 (.045)	.090 (.041)
		.15	.126 (.042)	.147 (.034)	.106 (.054)	.141 (.040)
		.20	.178 (.037)	.193 (.033)	.160 (.055)	.186 (.041)
	.5	.00	.007 (.017)	.028 (.028)	.004 (.013)	.025 (.027)
		.05	.023 (.030)	.051 (.033)	.015 (.027)	.046 (.034)
		.10	.076 (.038)	.099 (.031)	.060 (.043)	.092 (.037)
		.15	.132 (.036)	.147 (.030)	.118 (.048)	.142 (.036)
		.20	.176 (.034)	.188 (.031)	.162 (.048)	.182 (.037)
100	.1	.00	.003 (.011)	.031 (.032)	.001 (.009)	.029 (.032)
		.05	.007 (.018)	.050 (.034)	.003 (.012)	.046 (.035)
		.10	.047 (.040)	.095 (.032)	.032 (.037)	.090 (.036)
		.15	.112 (.038)	.144 (.026)	.097 (.045)	.141 (.030)
		.20	.171 (.030)	.193 (.026)	.160 (.039)	.190 (.029)
	.3	.00	.003 (.010)	.029 (.028)	.001 (.007)	.027 (.028)
		.05	.012 (.023)	.050 (.032)	.007 (.018)	.046 (.033)
		.10	.063 (.037)	.098 (.026)	.050 (.039)	.094 (.030)
		.15	.126 (.030)	.147 (.023)	.116 (.038)	.145 (.025)
		.20	.179 (.028)	.195 (.025)	.173 (.033)	.193 (.027)
	.5	.00	.003 (.011)	.024 (.024)	.002 (.008)	.023 (.024)
		.05	.016 (.024)	.050 (.027)	.013 (.022)	.048 (.028)
		.10	.074 (.030)	.098 (.021)	.067 (.032)	.097 (.024)
		.15	.132 (.026)	.146 (.022)	.126 (.031)	.145 (.023)
		.20	.178 (.024)	.189 (.022)	.173 (.029)	.188 (.024)

Note. Values not in parentheses are means of SD_ρ estimates. Values not in parentheses are standard deviations of SD_ρ estimates.

