**Appendix**

Derivation of results (5) and (6):

First, the distribution of  could be derived based on the following pivotal quantity:

 (A1)

The pivotal quantity (A1) is built on the central limit theorem with the belief that the simple estimator  should be unbiased for the true average treatment effect .

Given the unobserved treated sample size is  and the unobserved treated sample mean is , it is straightforward to write the ideal treated sample mean  as below:

 (A2)

Similarly, the ideal control sample mean  is written as follows:

 (A3)

The standard deviation associated with the simple estimator is derived as below:

 (A4)

Furthermore, because  and  in observational studies, we can simply the expressions in (A2) through (A3) as follows:

 (A5)

given  and . Finally, we can just plug (A5) in (A1), which yields the distribution (5) whose mean and variance are defined by (6).

Theorem 1 can also be proved under a Bayesian framework. Assuming the prior and the likelihood for the treated outcome is as follows:

 (A6)

And the prior and the likelihood for the control outcome is as follows:

 (A7)

The posterior distribution for  is as follows:

 (A8)

Furthermore, the posterior distribution for  is as follows:

 (A9)

Assuming the treated and the control outcomes are independent, the posterior distribution of  is equivalent to  and has the identical form as (5) and (6).

Derivation of results (7) and (8):

When a significant positive effect has been concluded, the PIV can be expressed by ,  and  as follows, drawing on (5) and (6):

 (A10)

From (A10), the probit model for PIV can be derived as identical to (7).

Likewise, when a significant negative effect has been concluded, the PIV is expressed as follows:

 (A11)

From (A11), the probit model for PIV can be derived as identical to (8).