WEB APPENDIX

PROOF OF LEMMA 1. Suppose that the referral fee is r and all sellers' retail prices are $p^*(r)$. In equilibrium, seller i will not change its retail price p_i away from $p^*(r)$. Seller i's profit is

$$\pi_i^S(p_i;r) = \frac{F_0(\bar{m} + \mu_K - p^*(r))}{n(1 - F(\bar{m}))} \cdot [1 - F(\bar{m} - p^*(r) + p_i)] \cdot [(1 - r)p_i - c].$$

The first-order condition (FOC) is $\frac{d\pi_i^S(p_i;r)}{dp_i} = \frac{F_0(\bar{m}+\mu_K-p^*(r))}{n(1-F(\bar{m}))} \cdot \{[1 - F(\bar{m} - p^*(r) + p_i)](1 - r) - f(\bar{m} - p^*(r) + p_i) \cdot [(1 - r)p_i - c]\} = 0$, i.e., $p_i = \frac{c}{1-r} + h(\bar{m} - p^*(r) + p_i)$. In a symmetric equilibrium, $p_i = p^*(r)$, therefore, $p^*(r) = \frac{c}{1-r} + h(\bar{m})$. We also need to check the second-order condition, i.e., $\frac{d^2\pi_i^S(p_i;r)}{dp_i^2}|_{p_i=p^*(r)} < 0$. This is equivalent to $-2f(\bar{m}) - f'(\bar{m})h(\bar{m}) < 0$. This is true because $h'(\bar{m}) < 0$ implies $-\frac{f(\bar{m})+h(\bar{m})f'(\bar{m})}{f(\bar{m})} < 0$, which implies $-2f(\bar{m}) - f'(\bar{m})h(\bar{m}) < 0$.

Substituting $p^*(r) = \frac{c}{1-r} + h(\bar{m})$ into expressions of $\tilde{\pi}_i^S$ and $\widetilde{D}$, we have $\tilde{\pi}_i^{S*} = \frac{F_0\left(\bar{m}+\mu_K-\frac{c}{1-r}-h(\bar{m})\right)}{n}(1 - r)h(\bar{m})$ and $\widetilde{D} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r} - h(\bar{m})\right)$. ■

PROOF OF LEMMA 2. These results are straightforward because $\frac{\partial\tilde{p}^*}{\partial\tau} = h'(\bar{m}) \cdot \frac{\partial\bar{m}}{\partial\tau} > 0$ and $\frac{\partial\widetilde{D}}{\partial\tau} = f_0\left(\bar{m} + q - \frac{c}{1-r} - h(\bar{m})\right)\left(1 - h'(\bar{m})\right) \cdot \frac{\partial\bar{m}}{\partial\tau} < 0$. ■

PROOF OF PROPOSITION 1. Since $\frac{\partial\bar{m}}{\partial\tau} < 0$, it is sufficient to show $\frac{d\pi^{P*}(\bar{m})}{d\bar{m}} > 0$. $\pi^{P*}(\bar{m}) = \pi^P\left(r^*(\bar{m})\right) = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) \cdot r^*(\bar{m}) \cdot \left(\frac{c}{1-r^*(\bar{m})} + h(\bar{m})\right)$. Because $r^*(\bar{m})$ maximizes the platform's profit π^P, it must follow the FOC that $\frac{\partial\pi^{P*}}{\partial r}|_{r=r^*} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r} - h(\bar{m})\right)\left[\frac{c}{(1-r)^2} + h(\bar{m})\right] - \frac{rc}{(1-r)^2}\left(\frac{c}{1-r} + h(\bar{m})\right)f_0\left(\bar{m} + \mu_K - \frac{c}{1-r} - h(\bar{m})\right) = 0$.

When τ decreases, $\bar{m}$ increases. By the envelope theorem, $\frac{d\pi^{P*}(\bar{m})}{d\bar{m}} = \frac{\partial \pi^P}{\partial \bar{m}}|_{r=r^*(\bar{m})} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) r^*(\bar{m}) h'(\bar{m}) + f_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right)(1 - h'(\bar{m})) \cdot r^*(\bar{m}) \cdot \left(\frac{c}{1-r^*(\bar{m})} + h(\bar{m})\right) = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) r^*(\bar{m}) h'(\bar{m}) + F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right)(1 - h'(\bar{m})) \cdot \frac{\frac{c}{(1-r^*(\bar{m}))^2} + h(\bar{m})}{\frac{c}{(1-r^*(\bar{m}))^2}} = F_0\left(\bar{m} + \mu_K - \frac{c}{1-r^*(\bar{m})} - h(\bar{m})\right) \cdot \left[\left(1 + \frac{h(\bar{m})(1-r^*(\bar{m}))^2}{c}\right) - h'(\bar{m})\left(1 - r^*(\bar{m}) + \frac{h(\bar{m})(1-r^*(\bar{m}))^2}{c}\right)\right] > 0$.

■

PROOF OF PROPOSITION 2. The platform's profit is $\pi^P = D(r, \bar{m}) \cdot (\frac{cr}{1-r} + rh(\bar{m}))$. The FOC is $\frac{\partial \pi^{P*}}{\partial r}|_{r=r^*(\bar{m})} = 0$ for all $\bar{m}$, thus $0 = \frac{d\frac{\partial \pi^P}{\partial r}|_{r=r^*(\bar{m})}}{d\bar{m}} = \frac{\partial^2 \pi^P}{\partial r^2}|_{r=r^*(\bar{m})} \cdot \frac{\partial r^*(\bar{m})}{\partial \bar{m}} + \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})}$. Because the second-order condition guarantees $\frac{\partial^2 \pi^P|_{r=r^*(\bar{m})}}{\partial r^2} < 0$, we know $\frac{\partial r^*(\bar{m})}{\partial \bar{m}} > 0$ if and only if $\frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})} > 0$. Moreover, $\frac{d\frac{\partial ln\pi^P}{\partial lnr}}{d\bar{m}} = \frac{d\left(\frac{\partial \pi^P}{\partial r} \cdot \frac{r}{\pi^P}\right)}{d\bar{m}} = \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}} \cdot \frac{r}{\pi^P} + \frac{\partial \pi^P}{\partial r} \cdot \frac{-r \cdot \frac{\partial \pi^P}{\partial \bar{m}}}{(\pi^P)^2}$. Thus $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} = \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})} \cdot \frac{r^*(\bar{m})}{\pi^P(r^*(\bar{m}))} + 0 = \frac{\partial^2 \pi^P}{\partial r \partial \bar{m}}|_{r=r^*(\bar{m})} \cdot \frac{r^*(\bar{m})}{\pi^P(r^*(\bar{m}))}$. Hence, $\frac{\partial r^*(\bar{m})}{\partial \bar{m}} > 0$ if and only if $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} > 0$.