Appendix G
Full Figures of Simulation Results

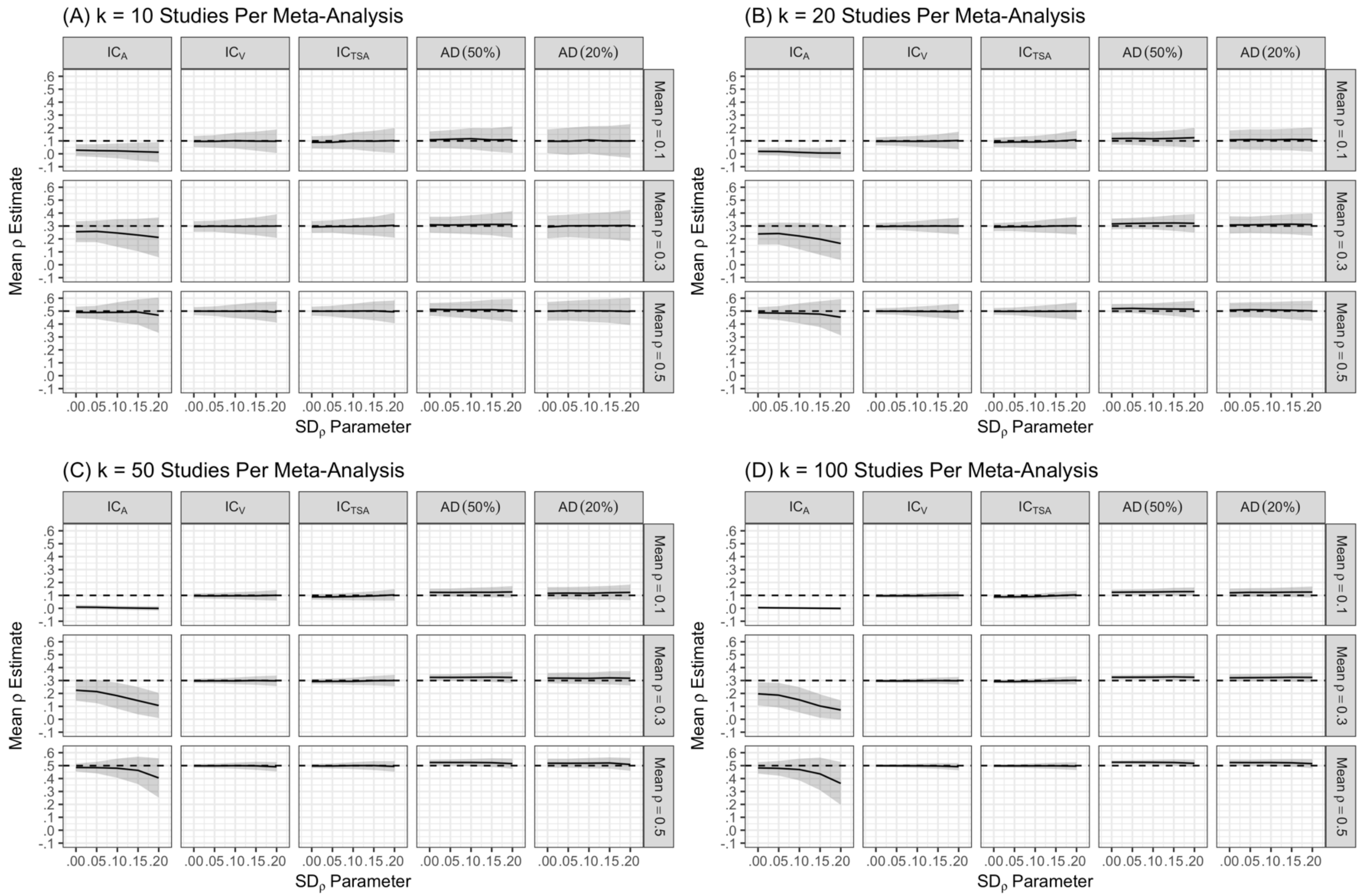

Figure G1. Mean ρ estimates by number of studies meta-analyzed and meta-analytic method. IC_A = traditional compound attenuation factor-based individual-correction method; IC_V = individual-correction method using equation-implied error variances and weights; IC_{TSA} = individual-correction method using Taylor series approximation to estimate corrected error variances and weights; AD (50%) = artifact distribution method with 50% of artifacts available; AD (20%) = artifact distribution method with 20% of artifacts available. AD results were identical across different AD computation methods, as interactive and Taylor series approximation methods both use mean artifacts to estimate mean ρ values. Dashed lines indicate true parameter values, solid lines indicate mean parameter estimates, and ribbons around solid lines indicate the parameter estimates values within 1 standard error of the mean parameter estimate (standard error was computed as the standard deviation of statistics from 1,000 replications).

