Appendix

It is assumed that the ground consists of a number, n, of parallel layers of different materials. The nth layer overlies a half-space or a rigid foundation, which is identified as 'layer' number $(n+1)$. For the jth layer the material constants are: elastic modulus, E_j, Poisson ratio, υ_j, density, ρ_j, loss factor, η_j and layer thickness, h_j. Each layer is only subjected to external forces on the boundaries, and the stress, deformation and displacement of each are all zero at the initial state. Thus, the stresses at the bottom of the jth layer with those at the top can be expressed as (Sheng et al., 1999)

$$\bar{\mathbf{s}}_{j1} = \mathrm{e}^{\alpha_{j1}h_j}\mathbf{A}_{j1}\mathbf{A}_{j0}^{-1}\bar{\mathbf{s}}_{j0} \tag{1}$$

where $\bar{\mathbf{s}}_{j0}$ is the (Fourier transformed) state vector containing displacements and stresses of the top interface of the jth layer, and $\bar{\mathbf{s}}_{j1}$ is the corresponding vector for the bottom; $\mathrm{e}^{\alpha_{j1}h_j}\mathbf{A}_{j1}\mathbf{A}_{j0}^{-1}$ is the transformed matrix of a single layer, and $\mathbf{A}_{j0}$, $\mathbf{A}_{j1}$ are 6×6 dynamic flexibility matrices dependent on wavenumber k_x and k_y, frequency Ω and material parameters. For the special case of $k_x = 0$, the detailed expressions for $\mathbf{A}_{j0}$, $\mathbf{A}_{j1}$ and other formulae can be found in (Sheng et al., 1999). For the case of $k_x \neq 0$, the matrices are given as follows.

(1) ω = 0

$$\mathbf{A}_{j0} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ \dfrac{\lambda_j + 3\mu_j}{2\mu_j\alpha_{j1}} & -\dfrac{\mathrm{i}k_x}{\alpha_{j1}} & -\dfrac{\mathrm{i}k_y}{\alpha_{j1}} & -\dfrac{\lambda_j + 3\mu_j}{2\mu_j\alpha_{j1}} & \dfrac{\mathrm{i}k_x}{\alpha_{j1}} & \dfrac{\mathrm{i}k_y}{\alpha_{j1}} \\ \dfrac{\mathrm{i}\mu_j k_x}{\alpha_{j1}} & \dfrac{\mu_j(k_x^2+\alpha_{j1}^2)}{\alpha_{j1}} & \dfrac{\mu_j k_x k_y}{\alpha_{j1}} & -\dfrac{\mathrm{i}\mu_j k_x}{\alpha_{j1}} & -\dfrac{\mu_j(k_x^2+\alpha_{j1}^2)}{\alpha_{j1}} & -\dfrac{\mu_j k_x k_y}{\alpha_{j1}} \\ \dfrac{\mathrm{i}\mu_j k_y}{\alpha_{j1}} & \dfrac{\mu_j k_x k_y}{\alpha_{j1}} & \dfrac{\mu_j(k_y^2+\alpha_{j1}^2)}{\alpha_{j1}} & -\dfrac{\mathrm{i}\mu_j k_y}{\alpha_{j1}} & -\dfrac{\mu_j k_x k_y}{\alpha_{j1}} & -\dfrac{\mu_j(k_y^2+\alpha_{j1}^2)}{\alpha_{j1}} \\ \lambda_j + 2\mu_j & -2\mathrm{i}\mu_j k_x & -2\mathrm{i}\mu_j k_y & \lambda_j + 2\mu_j & -2\mathrm{i}\mu_j k_x & -2\mathrm{i}\mu_j k_y \end{bmatrix} \tag{2}$$

$$\mathbf{A}_{j1} = \begin{bmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \\ a_{4j} \\ a_{5j} \\ a_{6j} \end{bmatrix} \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & \mathrm{e}^{-2\alpha_{j1}h_j} & & \\ & & & & \mathrm{e}^{-2\alpha_{j1}h_j} & \\ & & & & & \mathrm{e}^{-2\alpha_{j1}h_j} \end{bmatrix} \tag{3}$$