Note that $\frac{\partial ln\pi^P}{\partial lnr} = \epsilon_{D,r}(r, \bar{m}) + \frac{\partial\left(\frac{cr}{1-r} + rh(\bar{m})\right)}{\partial r} \cdot \frac{r}{\frac{cr}{1-r} + rh(\bar{m})} = \epsilon_{D,r}(r, \bar{m}) + \frac{c + h(\bar{m})(1-r)^2}{c + h(\bar{m})(1-r)}$. Thus, $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} > 0$ if and only if $\frac{\partial \epsilon_{D,r}(r^*, \bar{m})}{\partial \bar{m}} > \frac{ch'(\bar{m}) r^*(\bar{m})}{(c + h(\bar{m})(1-r^*(\bar{m})))^2}$. Because $\epsilon_{D,r}(r^*, \bar{m}) < 0$, $\frac{d\frac{\partial ln\pi^P}{\partial lnr}|_{r=r^*(\bar{m})}}{d\bar{m}} > 0$ if and only if $\frac{\partial |\epsilon_{D,r}(r^*, \bar{m})|}{\partial \bar{m}} < \frac{-ch'(\bar{m}) r^*(\bar{m})}{(c + h(\bar{m})(1-r^*(\bar{m})))^2}$. ■

PROOF OF LEMMA 3. Let $G(x) = \int_x^{m_{\max}} (m - x) f(m) dm$. Note that $G(x)$ is strictly decreasing and convex since $G'(x) = F(x) - 1 < 0$ and $G''(x) = f(x) > 0$. When filtering is not available, the consumer's acceptance aggregate match level threshold, $\bar{M}_N$, satisfies $\int_{\bar{M}_N}^{M_{\max}}(M-\bar{M}_N)f_M(M)dM=\tau$. Note that $f(x)=0$ when $x>m_{\max}$, so the left-hand-side can be written as:

$\tau=\int_{\bar{M}_N}^{m_{\max}+\mu_K}(M-\bar{M}_N)f_M(M)dM=\sum_{k=1}^{K}\phi_k\int_{\bar{M}_N-\mu_k}^{m_{\max}+(\mu_K-\mu_k)}[m-(\bar{M}_N-\mu_k)]f(m)dm=\sum_{k=1}^{K}\phi_k\int_{\bar{M}_N-\mu_k}^{m_{\max}}[m-(\bar{M}_N-\mu_k)]f(m)dm=\sum_{k=1}^{K}\phi_k G(\bar{M}_N-\mu_k)$.

Because $G''(x)>0$ and $\sum_{k=1}^{K}\phi_k\mu_k=0$, $G(\bar{M}_N)=G(\sum_{k=1}^{K}\phi_k(\bar{M}_N-\mu_k))<\sum_{k=1}^{K}\phi_k G(\bar{M}_N-\mu_k)=\tau$.

When filtering is available, consumers will search products with $m_{ij}=\mu_K$ and will buy a product if and only if $m_{ij}>\bar{m}$, where $\bar{m}$ is determined by $G(\bar{m})=\int_{\bar{m}}^{m_{\max}}(m-\bar{m})f(m)dm=\tau>G(\bar{M}_N)$. Because $G(x)$ is a strictly decreasing function, $\bar{M}_N>\bar{m}$. Thus, $\bar{M}-\bar{M}_N=\mu_K+\bar{m}-\bar{M}_N<\mu_K$. Moreover, $G(\bar{m})=\tau=\sum_{k=1}^{K}\phi_k G(\bar{M}_N-\mu_k)<\sum_{k=1}^{K}\phi_k G(\bar{M}_N-\mu_K)=G(\bar{M}_N-\mu_K)$, so $\bar{M}_N-\mu_K<\bar{m}$, i.e., $\bar{M}-\bar{M}_N>0$. ■

PROOF OF PROPOSITION 3. It is sufficient to show $F(\bar{m})<F_M(\bar{M}_N)=\mathbf{E}_\mu[F(\bar{M}_N-\mu)]$.

First, note that $G(x)=\int_x^{m_{\max}}\big(1-F(m)\big)dm$, so $\tau=\int_{\bar{m}}^{m_{\max}}\big(1-F(m)\big)dm=\mathbf{E}_\mu[\int_{\bar{M}_N-\mu}^{m_{\max}}\big(1-F(m)\big)dm]$. This implies $\mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\big(1-F(m)\big)dm\right]=0$.

Second, because $h(x)$ is a decreasing function, we know $(h(m)-h(\bar{m}))\cdot f(m)>0$ when $m<\bar{m}$, and $(h(m)-h(\bar{m}))\cdot f(m)<0$ when $m>\bar{m}$. Hence, $\int_x^{\bar{m}}\big(h(m)-h(\bar{m})\big)\cdot f(m)dm>0,\forall x\neq\bar{m}$.

This implies

$$\begin{aligned}
0&<\mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\big(h(m)-h(\bar{m})\big)\cdot f(m)dm\right]\\
&=\mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\big(h(m)f(m)-h(\bar{m})f(m)\big)dm\right]\\
&=\mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\Big(\big(1-F(m)\big)-h(\bar{m})f(m)\Big)dm\right]\\
&=\mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}\big(1-F(m)\big)dm\right]-h(\bar{m})\mathbf{E}_\mu\left[\int_{\bar{M}_N-\mu}^{\bar{m}}f(m)dm\right]\\
&=0-h(\bar{m})\mathbf{E}_\mu[F(\bar{m})-F(\bar{M}_N-\mu)].
\end{aligned}$$

Hence, $\mathbf{E}_\mu[F(\bar{m})-F(\bar{M}_N-\mu)]<0$, i.e., $F(\bar{m})<\mathbf{E}_\mu[F(\bar{M}_N-\mu)]=F_M(\bar{M}_N)$. ■

LEMMA A1. Suppose that $F(m)$ and its up-to-third derivatives are bounded and that $f(m) > C$ on an interval (m_L, m_H), where $C > 0$. If $\delta \to 0^+$, then $h'(m) < 0$ on (m_L, m_H) if and only if $h_M(m) < 0$ on (m_L, m_H).

PROOF: Let $\sigma_{\hat{\mu}}^2 = \mathbf{Var}(\hat{\mu})$, so $\mathbf{Var}(\mu) = \delta^2\sigma_{\hat{\mu}}^2$.

Let $g(m)$ be a twice-continuously-differentiable function on (m_L, m_H). $\mathbf{E}_\mu[g(m - \mu)] = \mathbf{E}_\mu\left[g(m) + g'(m)\mu + \frac{g''(m)}{2}\mu^2 + o(\mu^2)\right] = g(m) + \frac{g''(m)}{2}\delta^2\sigma_{\hat{\mu}}^2 + o(\delta^2)$.

Note that $h'(m) = -\frac{f'(m)(1-F(m))}{f^2(m)} - 1$, and $h_M'(m) = -\frac{\mathbf{E}_\mu[f'(m-\mu)]\mathbf{E}_\mu[1-F(m-\mu)]}{(\mathbf{E}_\mu[f(m-\mu)])^2} - 1 = -\frac{f'(m)(1-F(m)) + \frac{f'''(m)(1-F(m)) - f'^2(m)}{2}\delta^2\sigma_{\hat{\mu}}^2 + o(\delta^2)}{f^2(m) + f''(m)f(m)\delta^2\sigma_{\hat{\mu}}^2 + o(\delta^2)} - 1 = h'(m) + \frac{f'''(m)(1-F(m)) - 2f(m)f''(m) - f'^2(m)}{2f^2(m)}\delta^2\sigma_{\hat{\mu}}^2 + o(\delta^2)$. Hence, when $\delta \to 0^+$, $h'(m) < 0$ if and only if $h_M(m) < 0$. ■

PROOF OF PROPOSITION 4. Consider the marginal impact of filtering on the sellers' equilibrium price.