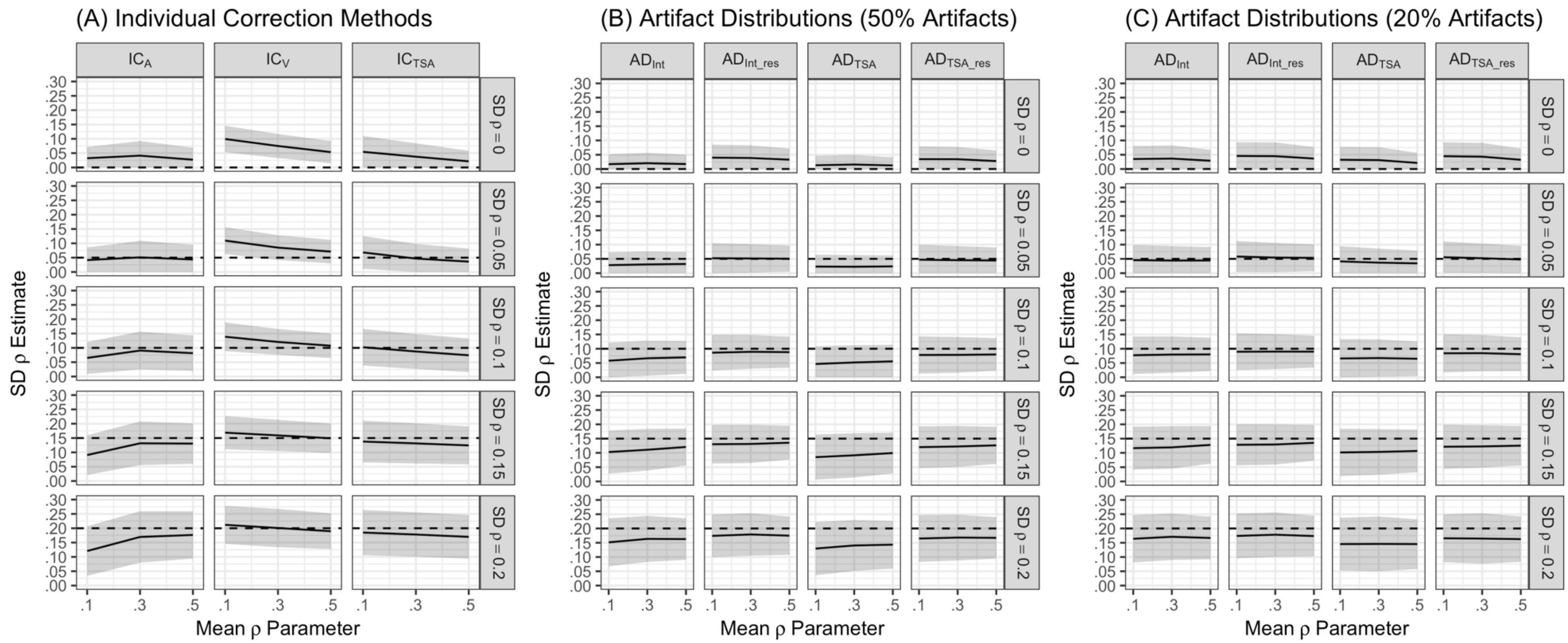

Figure G2. SD_ρ estimates by meta-analytic method for meta-analysis of 10 studies (k = 10). IC_A = traditional compound attenuation factor-based individual-correction method; IC_V = individual-correction method using equation-implied error variances and weights; IC_{TSA} = individual-correction method using Taylor series approximation to estimate corrected error variances and weights; AD_{Int} = interactive artifact distribution method; AD_{Int_res} = interactive artifact distribution method with shrunken artifact distributions; AD_{TSA} = Taylor series approximation artifact distribution method using observed artifact variances; AD_{TSA_res} = Taylor series approximation artifact distribution method using artifact variances residualized to remove the influence of predicted sampling error. Dashed lines indicate true parameter values, solid lines indicate mean parameter estimates, and ribbons around solid lines indicate the parameter estimates values within 1 standard error of the mean parameter estimate (standard error was computed as the standard deviation of statistics from 1,000 replications).

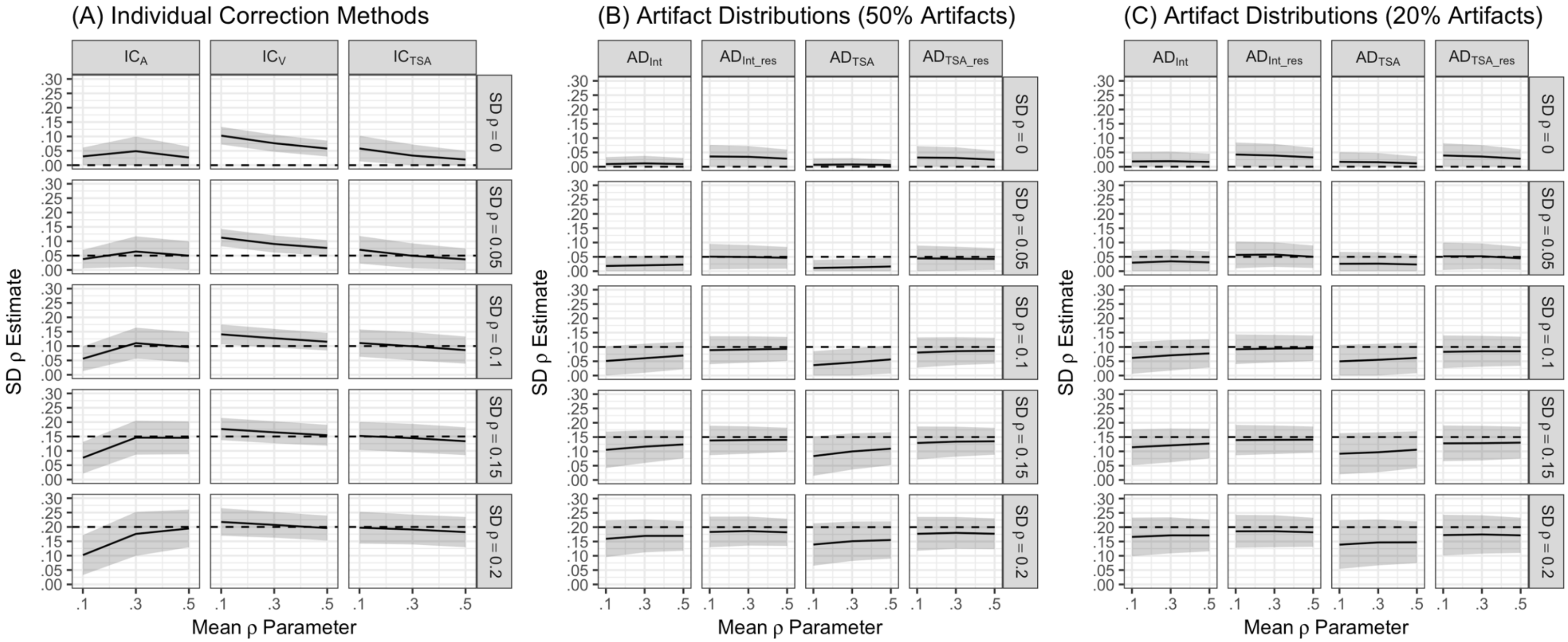

Figure G3. SD_{ρ} estimates by meta-analytic method for meta-analysis of 20 studies (k = 20). IC_A = traditional compound attenuation factor-based individual-correction method; IC_V = individual-correction method using equation-implied error variances and weights; IC_{TSA} = individual-correction method using Taylor series approximation to estimate corrected error variances and weights; AD_{Int} = interactive artifact distribution method; AD_{Int_res} = interactive artifact distribution method with shrunken artifact distributions; AD_{TSA} = Taylor series approximation artifact distribution method using observed artifact variances; AD_{TSA_res} = Taylor series approximation artifact distribution method using artifact variances residualized to remove the influence of predicted sampling error. Dashed lines indicate true parameter values, solid lines indicate mean parameter estimates, and ribbons around solid lines indicate the parameter estimates values within 1 standard error of the mean parameter estimate (standard error was computed as the standard deviation of statistics from 1,000 replications).