where

$$a_{1j}=\left[\frac{\mathrm{i}(\mu_j+\lambda_j)k_xh_j}{2\mu_j\alpha_{j1}}\quad 1\quad 0\quad \frac{\mathrm{i}(\mu_j+\lambda_j)k_xh_j}{2\mu_j\alpha_{j1}}\quad 1\quad 0\right]$$

$$a_{2j}=\left[\frac{\mathrm{i}(\mu_j+\lambda_j)k_yh_j}{2\mu_j\alpha_{j1}}\quad 0\quad 1\quad \frac{\mathrm{i}(\mu_j+\lambda_j)k_yh_j}{2\mu_j\alpha_{j1}}\quad 0\quad 1\right]$$

$$a_{3j}=\left[\left(\frac{3\mu_j+\lambda_j}{2\mu_j\alpha_{j1}}-\frac{\mu_j+\lambda_j}{2\mu_j}\right)\quad -\frac{\mathrm{i}k_x}{\alpha_{j1}}\quad -\frac{\mathrm{i}k_y}{\alpha_{j1}}\quad \left(-\frac{3\mu_j+\lambda_j}{2\mu_j\alpha_{j1}}-\frac{\mu_j+\lambda_j}{2\mu_j}\right)\quad \frac{\mathrm{i}k_x}{\alpha_{j1}}\quad \frac{\mathrm{i}k_y}{\alpha_{j1}}\right]$$

$$a_{4j}=\left[\mathrm{i}k_x\left(-\lambda_jh_j-\mu_jh_j+\frac{\mu_j}{\alpha_{j1}}\right)\quad \mu_j\left(\frac{k_x^2}{\alpha_{j1}}+\alpha_{j1}\right)\quad \frac{k_xk_y\mu_j}{\alpha_{j1}}\quad \mathrm{i}k_x\left(-\lambda_jh_j-\mu_jh_j-\frac{\mu_j}{\alpha_{j1}}\right)\quad \mu_j\left(\frac{k_x^2}{\alpha_{j1}}+\alpha_{j1}\right)\quad -\frac{k_xk_y\mu_j}{\alpha_{j1}}\right]$$

$$a_{5j}=\left[\mathrm{i}k_y\left(-\lambda_jh_j-\mu_jh_j+\frac{\mu_j}{\alpha_{j1}}\right)\quad \frac{k_xk_y\mu_j}{\alpha_{j1}}\quad \mu_j\left(\frac{k_y^2}{\alpha_{j1}}+\alpha_{j1}\right)\quad \mathrm{i}k_y\left(-\lambda_jh_j-\mu_jh_j-\frac{\mu_j}{\alpha_{j1}}\right)\quad \frac{k_xk_y\mu_j}{\alpha_{j1}}\quad -\mu_j\left(\frac{k_y^2}{\alpha_{j1}}+\alpha_{j1}\right)\right]$$

$$a_{6j}=\left[(\lambda_j+2\mu_j)-\alpha_{j1}h_j(\lambda_j+\mu_j)\quad -2\mathrm{i}\mu_jk_x\quad -2\mathrm{i}\mu_jk_y\quad (\lambda_j+2\mu_j)+\alpha_{j1}h_j(\lambda_j+\mu_j)\quad -2\mathrm{i}\mu_jk_x\quad -2\mathrm{i}\mu_jk_y\right]$$