First, we determine the relationship between $\bar{M}_N$ and $\bar{m}$ using second-order Taylor's expansion. Observe that

$$\tau = G(\bar{m}) = \mathbf{E}_\mu G(\bar{M}_N - \mu) = \mathbf{E}_\mu G\big(\bar{m} + (\bar{M}_N - \bar{m} - \mu)\big)$$

$$= \mathbf{E}_\mu\left[G(\bar{m}) + G'(\bar{m})(\bar{M}_N - \bar{m} - \mu) + \frac{G''(\bar{m})}{2}(\bar{M}_N - \bar{m} - \mu)^2 + o((\bar{M}_N - \bar{m} - \mu)^2)\right]$$

$$= G(\bar{m}) + G'(\bar{m})(\bar{M}_N - \bar{m}) + \frac{G''(\bar{m})}{2}\left[(\bar{M}_N - \bar{m})^2 + \delta^2\sigma_{\hat{\mu}}^2\right] + o((\bar{M}_N - \bar{m})^2) + o(\delta^2)$$

$$= G(\bar{m}) + G'(\bar{m})(\bar{M}_N - \bar{m}) + \frac{G''(\bar{m})}{2}\sigma_\mu^2 + o(\bar{M}_N - \bar{m}) + o(\delta^2) .$$

Thus $\bar{M}_N - \bar{m} \cong -\frac{G''(\bar{m})}{G'(\bar{m})} \cdot \frac{\sigma_\mu^2}{2} = \frac{\sigma_\mu^2}{2} \cdot \frac{f(\bar{m})}{1-F(\bar{m})} = \frac{\sigma_{\hat{\mu}}^2}{2} \cdot \frac{1}{h(\bar{m})}\delta^2$

Substituting the above into the expression of $\frac{h(\bar{m})}{h_M(\bar{M})}$, we have

$$\frac{h(\bar{m})}{h_M(\bar{M}_N)} = h(\bar{m}) \cdot \frac{\mathbf{E}_\mu f(\bar{M}_N - \mu)}{\mathbf{E}_\mu[1 - F(\bar{M}_N - \mu)]}$$

$$= h(\bar{m}) \cdot \frac{\mathbf{E}_\mu f\left(\bar{m} + (\bar{M}_N - \bar{m} - \mu)\right)}{\mathbf{E}_\mu\left[1 - F\left(\bar{m} + (\bar{M}_N - \bar{m} - \mu)\right)\right]}$$

$$= h(\bar{m}) \cdot \frac{\mathbf{E}_\mu\left[f(\bar{m}) + f'(\bar{m})(\bar{M}_N - \bar{m} - \mu) + \frac{f''(\bar{m})}{2}(\bar{M}_N - \bar{m} - \mu)^2 + o(\bar{M}_N - \bar{m}) + o(\delta^2)\right]}{\mathbf{E}_\mu\left[\left(1 - F(\bar{m})\right) - f(\bar{m})(\bar{M}_N - \bar{m} - \mu) - \frac{f'(\bar{m})}{2}(\bar{M}_N - \bar{m} - \mu)^2 + o(\bar{M}_N - \bar{m}) + o(\delta^2)\right]}$$

$$\cong \frac{1 - F(\bar{m})}{f(\bar{m})} \cdot \frac{f(\bar{m}) + f'(\bar{m})(\bar{M}_N - \bar{m}) + \frac{f''(\bar{m})}{2}\delta^2\sigma_{\hat{\mu}}^2}{\left(1 - F(\bar{m})\right) - f(\bar{m})(\bar{M}_N - \bar{m}) - \frac{f'(\bar{m})}{2}\delta^2\sigma_{\hat{\mu}}^2}$$

$$= \frac{1 + \frac{f'(\bar{m})}{f(\bar{m})} \cdot \frac{\delta^2\sigma_{\hat{\mu}}^2}{2} \cdot \frac{f(\bar{m})}{1 - F(\bar{m})} + \frac{f''(\bar{m})}{f(\bar{m})} \cdot \frac{\delta^2\sigma_{\hat{\mu}}^2}{2}}{1 - \frac{f(\bar{m})}{1 - F(\bar{m})} \cdot \frac{\delta^2\sigma_{\hat{\mu}}^2}{2} \cdot \frac{f(\bar{m})}{1 - F(\bar{m})} - \frac{f'(\bar{m})}{1 - F(\bar{m})} \cdot \frac{\delta^2\sigma_{\hat{\mu}}^2}{2}}$$

$$\cong 1 + \frac{\sigma_{\hat{\mu}}^2}{2}\left[\frac{f''(\bar{m})}{f(\bar{m})} + \frac{f(\bar{m})}{1 - F(\bar{m})} \cdot \frac{f(\bar{m})}{1 - F(\bar{m})} + \frac{2f'(\bar{m})}{1 - F(\bar{m})}\right] \cdot \delta^2$$

The above expression is greater than 1 if and only if $f''(\bar{m}) \cdot h^2(\bar{m}) + 2f'(\bar{m}) \cdot h(\bar{m}) + f(\bar{m}) > 0$. ■

One can calculate $f(\bar{m}) - f_M(\bar{M}_N) = f(\bar{m}) - \left[f(\bar{m}) + f'(\bar{m})(\bar{M}_N - \bar{m}) + \frac{f''(\bar{m})}{2} \cdot \delta^2\sigma_{\hat{\mu}}^2\right] + o(\bar{M}_N - \bar{m}) + o(\delta^2) \cong -\frac{\sigma_{\hat{\mu}}^2}{2}\left(\frac{f'(\bar{m})}{h(\bar{m})} + f''(\bar{m})\right)\delta^2$.

One can also calculate the marginal effect of filtering on a consumer's equilibrium probability of buying a product after searching it, which increases from $1 - F_M(\bar{M}_N)$ to $1 - F(\bar{m})$. Their difference equals to

$$1 - F(\bar{m}) - \left(1 - F_M(\bar{M}_N)\right)$$
$$= F_M(\bar{M}_N) - \mathrm{F}(\bar{m})$$
$$= F(\bar{m}) + F'(\bar{m})(\bar{M}_N - \bar{m}) + \frac{F''(\bar{m})}{2} \cdot \delta^2\sigma_{\hat{\mu}}^2 - F(\bar{m}) + o(\bar{M}_N - \bar{m}) + o(\delta^2)$$
$$\cong \frac{\sigma_{\hat{\mu}}^2}{2}\left(\frac{f(\bar{m})}{h(\bar{m})} + f'(\bar{m})\right)\delta^2 = \frac{\sigma_{\hat{\mu}}^2}{2} \cdot \frac{f^2(\bar{m}) + [1 - F(\bar{m})]f'(\bar{m})}{1 - F(\bar{m})}\delta^2,$$

which is strictly positive (because $h'(\bar{m}) = -\frac{f^2(\bar{m}) + [1 - F(\bar{m})]f'(\bar{m})}{f^2(\bar{m})} < 0$) and increases with $f(\bar{m})$ and $f'(\bar{m})$.