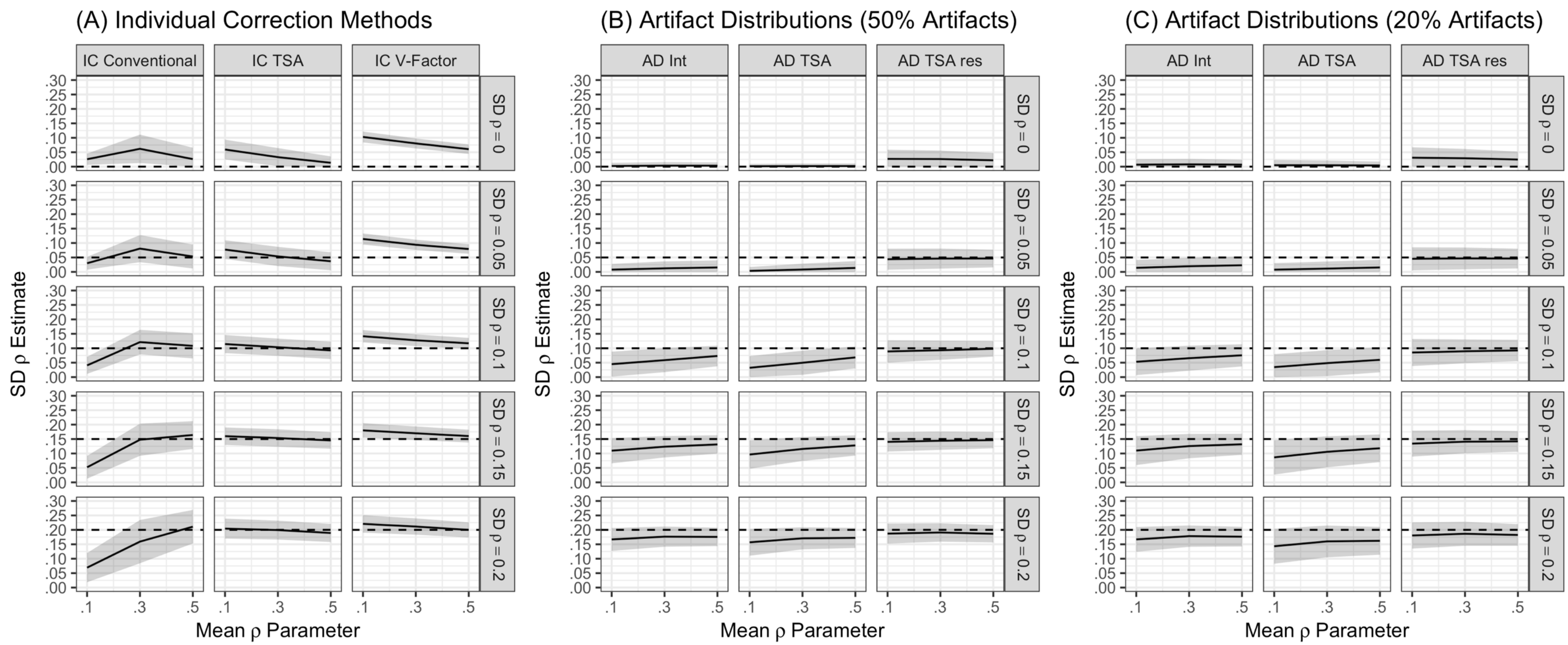

Figure G4. SD_ρ estimates by meta-analytic method for meta-analysis of 50 studies (k = 50). IC_A = traditional compound attenuation factor-based individual-correction method; IC_V = individual-correction method using equation-implied error variances and weights; IC_{TSA} = individual-correction method using Taylor series approximation to estimate corrected error variances and weights; AD_{Int} = interactive artifact distribution method; AD_{Int_res} = interactive artifact distribution method with shrunken artifact distributions; AD_{TSA} = Taylor series approximation artifact distribution method using observed artifact variances; AD_{TSA_res} = Taylor series approximation artifact distribution method using artifact variances residualized to remove the influence of predicted sampling error. Dashed lines indicate true parameter values, solid lines indicate mean parameter estimates, and ribbons around solid lines indicate the parameter estimates values within 1 standard error of the mean parameter estimate (standard error was computed as the standard deviation of statistics from 1,000 replications).