(2) $\omega\neq 0$

$$\mathbf{A}_{j0}=\begin{bmatrix}\frac{\mathrm{i}k_x}{\xi_{j1}^2} & 1 & 0 & -\frac{\mathrm{i}k_x}{\xi_{j1}^2} & 1 & 0\\ \frac{\mathrm{i}k_y}{\xi_{j1}^2} & 0 & 1 & -\frac{\mathrm{i}k_y}{\xi_{j1}^2} & 0 & 1\\ -\frac{\alpha_{j1}}{\xi_{j1}^2} & -\frac{\mathrm{i}k_x}{\alpha_{j2}} & -\frac{\mathrm{i}k_y}{\alpha_{j2}} & \frac{\alpha_{j1}}{\xi_{j1}^2} & \frac{\mathrm{i}k_x}{\alpha_{j2}} & \frac{\mathrm{i}k_y}{\alpha_{j2}}\\ -\frac{2\mathrm{i}\mu_jk_x\alpha_{j1}}{\xi_{j1}^2} & \frac{\mu_j(k_x^2+\alpha_{j2}^2)}{\alpha_{j2}} & \frac{\mu_jk_xk_y}{\alpha_{j2}} & \frac{2\mathrm{i}\mu_jk_x\alpha_{j1}}{\xi_{j1}^2} & -\frac{\mu_j(k_x^2+\alpha_{j2}^2)}{\alpha_{j2}} & -\frac{\mu_jk_xk_y}{\alpha_{j2}}\\ -\frac{2\mathrm{i}\mu_jk_y\alpha_{j1}}{\xi_{j1}^2} & \frac{\mu_jk_xk_y}{\alpha_{j2}} & \frac{\mu_j(k_y^2+\alpha_{j2}^2)}{\alpha_{j2}} & \frac{2\mathrm{i}\mu_jk_y\alpha_{j1}}{\xi_{j1}^2} & -\frac{\mu_jk_xk_y}{\alpha_{j2}} & -\frac{\mu_j(k_y^2+\alpha_{j2}^2)}{\alpha_{j2}}\\ \lambda_j-\frac{2\mu_j\alpha_{j1}^2}{\xi_{j1}^2} & -2\mathrm{i}\mu_jk_x & -2\mathrm{i}\mu_jk_y & \lambda_j-\frac{2\mu_j\alpha_{j1}^2}{\xi_{j1}^2} & -2\mathrm{i}\mu_jk_x & -2\mathrm{i}\mu_jk_y\end{bmatrix} \tag{4}$$

$$\mathbf{A}_{j1}=\mathbf{A}_{j0}\begin{bmatrix}1 & & & & & \\ & \mathrm{e}^{(\alpha_{j2}-\alpha_{j1})h_j} & & & & \\ & & \mathrm{e}^{(\alpha_{j2}-\alpha_{j1})h_j} & & & \\ & & & \mathrm{e}^{-2\alpha_{j1}h_j} & & \\ & & & & \mathrm{e}^{-(\alpha_{j2}+\alpha_{j1})h_j} & \\ & & & & & \mathrm{e}^{-(\alpha_{j2}+\alpha_{j1})h_j}\end{bmatrix} \tag{5}$$

where

$$\lambda_j=\frac{\upsilon_jE_j[1+\mathrm{i}\eta_j\mathrm{sgn}(\Omega)]}{(1+\upsilon_j)(1-2\upsilon_j)},\mu_j=\frac{E_j[1+\mathrm{i}\eta_j\mathrm{sgn}(\Omega)]}{2(1+\upsilon_j)}$$

$$c_{j1}=\sqrt{\frac{(\lambda_j+2\mu_j)}{\rho_j}},c_{j2}=\sqrt{\frac{\mu_j}{\rho_j}}$$

$$\xi_{j1}^2=\frac{\Omega^2}{c_{j1}^2},\xi_{j2}^2=\frac{\Omega^2}{c_{j2}^2}$$

$$\alpha_{j1}^2=k_x^2+k_y^2-\xi_{j1}^2,\alpha_{j2}^2=k_x^2+k_y^2-\xi_{j2}^2$$

where k_y denotes the wavenumber in the y-direction, rad/m; λ_j and μ_j denote the Lame constants of the jth layer, respectively; c_{j1} and c_{j2} denote the compression wave velocity and shear wave velocity of the jth layer, respectively, and the corresponding wavenumbers are ξ_{j1} and ξ_{j2}.

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