PROOF OF PROPOSITION 5. A seller's equilibrium profit is $\tilde{\pi}_N^{S*} = \frac{1}{n} \cdot F_0(\bar{M}_N - \tilde{p}_N^*) \cdot [(1 - r)\tilde{p}_N^* - c]$ without filtering and is $\tilde{\pi}^{S*} = \frac{1}{n} \cdot F_0(\bar{M} - \tilde{p}^*) \cdot [(1 - r)\tilde{p}^* - c]$ with filtering. Substituting equation (3) and (4) into the expression of $\tilde{\pi}^{S*}$ and then expanding the expression of $F_0(\cdot)$ at point $\bar{M}_N - \tilde{p}_N^*$:

$$\begin{aligned}
\tilde{\pi}^{S*} &= \frac{1}{n} \cdot F_0\big(\bar{M}_N - \tilde{p}_N^* + \hat{\mu}_K\delta + O(\delta^2)\big) \cdot [(1-r)\tilde{p}_N^* + O(\delta^2) - c] \\
&= \frac{1}{n} \cdot [F_0(\bar{M}_N - \tilde{p}_N^*) + f_0(\bar{M}_N - \tilde{p}_N^*) \cdot \hat{\mu}_K\delta][(1-r)\tilde{p}_N^* - c] + o(\delta) \\
&\cong \tilde{\pi}_N^{S*} + \frac{1}{n} \cdot f_0(\bar{M}_N - \tilde{p}_N^*)\hat{\mu}_K[(1-r)\tilde{p}_N^* - c]\delta \\
&> \tilde{\pi}_N^{S*}.
\end{aligned}$$

The platform's equilibrium profit is $\tilde{\pi}_N^{P*} = F_0(\bar{M}_N - \tilde{p}_N^*) \cdot r\tilde{p}_N^*$ without filtering and is $\tilde{\pi}^{S*} = \frac{1}{n} \cdot F_0(\bar{M} - \tilde{p}^*) \cdot r\tilde{p}^*$ with filtering. Substituting equation (3) and (4) into the expression of $\tilde{\pi}^{P*}$ and then expanding the expression of $F_0(\cdot)$ at point $\bar{M}_N - \tilde{p}_N^*$:

$$\begin{aligned}
\tilde{\pi}^{S*} &= \frac{1}{n} \cdot F_0\big(\bar{M}_N - \tilde{p}_N^* + \hat{\mu}_K\delta + O(\delta^2)\big) \cdot r[\tilde{p}_N^* + O(\delta^2)] \\
&= \frac{1}{n} \cdot [F_0(\bar{M}_N - \tilde{p}_N^*) + f_0(\bar{M}_N - \tilde{p}_N^*) \cdot \hat{\mu}_K\delta]r\tilde{p}_N^* + o(\delta) \\
&\cong \tilde{\pi}_N^{P*} + \frac{1}{n} \cdot f_0(\bar{M}_N - \tilde{p}_N^*)\hat{\mu}_K r\tilde{p}_N^*\delta \\
&> \tilde{\pi}_N^{P*}.
\end{aligned}$$

The consumer surplus is $\widetilde{CS}_N = \mathbf{E}_{u_0}[\max\{u_0, \bar{M}_N - \tilde{p}_N^*\}]$ without filtering and is $\widetilde{CS}^* = \mathbf{E}_{u_0}[\max\{u_0, \bar{M} - \tilde{p}^*\}]$ with filtering. Because $\bar{M} - \tilde{p}^* \cong \bar{M}_N - \tilde{p}_N^* + \hat{\mu}_K\delta$, filtering will increase consumer surplus. ■

PROOF OF PROPOSITION 6. A seller's profit as a function of its price p_i is given by $\pi_i = \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-p^*(r))}{n(1-F(\bar{m}(\tau)))}[1-F(\bar{m}(\tau)-p^*+p_i)][(1-r)p_i-c]\right]$.

The seller's optimal price $\tilde{p}^*$ satisfies the FOC, i.e., $\frac{\partial \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{n(1-F(\bar{m}(\tau)))}[1-F(\bar{m}(\tau)-\tilde{p}^*+p_i)][(1-r)p_i-c]\right]}{\partial p} = 0$ when $p_i = \tilde{p}^*$. Rearranging terms, we get $0 = \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{\frac{(1-F(\bar{m}(\tau)))}{f(\bar{m}(\tau))}}(\tilde{p}^* - \tilde{p}_\tau^*)\right] = \mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{h(\bar{m}(\tau))}(\tilde{p}^* - \tilde{p}_\tau^*)\right]$. Let $a(\tau) =$

$\frac{\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{h(\bar{m}(\tau))}}{\mathbf{E}_\tau\left[\frac{F_0(\bar{m}(\tau)+\mu_K-\tilde{p}^*)}{h(\bar{m}(\tau))}\right]}$, which is a strict decreasing function of τ, because $F_0\big(\bar{m}+\mu_K-p^*(r)\big)$ increases with $\bar{m}$, $h(\bar{m})$ decreases with $\bar{m}$, and $\bar{m}$ decreases with τ. Note $\mathbf{E}_\tau[a(\tau)]=1$, so $\tilde{p}^*=\mathbf{E}_\tau[a(\tau)\tilde{p}_\tau^*]$. Note that $\tilde{p}_\tau^*$ strictly increases with τ, so $\tilde{p}^*=\mathbf{E}_\tau[a(\tau)\tilde{p}_\tau^*]=\mathbf{E}_\tau[a(\tau)]\mathbf{E}_\tau[\tilde{p}_\tau^*]+\mathbf{Cov}_\tau(a(\tau),\tilde{p}_\tau^*)<\mathbf{E}_\tau[\tilde{p}_\tau^*]$. ∎

Heterogeneous search cost: Special case. Consider the case with $\tau\sim\text{Uniform}(0,\tau_{\max})$, where $\tau_{\max}<\frac{1}{8}\left(1-\frac{c}{1-r}\right)^2$. The FOC can be simplified as $\int_0^{\tau_{\max}}\frac{1}{\tau_{\max}}\cdot\frac{1-\sqrt{2\tau}-\tilde{p}^*}{\sqrt{2\tau}}\left(\tilde{p}^*-\frac{c}{1-r}-\sqrt{2\tau}\right)d\tau=0$. Let $s=\sqrt{2\tau}$, so $\int_0^{\sqrt{2\tau_{\max}}}(1-s-\tilde{p}^*)\left(\tilde{p}^*-\frac{c}{1-r}-s\right)ds=0$. One can derive that $\tilde{p}^*=\frac{1-\frac{c}{1-r}}{2}-\sqrt{\frac{1}{3}\left(\sqrt{2\tau_{\max}}-\frac{3}{2}\frac{1-\frac{c}{1-r}}{2}\right)^2+\left(\frac{1-\frac{c}{1-r}}{2}\right)^2}$, which decreases with $\tau_{\max}$. ∎

Fixed Referral Fee. Seller i's profit is given by $\pi_i^S(p_i;d)=\frac{F_0(\bar{m}+\mu_K-p^*(d))}{n(1-F(\bar{m}))}\cdot[1-F(\bar{m}-p^*(d)+p_i)]\cdot[p_i-d-c]$.