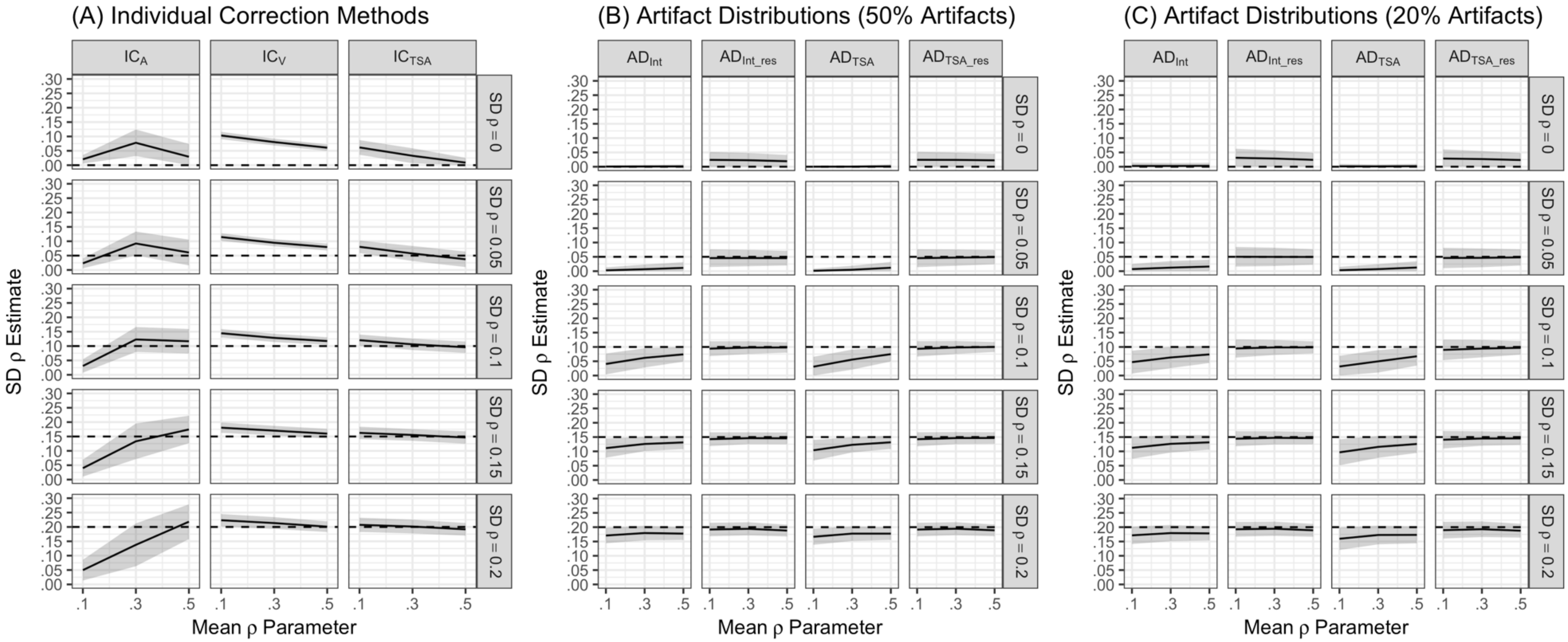

Figure G5. SD_ρ estimates by meta-analytic method for meta-analysis of 100 studies (k = 100). IC$_A$ = traditional compound attenuation factor-based individual-correction method; IC$_V$ = individual-correction method using equation-implied error variances and weights; IC$_{TSA}$ = individual-correction method using Taylor series approximation to estimate corrected error variances and weights; AD$_{Int}$ = interactive artifact distribution method; AD$_{Int_res}$ = interactive artifact distribution method with shrunken artifact distributions; AD$_{TSA}$ = Taylor series approximation artifact distribution method using observed artifact variances; AD$_{TSA_res}$ = Taylor series approximation artifact distribution method using artifact variances residualized to remove the influence of predicted sampling error. Dashed lines indicate true parameter values, solid lines indicate mean parameter estimates, and ribbons around solid lines indicate the parameter estimates values within 1 standard error of the mean parameter estimate (standard error was computed as the standard deviation of statistics from 1,000 replications).