The equilibrium retail price is $\tilde{p}^*(d)=d+c+h(\bar{m})$, the seller's equilibrium profit is $\tilde{\pi}_i^{S*}=\frac{F_0(\bar{m}+\mu_K-d-c-h(\bar{m}))}{n}h(\bar{m})$. The total demand on the retail platform is $\widetilde{D}=F_0\big(\bar{m}+\mu_K-d-c-h(\bar{m})\big)$. The platform's profit is $\tilde{\pi}^{P*}=F_0\big(\bar{m}+\mu_K-d-c-h(\bar{m})\big)\cdot d$.

If the platform endogenously chooses its referral fee, the platform will earn a strictly higher profit when it charges a percentage referral fee than when it charges a fixed per-unit referral fee. The proof is below.

Suppose that in the case with a fixed per-unit referral fee, the platform's optimal referral fee is d^*, so its profit is $F_0\big(\bar{m}+\mu_K-d^*-c-h(\bar{m})\big)\cdot d^*$. Let us consider the case with a percentage referral fee and set the referral fee to $r=\frac{d^*}{c+d^*}$. The platform's profit is given by $r\cdot\left(\frac{c}{1-r}+h(\bar{m})\right)F_0\left(\bar{m}+\mu_K-\frac{c}{1-r}-h(\bar{m})\right)=F_0\big(\bar{m}+\mu_K-d^*-c-h(\bar{m})\big)\cdot d^*\cdot\frac{c+d^*+h(\bar{m})}{c+d^*}>F_0\big(\bar{m}+\mu_K-d^*-c-h(\bar{m})\big)\cdot d^*$. Thus, the platform's profit under the optimal percentage referral fee is strictly higher than its profit under the optimal fixed per-unit referral fee.

Heterogeneous Product Quality

LEMMA A2. *The difference in equilibrium prices between the premium product and the non-premium product is smaller than their difference in base quality, i.e.,* $p_1^*(r) - p^*(r) < \Delta q$. *Consumers will always search the premium seller's (seller 1's) product first if they search on the platform.*

Lemma A2 shows that the premium seller 1 will not set its price at a level that exceeds the non-premium sellers' prices by their base quality difference. Note that in equilibrium, a consumer's utility of buying the premium product (product 1) is $u_{1j} = q_H - p_1^*(r) + m_{1j} = q_L + \Delta q - p_1^*(r) + m_{1j}$, and her expected utility of buying a non-premium product $i \neq 1$ is $u_{ij} = q_L - p^*(r) + m_{ij}$. Therefore, the condition $p_1^*(r) - p^*(r) < \Delta q$ suggests that seller 1 will set its price such that consumers will first search seller 1 instead of other sellers. Otherwise, if consumers search the non-premium sellers first, because the number of non-premium sellers, n, is large, the chance that consumers will ever search seller 1 will be negligible.

LEMMA A3. *Define* $u_0^a = q_H - p_1^*(r) + \bar{m}$ *and* $u_0^b = q_L - p^*(r) + \bar{m}$, *where* $u_0^a > u_0^b$. *Let* $p_1^*(r)$ *be the equilibrium retail price of the premium seller (seller 1), and* p_1 *be the price that seller 1 actually charges (in equilibrium,* $p_1^*(r) = p_1$*). A consumer's outside option is* u_{0j}.

(1)If $u_{0j} \geq u_0^a$, *she will choose her outside option and not search on the platform.*

(2)If $u_0^a > u_{0j} \geq u_0^b$, *the consumer will search seller 1 first. If she finds that the match level of seller 1,* m_{1j}, *is higher than* $u_{0j} - q_H + p_1$, *she will purchase product 1. If* $m_{1j} < u_{0j} - q_H + p_1$, *she will stop searching and choose her outside option.*

(3)If $u_{0j} < u_0^b$, *she will search seller 1 first and buy its product if* $m_{1j} > p_1 - p^*(r) - \Delta q + \bar{m}$. *If* $m_{1j} < p_1 - p^*(r) - \Delta q + \bar{m}$, *she will continue to search other non-premium sellers.*

The second part of Lemma A3 shows that when products vary in base quality, some consumers may search only for the premium product and will never consider the non-premium products, even when the match level of the premium product is low. It can happen when the utilities of consumers' outside options are in the middle range ($u_0^a > u_{0j} \geq u_0^b$). These consumers will search the premium product because their outside options do not provide a high enough utility. However, these consumers will not search the non-premium products because it is unlikely that a low-quality product will have such a high match level that can significantly improve upon the outside option, so the benefit of searching cannot justify the search cost.

Next we discuss sellers' pricing strategies. We still fix the platform's referral fee at r for now.

LEMMA A4. *The equilibrium retail price of the non-premium products is* $p^*(r) = \frac{c}{1-r} + h(\bar{m})$, *which is independent of the base quality level of the premium product.*

PROOF. According to Lemma 4, the consumers will search the non-premium products if and only if $u_0 < q_L - p^*(r) + \bar{m}$ and $m_{1j} < p_1 - p^*(r) - \Delta q + \bar{m}$. Following Wolinsky (1986), the profit of a non-premium seller's profit is $\pi_i^S(p_i) = F_0(q_L - p^*(r) + \bar{m}) \cdot F(p_1 - p^*(r) - \Delta q + \bar{m}) \cdot \frac{[1-F(\bar{m}-p^*(r)+p_i)]\cdot[(1-r)p_i - c]}{n-1}$. Note that the first two terms is positive and independent of p_i, so the optimal p_i should satisfy $\frac{\partial[1-F(\bar{m}-p^*(r)+p_i)]\cdot[(1-r)p_i-c]}{\partial p_i} = 0$. The FOC implies that $p^*(r) = \frac{c}{1-r} + h(\bar{m})$. ■

Lemma A4 shows that the equilibrium price of the non-premium products is independent of q_H, the base quality level of the premium product. The intuition is as follows. The price and the quality level of the premium product will affect the profit of non-premium brands by changing the number of consumers who will search the non-premium products. However, given that a consumer has searched the premium product and decides to continue to search the non-premium sellers' products, she will exclude the premium product from her consideration set and will never buy the premium product. Thus, the premium product's base quality and price will not affect this consumer's probability of buying a non-premium product if she decides to search non-premium products. In addition, consumers will search the non-premium products only after they have searched the premium product. Thus, deviation to an off-equilibrium price for non-premium sellers will not affect consumers' decisions on whether to search the premium product and whether to continue searching non-premium products. Therefore, the optimal price of non-premium products will be independent of the price and base quality of the premium product.

Next we examine the premium seller's optimal price. According to Lemma A3 and, if the premium seller's price is p_1, the demand of product 1 is:

$$D_1(p_1; r) = \int_{q_L - p^*(r)+\bar{m}}^{q_H - p_1^*(r)+\bar{m}} \int_{u_0 - q_{\mathrm{H}} + p_1}^{\bar{M}} dF(m)\ dF_0(u_0) + \int_{\underline{U}}^{q_L - p^*(r)+\bar{m}} \int_{p_1 - p^*(r)+\bar{m}-\Delta q}^{\bar{M}} dF(m)\ dF_0(u_0)$$

$= \int_{q_L - p^*(r)+\bar{m}}^{q_{\mathrm{H}} - p_1^*(r)+\bar{m}} [1 - F(u_0 - q_{\mathrm{H}} + p_1)]\, dF_0(u_0) + [1 - F(p_1 - p^*(r) + \bar{m} - \Delta q)] \cdot F_0(q_L - p^*(r) + \bar{m})$.

For tractability, we follow the assumption in example 1 and 3 that both u_{0j} and m_{ij} follow exponential distributions with mean $\frac{1}{\theta}$. The cumulative distribution of u_{0j} and m_{ij} are $F_0(u) = 1 - e^{-\theta u}$ and $F(m) = 1 - e^{-\theta m}$, respectively.

LEMMA A5. The non-premium product's optimal price is $p^*(r) = \frac{c}{1-r} + \frac{1}{\theta}$. The premium product's optimal price is $p_1^*(r) = p^*(r) + \min\{\frac{c_1 - c}{1-r}, \Delta q\}$.

PROOF. When $F_0(u) = 1 - e^{-\theta u}$ and $F(m) = 1 - e^{-\theta m}$, the premium sellers' demand is:

$$D_1(p_1; r) = \int_{q_L - p^*(r) + \bar{m}}^{q_H - p_1^*(r) + \bar{m}} \int_{u_0 - q_H + p_1}^{\bar{M}} dF(m)\ dF_0(u_0) + \int_{\underline{U}}^{q_L - p^*(r) + \bar{m}} \int_{p_1 - p^*(r) + \bar{m} - \Delta q}^{\bar{M}} dF(m)\ dF_0(u_0)$$

$$= \int_{q_L - p^*(r) + \bar{m}}^{q_H - p_1^*(r) + \bar{m}} [1 - F(u_0 - q_H + p_1)]\, dF_0(u_0) + [1 - F(p_1 - p^*(r) + \bar{m} - \Delta q)] \cdot F_0(q_L - p^*(r) + \bar{m})$$

$$= \int_{q_L - p^*(r) + \bar{m}}^{q_H - p_1^*(r) + \bar{m}} \theta e^{-\theta(u_0 - q_H + p_1)} \cdot e^{-\theta u_0}\, du_0 + e^{-\theta(p_1 - p^*(r) + \bar{m} - \Delta q)} \cdot \left[1 - e^{\theta(q_L - p^*(r) + \bar{m})}\right]$$

$$= \frac{1}{2}\left\{e^{-\theta[2q_L - q_H + p_1 - 2p^*(r) + 2\bar{m}]} - e^{-\theta[q_H - 2p_1^*(r) + p_1 + 2\bar{m}]}\right\} + e^{-\theta(p_1 - p^* + \bar{m} - q_H + q_L)} - e^{-\theta[2q_L - q_H + p_1 - 2p^*(r) + 2\bar{m}]}$$

$$= \frac{1}{2}\left\{2e^{-\theta(p_1 - p^* + \bar{m} - q_H + q_L)} - e^{-\theta[2q_L - q_H + p_1 - 2p^*(r) + 2\bar{m}]} - e^{-\theta[q_H - 2p_1^*(r) + p_1 + 2\bar{m}]}\right\} > 0.$$

The premium seller's profit is $\pi_1(p_1; r) = D_1(p_1; r)[(1-r)p_1 - c_1]$. For now, we assume that the condition $p_1 - p^*(r) < q_H - q_L$ is not binding. It is easy to verify that $\pi_1(p_1; r) < 0$ when $p_1 = 0$, $\lim_{p \to +\infty} \pi_1(p_1; r) = 0$, and $\pi_1(p_1; r) > 0$ when $p_1 > \frac{c_1}{1-r}$, so optimal p_1 that maximizes $\pi_1(p_1; r)$ satisfies the FOC, which is:

$$\frac{\partial \pi_1(p_1; r)}{\partial p_1} = \frac{\theta}{2}\left\{2e^{-\theta(p_1 - p^* + \bar{m} - q_H + q_L)} - e^{-\theta[2q_L - q_H + p_1 - 2p^*(r) + 2\bar{m}]} - e^{-\theta[q_H - 2p_1^*(r) + p_1 + 2\bar{m}]}\right\}\left[c_1 + \frac{1}{\theta} - (1-r)p_1\right] = 0,$$

which has a unique solution $p_1^*(r) = \frac{c_1}{1-r} + \frac{1}{\theta}$. Further, note that $\frac{\partial \pi_1(p_1; r)}{\partial p_1} > 0$ when $p_1 < \frac{c_1}{1-r} + \frac{1}{\theta}$, and $\frac{\partial \pi_1(p_1; r)}{\partial p_1} < 0$ when $p_1 > \frac{c_1}{1-r} + \frac{1}{\theta}$, so $p_1^*(r)$ maximizes $\pi_1(p_1; r)$.

Next we consider whether $p_1^*(r) - p^*(r) < q_H - q_L$ is satisfied. It is satisfied when $\frac{c_1 - c}{1-r} < \Delta q$, so $p_1^*(r) = \frac{c_1}{1-r} + \frac{1}{\theta}$ is the maximizer of $\pi_1(p_1; r)$ indeed. When $\frac{c_1 - c}{1-r} \geq \Delta q$, the condition $p_1^*(r) - p^*(r) < q_H - q_L$ is binding, so the premium seller should charge (ϵ lower than) $p^*(r) + \Delta q$. ■

Lemma A5 summarizes the equilibrium prices of the premium product and the non-premium product when the platform's referral fee is r. If the premium product has a higher marginal cost than the non-premium product, the premium seller will charge a higher price to cover its cost. However, the premium seller needs to keep its price below $p^*(r) + \Delta q$ so consumers will search its product first, as is shown in Lemma 3. In the rest of this subsection, we assume that the cost difference between the premium product and the non-premium product is not too large, i.e., $c_1 - c < (1-r)\Delta q$. Under this assumption, $p_1^*(r) = p^*(r) + \frac{c_1 - c}{1-r}$.

Lemma A6 summarizes how the consumer's search cost will affect the profit of the premium seller and non-premium sellers when the platform's referral fee is exogenous.