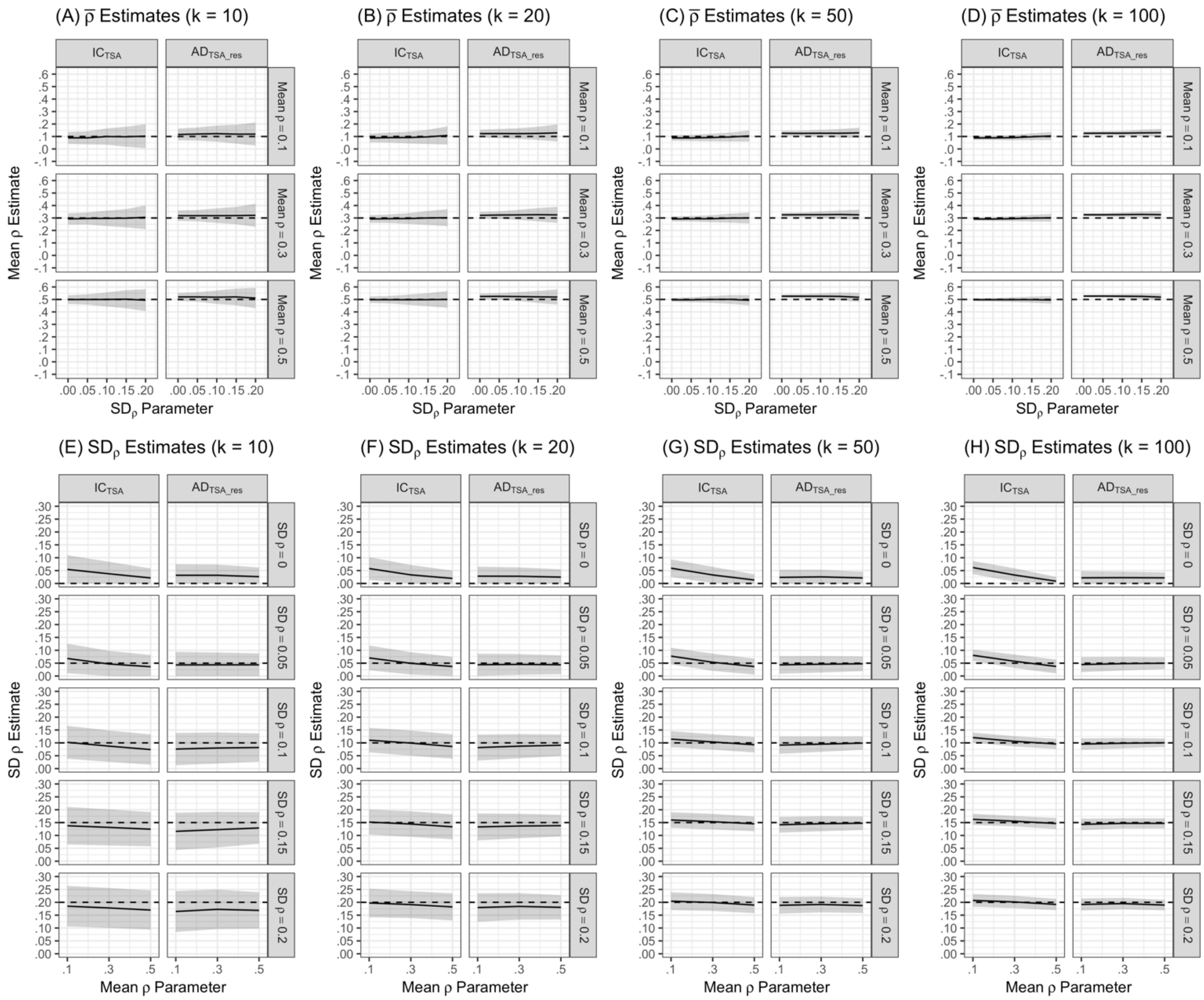

Figure G6. Mean ρ and SD_ρ estimates for IC_{TSA} (individual-correction method using Taylor series approximation to estimate corrected error variances and weights) and AD_{TSA_res} (Taylor series approximation artifact distribution method using artifact variances residualized to remove the influence of predicted sampling error; using 100% of artifact information) methods by number of studies meta-analyzed.

Appendix H
R Functions for BVIRR Corrections, Error Variance Estimates, and Taylor Series Approximation Artifact-Distribution Meta-Analysis Random-Effects Variance Estimates

R code for simplified versions of key functions for key BVIRR methods are presented in this section. More comprehensive and robust functions to implement BVIRR methods are available in the *psychmeta* package for *R* (Dahlke & Wiernik, 2018, 2017/2019).

The correction (or attenuation) for BVIRR can be applied using the `correct_r_bvirr` function defined below. The arguments to this function are:

- `rxyi`: Observed range-restricted sample correlation between *X* and *Y*. If `attenuate` is set to `TRUE`, supply the unrestricted true-score correlation as `rxyi`.
- `qxi`: Square root of observed range-restricted reliability for *X*.
- `qyi`: Square root of observed range-restricted reliability for *Y*.
- `ux`: Ratio of sample standard deviation to unrestricted standard deviation for *X*.
- `uy`: Ratio of sample standard deviation to unrestricted standard deviation for *Y*.
- `sign_rxz`: Sign of the unrestricted correlation between *X* and the selection mechanism.
- `sign_ryz`: Sign of the unrestricted correlation between *Y* and the selection mechanism.
- `attenuate`: Logical scalar that determines whether a correction is performed (`FALSE`; default) or attenuation is performed (`TRUE`).
- `vare_from_means`: Logical scalar that determines whether sampling variances are estimated using mean sample statistics (`TRUE`; default) or individual sample statistics (`FALSE`).

```
correct_r_bvirr<-function(rxyi,
                          ux=1, uy=1, qxi=1, qyi=1,
                          sign_rxz=1, sign_ryz=1, attenuate=FALSE){
     ux_prime<-ux
     uy_prime<-uy
     ux_prime[ux>1/ux]<-1/ux[ux>1/ux]
     uy_prime[uy>1/uy]<-1/uy[uy>1/uy]
     sign_x<-sign(ux-1)
     sign_y<-sign(uy-1)
     sign_x<-sign(1-ux)
     sign_y<-sign(1-uy)
     lambda<-sign_x*sign_y*sign(sign_rxz*sign_ryz)*
          (sign_x*ux_prime+sign_y*uy_prime)/(ux_prime+uy_prime)
     qxa<-(1-ux^2*(1-qxi^2))^.5
     qya<-(1-uy^2*(1-qyi^2))^.5
     if(attenuate){
          (rxyi*qxa*qya-lambda*sqrt(abs(1-ux^2)*abs(1-uy^2)))/(ux*uy)
     }else{
          (rxyi*ux*uy+lambda*sqrt(abs(1-ux^2)*abs(1-uy^2)))/(qxa*qya)
     }
}
```