LEMMA A6. *When the consumer's search cost (τ) decreases, the premium seller's profit decreases if $\tau \leq \tau^*$, and increases if $\tau > \tau^*$. The non-premium seller's profit will always increase. The platform's profit and the total profit of sellers may either increase or decrease.*

PROOF. Under the assumption that $\frac{c_1-c}{1-r} < \Delta q$, The premium seller's profit is $\pi_1^* = \frac{1-r}{2}\{2e^{-\theta\left(\bar{m}-\Delta q+\frac{c_1-c}{1-r}\right)} - e^{-\theta\left(q_H+2\bar{m}-\frac{c_1}{1-r}-\frac{1}{\theta}\right)} - e^{-\theta\left(q_L-\Delta q+2\bar{m}+\frac{c_1-2c}{1-r}-\frac{1}{\theta}\right)}\}$. $\frac{\partial \pi_1^*}{\partial \bar{m}} = (1-r)\theta[e^{-\theta\left(q_H+2\bar{m}-\frac{c_1}{1-r}-\frac{1}{\theta}\right)} + e^{-\theta\left(q_L-\Delta q+2\bar{m}+\frac{c_1-2c}{1-r}-\frac{1}{\theta}\right)} - e^{-\theta\left(\bar{m}-\Delta q+\frac{c_1-c}{1-r}\right)}]$. One can show that $\frac{\partial \pi_1^*}{\partial \bar{m}} > 0$ when $\bar{m} < \bar{m}^* = \frac{1}{\theta}\ln[1 + e^{-2\theta(\Delta q - p_1^*(r) + p^*(r))}] - q_L + p^*(r)$, and $\frac{\partial \pi_1^*}{\partial \bar{m}} < 0$ when $\bar{m} > \bar{m}^*$. τ^* is hence implicitly defined by $\bar{m}(\tau^*) = \bar{m}^*$. ■

On the one hand, the number of consumers searching on the platform will be higher when the consumer's search cost declines, benefiting both the premium seller and the non-premium sellers. On the other hand, a decrease in the search cost makes consumers more likely to continue to search the non-premium products after searching the premium product, which benefits the non-premium sellers but hurts the premium seller. Therefore, a decrease in the consumer search cost has a non-monotonic effect on the premium seller's profit, but will always benefit the non-premium sellers.

When the platform endogenously chooses its referral fee r, we find that the platform's profit will always be higher when the consumer's search cost decreases, which is consistent with our finding in Proposition 1. Moreover, the platform's profit will always increase with the base quality levels of the premium product and the non-premium product, q_H and q_L. Thus, our results in the main model are robust even if products have heterogeneous base quality levels.

Platform competition

In this numerical example, we consider a model with two competing retail platforms (denoted as A and B) to examine how the consumer's search cost affects competition between platforms. Consumers are uniformly distributed on a Hotelling line of [0,1], and platforms A and B are respectively located at 0 and 1 on the line. Consumer i at location $x_i \in [0,1]$ has a valuation $v_{ij,A} = u_{ij} - t \cdot x_i$ for product j on platform A and a valuation $v_{ij,B} = u_{ij} - t \cdot (1 - x_i)$ for product j on platform B, where $u_{ij} = \mu_{ij} + m_{ij} - p_j$ is defined the same as in the main model. Each seller sells on only one platform. First, the platforms simultaneously set their referral fees, r_A and r_B, respectively. Second, sellers simultaneously set their retail prices. Then, consumers choose a platform to shop on or the outside option. Consumers' search costs on the two platforms are τ_A and τ_B, respectively, and the two platforms are otherwise the same. We continue to adopt the assumptions for the model parameters in Examples 1 and 2: $\mu_K = 0.5$ and m_{ij}~Uniform$(-0.5,0.5)$. All other assumptions are the same as in the main model.

Suppose that the referral fee is r_l for platform l ($l = 1,2$). One can easily see that the equilibrium price for a seller on platform l will be $p_l^* = \frac{c}{1-r_l} + h(\bar{m}(\tau_l))$, the same expression as in the main model. Consumer i's expected utility of shopping on platform A is $v_{iA} = \mu_K + \bar{m}(\tau_A) - p_A^* - t \cdot x_i = 1 - 2\sqrt{2\tau_A} - \frac{c}{1-r_l} - t \cdot x_i$, her expected utility of shopping on platform B is $v_{iB} = \mu_K + \bar{m}(\tau_B) - p_B^* - t \cdot (1 - x_i) = 1 - 2\sqrt{2\tau_B} - \frac{c}{1-r_B} - t \cdot (1 - x_i)$, and her utility from the outside option is u_{i0}. Hence, consumer i will shop on platform A if and only if $x_i \leq \frac{1}{2} + \frac{2(\sqrt{2\tau_B} - \sqrt{2\tau_A}) + (\frac{c}{1-r_B} - \frac{c}{1-r_A})}{2t}$ and $v_{iA} \geq u_{i0}$, and will shop on platform B if and only if $x_i > \frac{1}{2} + \frac{2(\sqrt{2\tau_B} - \sqrt{2\tau_A}) + (\frac{c}{1-r_B} - \frac{c}{1-r_A})}{2t}$ and $v_{iB} \geq u_{i0}$. Let $x^* = \frac{1}{2} + \frac{2(\sqrt{2\tau_B} - \sqrt{2\tau_A}) + (\frac{c}{1-r_B} - \frac{c}{1-r_A})}{2t}$. Because a consumer who decides to shop on a platform will eventually buy from that platform, platform A's profit is

$$\pi_A = \int_0^{x^*} F_0(v_{iA}) dx \cdot r_A \left(\frac{c}{1 - r_A} + \sqrt{2\tau_A} \right)$$
$$= \int_0^{x^*} \left(1 - 2\sqrt{2\tau_A} - \frac{c}{1-r_l} - t \cdot x \right) dx \cdot r_A \left(\frac{c}{1-r_A} + \sqrt{2\tau_A} \right)$$
$$= x^* \left(1 - 2\sqrt{2\tau_A} - \frac{c}{1-r_A} - \frac{t}{2} x^* \right) r_A \left(\frac{c}{1-r_A} + \sqrt{2\tau_A} \right),$$

and platform B's profit is

$$\pi_B = \int_{x^*}^{1} F_0(v_{iB}) dx \cdot r_B \left(\frac{c}{1 - r_B} + \sqrt{2\tau_B} \right)$$
$$= (1 - x^*) \left(1 - 2\sqrt{2\tau_B} - \frac{c}{1-r_B} - \frac{t}{2}(1 - x^*) \right) r_B \left(\frac{c}{1-r_B} + \sqrt{2\tau_B} \right).$$

Denote $w_A = \frac{c}{1-r_A}$, $w_B = \frac{c}{1-r_B}$, $s_A = \sqrt{2\tau_A}$, and $s_B = \sqrt{2\tau_B}$. Then $x^* = \frac{1}{2} + \frac{2(s_B - s_A) + (w_B - w_A)}{2t}$, and it is equivalent to consider that platform l chooses w_l ($l = A, B$) instead of choosing r_l. Platform A's and platform B's profits can be respectively written as