The sampling variance of a correlation corrected for BVIRR can be estimated using the `var_error_r_bvirr` function defined below. The arguments to the `var_error_r_bvirr` are the same as the `correct_r_bvirr` function, but with two additional arguments:

- `dependent_sds_x`: Logical scalar or vector determining whether supplied `ux` values were computed using dependent samples (`TRUE`; i.e., the restricted sample is a subset of the unrestricted sample) or independent samples (`FALSE`; i.e., the restricted sample and the unrestricted sample are observed in separate studies; default).
- `dependent_sds_y`: Same as `dependent_sds_x`, but for `uy` values.

```
var_error_r_bvirr<-function(rxyi, ni, na=NA,
                            ux=1, dependent_sds_x=FALSE,
                            uy=1, dependent_sds_y=FALSE,
                            qxi=1, qyi=1,
                            sign_rxz=1, sign_ryz=1,
                            vare_from_means=TRUE){
     ux_prime<-ux
     uy_prime<-uy
     ux_prime[ux>1/ux]<-1/ux[ux>1/ux]
     uy_prime[uy>1/uy]<-1/uy[uy>1/uy]
     sign_x<-sign(ux-1)
     sign_y<-sign(uy-1)
     sign_x<-sign(1-ux)
     sign_y<-sign(1-uy)
     lambda<-sign_x*sign_y*sign(sign_rxz*sign_ryz)*
          (sign_x*ux_prime+sign_y*uy_prime)/(ux_prime+uy_prime)
     if(vare_from_means){
          weighted_mean<-function(x,wt=rep(1,length(x))){
               x[is.na(wt)]<-NA
               wt[is.na(x)]<-NA
               sum(as.numeric(x*wt),na.rm=TRUE)/
                    sum(as.numeric(wt),na.rm=TRUE)
          }
          mean_rxyi<-weighted_mean(x=rxyi,wt=ni)
          mean_ux<-weighted_mean(x=ux,wt=ni)
          mean_uy<-weighted_mean(x=uy,wt=ni)
          mean_qxi<-weighted_mean(x=qxi,wt=ni)
          mean_qyi<-weighted_mean(x=qxi,wt=ni)
     }else{
          mean_rxyi<-rxyi
          mean_ux<-ux
          mean_uy<-uy
          mean_qxi<-qxi
          mean_qyi<-qyi
     }
     n_term_x<-n_term_y<-1/(ni-1)
     sign_xa<-sign_ya<-rep(1,length(n_term_x))
     sign_xa[dependent_sds_x]<-sign_ya[dependent_sds_y]<--1
     n_term_x[!is.na(na)]<-n_term_x[!is.na(na)]+sign_xa*1/(na[!is.na(na)]-1)
     n_term_y[!is.na(na)]<-n_term_y[!is.na(na)]+sign_ya*1/(na[!is.na(na)]-1)
     var_e<-(1-mean_rxyi^2)^2/(ni-1)
     var_e_ux<-0.5*mean_ux^2*n_term_x
     var_e_uy<-0.5*mean_uy^2*n_term_y
```

```
    var_e_qxi<-(1-mean_qxi^2)^2/(ni-1)
    var_e_qyi<-(1-mean_qyi^2)^2/(ni-1)
    var_e_qxa<-(mean_qxi*ux^2)^2/((mean_qxi^2-1)*ux^2+1)*var_e_qxi
    var_e_qya<-(mean_qyi*uy^2)^2/((mean_qyi^2-1)*uy^2+1)*var_e_qyi
    qxa<-(1-ux^2*(1-qxi^2))^.5
    qya<-(1-uy^2*(1-qyi^2))^.5

    rtpa<-(rxyi*ux*uy+lambda*sqrt(abs(1-ux^2)*abs(1-uy^2)))/(qxa*qya)
    b_rxyi<-(ux*uy)/(qxa*qya)
    b_qxa<--rtpa/qxa
    b_qya<--rtpa/qya
    b_ux<-(rxyi*uy-(lambda*ux*(1-ux^2)*sqrt(abs(1-uy^2)))/abs(1-ux^2)^1.5)/(qxa*qya)
    b_uy<-(rxyi*ux-(lambda*uy*(1-uy^2)*sqrt(abs(1-ux^2)))/abs(1-uy^2)^1.5)/(qxa*qya)
    as.numeric(b_qxa^2*var_e_qxa+b_qya^2*var_e_qya+
                    b_ux^2*var_e_ux+b_uy^2*var_e_uy+b_rxyi^2*var_e)
}
```

The random-effects variance of correlations corrected for BVIRR can be estimated using the `estimate_var_rho_tsa_bvirr` function defined below. The arguments to the `estimate_var_rho_tsa_bvirr` function are:

- `mean_rtpa`: Optional mean unrestricted true-score correlation between *X* and *Y*.
- `mean_rxyi`: Mean observed range-restricted sample correlation between *X* and *Y*.
- `var_rxyi`: Variance of observed range-restricted sample correlations between *X* and *Y*.
- `mean_ni`: Mean sample size.
- `mean_ux`: Mean ratio of sample standard deviation to unrestricted standard deviation for *X*.
- `var_ux`: Variance of ratios of sample standard deviations to unrestricted standard deviations for *X*.
- `mean_uy`: Mean ratio of sample standard deviation to unrestricted standard deviation for *Y*.
- `var_uy`: Variance of ratios of sample standard deviations to unrestricted standard deviations for *Y*.
- `mean_qxi`: Mean square root of observed range-restricted reliability for *X*.
- `var_qxi`: Variance of square roots of observed range-restricted reliabilities for *X*.
- `mean_qyi`: Mean square root of observed range-restricted reliability for *Y*.
- `var_qyi`: Variance of square roots of observed range-restricted reliabilities for *Y*.
- `sign_rxz`: Sign of the unrestricted correlation between *X* and the selection mechanism.
- `sign_ryz`: Sign of the unrestricted correlation between *Y* and the selection mechanism.

```
estimate_var_rho_tsa_bvirr <- function(mean_rtpa=NULL, mean_rxyi, var_rxyi=0,
mean_ni,
                                    mean_ux=1, var_ux=0,
                                    mean_uy=1, var_uy=0,
                                    mean_qxi=1, var_qxi=0,
                                    mean_qyi=1, var_qyi=0,
                                    sign_rxz=1, sign_ryz=1,
                                    residualize_ads=TRUE){
     ux_prime<-mean_ux
     uy_prime<-mean_uy
     ux_prime[mean_ux>1/mean_ux]<-1/mean_ux[mean_ux>1/mean_ux]
     uy_prime[mean_uy>1/mean_uy]<-1/mean_uy[mean_uy>1/mean_uy]
     sign_x<-sign(mean_ux-1)
     sign_y<-sign(mean_uy-1)
     sign_x<-sign(1-mean_ux)
     sign_y<-sign(1-mean_uy)
     lambda<-sign_x*sign_y*sign(sign_rxz*sign_ryz)*
          (sign_x*ux_prime+sign_y*uy_prime)/(ux_prime+uy_prime)

     if(residualize_ads){
          var_ux<-var_ux-0.5*mean_ux^2*1/(mean_ni-1)
          var_uy<-var_uy-0.5*mean_uy^2*1/(mean_ni-1)
          var_qxi<-var_qxi-(1-mean_qxi^2)^2/(mean_ni-1)
          var_qyi<-var_qyi-(1-mean_qyi^2)^2/(mean_ni-1)
     }
     var_ux[is.na(var_ux)]<-var_uy[is.na(var_uy)]<-
```

```
          var_qxi[is.na(var_qxi)]<-var_qyi[is.na(var_qyi)]<-0
     var_ux[var_ux<0]<-var_uy[var_uy<0]<-
          var_qxi[var_qxi<0]<-var_qyi[var_qyi<0]<-0
     mean_qxa<-(1-mean_ux^2*(1-mean_qxi^2))^.5
     mean_qya<-(1-mean_uy^2*(1-mean_qyi^2))^.5
     var_qxa<-(mean_qxi*mean_ux^2)^2/(mean_qxi^2*mean_ux^2-mean_ux^2+1)*var_qxi
     var_qya<-(mean_qyi*mean_uy^2)^2/(mean_qyi^2*mean_uy^2-mean_uy^2+1)*var_qyi

     if(is.null(mean_rtpa))
          mean_rtpa<-(mean_rxyi*mean_ux*mean_uy+
                        lambda*sqrt(abs(1-mean_ux^2)*
                                    abs(1-mean_uy^2)))/(mean_qxa*mean_qya)
     var_e<-(1-mean_rxyi^2)^2/(mean_ni-1)

     b_qxa<-mean_rtpa*mean_qya/(mean_ux*mean_uy)
     b_qya<-mean_rtpa*mean_qxa/(mean_ux*mean_uy)
     b_ux<-(lambda*(1-mean_ux^2)*sqrt(abs(1-mean_uy^2)))/
          (mean_uy*abs(1-mean_ux^2)^1.5)-mean_rxyi/mean_ux
     b_uy<-(lambda*(1-mean_uy^2)*sqrt(abs(1-mean_ux^2)))/
          (mean_ux*abs(1-mean_uy^2)^1.5)-mean_rxyi/mean_uy
     b_rtpa<-(mean_qxa*mean_qya)/(mean_ux*mean_uy)

     var_art<-b_qxa^2*var_qxa+b_qya^2*var_qya+b_ux^2*var_ux+b_uy^2*var_uy
     var_pre<-var_e+var_art
     var_res<-var_rxyi-var_pre
     var_rho<-var_res/b_rtpa^2
     data.frame(var_art=as.numeric(var_art),
                var_pre=as.numeric(var_pre),
                var_res=as.numeric(var_res),
                var_rho=as.numeric(var_rho))
}
```