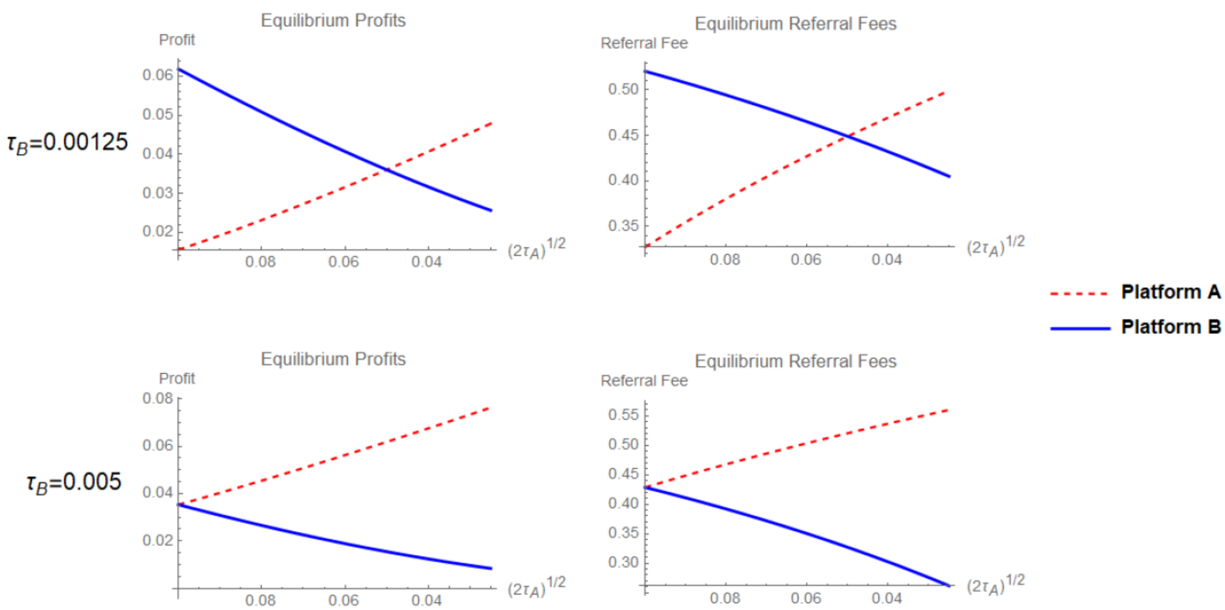
$$\pi_A = \frac{(t + 2s_B - 2s_A + w_B - w_A)(4 - 6s_A - 3w_A - t - 2s_B - w_B)(w_A - c)(w_A + s_A)}{8tw_A},$$

and

$$\pi_B = \frac{(t + 2s_A - 2s_B + w_A - w_B)(4 - 6s_B - 3w_B - t - 2s_A - w_A)(w_B - c)(w_B + s_B)}{8tw_B}.$$

We derive the platforms' profit functions in the Web Appendix. Figure WA1 shows how the equilibrium referral fees and the platforms' profits change with platform A's search cost when platform B's search cost is 0.00125 and when it is 0.005. Figure WA1 indicates that a decrease in the search cost on a platform will increase its equilibrium referral fee and profit, and reduce the competing platform's equilibrium referral fee and profit.

Figure WA1 Numerical Example with Platform Competition



Note. The figures are plotted using $c = 0.1$ and $t = 0.1$.

Effect of Outside Options

For analytical tractability, we assume that the platform will endogenously choose its fixed per-unit referral fee, d. We adopt the distribution assumptions in Examples 1 and 2: $\mu_K = 0.5$ and $m_{ij} \sim \text{Uniform}(-0.5, 0.5)$. The consumer's outside option, u_0, follows a uniform distribution between l and $1 + l$. A larger l indicates that the outside option tends to be more attractive. We consider the nontrivial case with $-2\sqrt{2\tau} - c < l < 1 - 2\sqrt{2\tau} - c$, otherwise either no consumers or all consumers will shop on the platform.

We show that in equilibrium, the platform's referral fee is $d^* = \frac{1-2\sqrt{2\tau}-c-l}{2}$, a seller's profit is $\pi_i^{S*} = \frac{\sqrt{2\tau}(1-2\sqrt{2\tau}-c-l)}{2n}$, and the platform's profit is $\pi^{P*} = \frac{(1-2\sqrt{2\tau}-c-l)^2}{4}$.

The platform can always benefit from a lower search cost: $\frac{\partial \pi^{S*}}{\partial \tau} = -\frac{1-2\sqrt{2\tau}-c-l}{\sqrt{\tau}} < 0$. When consumers have better outside options, the platform benefits less from a decrease in the search cost, because the absolute value of $\frac{\partial \pi^{S*}}{\partial \tau}$ decreases with l.

Effect of consumers' search quality

Specifically, a consumer has probability ρ of being able to learn a product's unfilterable match value, m_{ij}, after searching product i—a higher ρ indicates better search quality of the consumer's search. For analytical tractability, we assume that, if a consumer fails to learn a product's m_{ij} after searching the product (which occurs with probability $1-\rho$), her expectation of this product's unfilterable match value will remain at her prior expectation, $\mathbf{E}[m]$. The main model of this paper is essentially the special case with $\rho = 1$.

The ensuing analysis assumes the consumer's search cost $\tau < \rho \cdot \int_{\mathbf{E}[m]}^{m_{\max}} (m - \mathbf{E}[m]) dF(m)$ to exclude the uninteresting case in which consumers will buy a product even when they fail to learn its unfilterable match value after searching the product. Suppose that in equilibrium consumers expect all sellers to set prices at p^*. Consider the scenario in which after searching a product, the consumer successfully learns the product's unfilterable match value m_{ij} and its price p_i. Hence, the consumer's utility of buying this product will be $u_{ij} = \mu_K + m_{ij} - p_i$. Following Wolinsky (1986), the consumer's optimal search strategy in our case is to buy this product if and only if her expected utility increase from searching the next product is smaller than the search cost; otherwise the consumer will continue searching. If the consumer continues to search another product (indexed by i') with filterable match value $\mu_{i'j} = \mu_K$, with probability ρ she will successfully learn the product's unfilterable match value $m_{i'j}$, and with probability $1-\rho$ she will fail to learn it. Hence, her expected utility increase from searching another product after searching product i will be $\rho \int_{m_{ij}}^{m_{\max}} \left(m - m_{ij} + (p_i - p^*)\right) dF(m) + (1-\rho) \max\{\mathbf{E}[m] - m_{ij} + (p_i - p^*), 0\}$, and the consumer will continue searching if it is higher than τ. Notice that the expression strictly decreases with m_{ij} and it is greater than τ when $m_{ij} = \mathbf{E}[m]$ and $p_i = p^*$. Hence, in equilibrium where $p_i = p^*$, a consumer will stop searching and buy product i if and only if $m_{ij} \geq \bar{m}$, where $\bar{m}$ is implicitly defined by $\int_{\bar{m}}^{m_{\max}} (m - \bar{m}) dF(m) = \tau/\rho$. Note that ρ and τ affect the sellers' and the platform's decisions and profits only via affecting $\bar{m}$, so the equilibrium outcomes of our extended model will be the same as those of our main model with the search cost being τ/ρ. This result suggests that, as long as $\tau <$

$\rho \cdot \int_{\mathbf{E}[m]}^{m_{\max}} (m - \mathbf{E}[m]) dF(m)$, an increase in the consumer's search quality (ρ) is equivalent to a decrease in the consumer's per-product search cost (τ).

Example: $\boldsymbol{r^*}$ decreases when $\boldsymbol{\tau}$ decreases. Suppose that $\mu_K = 2, c = 1$, u_{0j} follows the student-t distribution with degree of freedom 5, and m_{ij} follows a logistic distribution with c.d.f. $F(m) = \frac{1}{1+e^{-5m}}$. The platform's optimal referral fee is shown in Figure WA2. When $\overline{m} < 3.5$, the platform's optimal referral fee will decrease as the consumer's search cost decreases.

Figure WA2 Optimal Referral Fee