Voting Behavior under Proportional Representation

Supplemental Appendix

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Bounds on vote shares

Let $v_A(\Delta, \xi)$ be defined as the vote share of party A as a function of the payoff difference Δ and the aggregate shock ξ:

$$v_A(\Delta, \xi) = \frac{1}{2} + \phi(\Delta - \xi),$$

where we recall $\Delta(l_A, l_B) \equiv V_A(l_A, l_B) - V_B(l_A, l_B)$. We assume that the density of the idiosyncratic shock is small enough so that $v_A(\Delta, \xi)$ is always interior: $v_A(\Delta, \xi) \in (0, 1)$ for all $(\Delta, \xi) \in [-1, 1] \times \left[-\frac{1}{2\psi}, \frac{1}{2\psi}\right]$. In addition, we assume that both parties have always a chance of securing two seats, which is equivalent to imposing that $[1 - \pi^*, \pi^*] \subset supp(v_A(\Delta, \xi))$ for all $\Delta \in [-1, 1]$.

Recall that $\tilde{\pi} \equiv \phi^{-1}(\pi^* - .5)$. After rearranging, these two assumptions are equivalent to imposing

Assumption 1. *ϕ and $\tilde{\pi}$ are such that*

$$\frac{1}{2\phi} \geq \frac{1}{2\psi} + 1$$
$$\tilde{\pi} \leq \frac{1}{2\psi} - 1$$

Notice that this assumption implies that $2\psi > 1$.

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Proof of Proposition 1

For any voter, the probability of being decisive for the assignment of party A's first seat, conditional on being pivotal, and given other voters' computed $\Delta(l_A, l_B) \equiv V_A(l_A, l_B) - V_B(l_A, l_B)$ is given by Equation 15. Substituting (13) into (15), we obtain that $\Delta(l_A, l_B)$ must be a root of the mapping $\mathcal{V} : [-1, 1] \to [-1, 1]$, where

$$\mathcal{V}(\Delta) \equiv \frac{l_A(2) - l_B(1)}{2} + \frac{l_A(1) + l_B(1) - l_A(2) - l_B(2)}{2} \frac{\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi})}{2\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi}) + \theta \tilde{f}(\Delta - \tilde{\pi})} - \Delta.$$

We argue that when θ is small enough, this mapping has a unique root. To see this, notice that we have have four possible cases:

I: $l_A(1) = l_B(1) = 2$, in which case

$$\mathcal{V}(\Delta) = \mathcal{V}_I(\Delta) \equiv -\frac{1}{2} + \frac{\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi})}{2\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi}) + \theta \tilde{f}(\Delta - \tilde{\pi})} - \Delta.$$

II: $l_A(1) = l_B(1) = 1$, in which case

$$\mathcal{V}(\Delta) = \mathcal{V}_{II}(\Delta) \equiv \frac{1}{2} - \frac{\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi})}{2\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi}) + \theta \tilde{f}(\Delta - \tilde{\pi})} - \Delta.$$

III: $l_A(1) = l_B(2) = 2$, in which case $\mathcal{V}(\Delta) = -\Delta$, which implies $\Delta(l_A, l_B) = 0$.

IV: $l_A(2) = l_B(1) = 2$, in which case $\mathcal{V}(\Delta) = -\Delta$, which implies $\Delta(l_A, l_B) = 0$.

It is immediate that in cases III and IV there is a unique root. Let's consider case I (case II is analogous). We have that

$$\frac{d\mathcal{V}}{d\Delta} = \theta \frac{(1-\theta)\psi\big[\tilde{f}'(\Delta + \tilde{\pi}) - \tilde{f}'(\Delta - \tilde{\pi})\big] + \theta\big[\tilde{f}'(\Delta + \tilde{\pi})\tilde{f}(\Delta - \tilde{\pi}) - \tilde{f}'(\Delta - \tilde{\pi})\tilde{f}(\Delta + \tilde{\pi})\big]}{\Big[2\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi}) + \theta \tilde{f}(\Delta - \tilde{\pi})\Big]^2}.$$

Since $\tilde{f}'$ is finite, $\lim_{\theta \to 0} \frac{d\mathcal{V}}{d\Delta} = 0$, there exists $\theta^* \in (0, 1]$ such that $\forall \theta \leq \theta^*$, $\frac{d\mathcal{V}}{d\Delta} < 1$.
Finally, we show that $\tau^{CL}(\Delta(l_A, l_B)) > \frac{1}{2}$, where

$$\tau^{CL}(\Delta) = \frac{\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi})}{2\psi(1-\theta) + \theta \tilde{f}(\Delta + \tilde{\pi}) + \theta \tilde{f}(\Delta - \tilde{\pi})}.$$

Suppose, to the contrary, $\tau^{CL} \leq \frac{1}{2}$. Then we must have $\tilde{f}(\tilde{\pi}+\Delta) \leq \tilde{f}(-\tilde{\pi}+\Delta)$. Since $\tilde{f}$ is single peaked and symmetrically distributed around $\overline{\xi} > 0$, it must be that $\Delta(l_A, l_B) \geq \overline{\xi}$.
First, notice that under cases III and IV, we obtain $\Delta(l_A, l_B) = 0 < \overline{\xi}$, a contradiction.
Second, consider case I. In this case, $\Delta(l_A, l_B)$ is the unique root of $\mathcal{V}_I = \tau^{CL}(\Delta) - \frac{1}{2} - \Delta$. Since we must have $\tau^{CL}(\Delta) - \frac{1}{2} \leq 0$, we obtain $\Delta \leq 0 < \overline{\xi}$, a contradiction.
Finally, consider case II. In this case, $\Delta(l_A, l_B)$ is the unique root of $\mathcal{V}_{II} = \frac{1}{2} - \tau^{CL}(\Delta) - \Delta$. Notice that $\mathcal{V}_{II}(\Delta)$ is continuous. Moreover, we have $\mathcal{V}_{II}(-1) = \frac{3}{2} - \tau^{CL}(-1) > \frac{1}{2} > 0$ and $\mathcal{V}_{II}(0) = \frac{1}{2} - \tau^{CL}(0) < 0$—by symmetry of $\tilde{f}$ and $\overline{\xi} > 0$. By the intermediate value theorem, it must be that the unique root of $\mathcal{V}_{II}$ is in $(-1, 0)$, which again contradicts that $\Delta(l_A, l_B) \geq \overline{\xi} > 0$. This completes the proof. $\square$

Proof of Observation 2

Applying the implicit function theorem to $\mathcal{V} = 0$, we obtain

$$\frac{\partial \Delta(l_A, l_B)}{\partial l_A(1)} \Bigg/ \frac{\partial(-\Delta(l_A, l_B))}{\partial l_B(1)} = -\frac{\frac{\partial \mathcal{V}}{\partial l_A(1)}}{\frac{\partial \mathcal{V}}{\partial \Delta}} \Bigg/ -\frac{\frac{\partial \mathcal{V}}{\partial l_B(1)}}{\frac{\partial \mathcal{V}}{\partial(-\Delta)}} = \frac{\partial \mathcal{V}}{\partial l_A(1)} \Bigg/ \frac{\partial(-\mathcal{V})}{\partial l_B(1)} = \frac{\tau^{CL}}{1-\tau^{CL}} > 1.$$

A Model of Open List PR

To model open list PR in the simplest possible way, we assume that each party has a mass λ of *loyal voters* who cast a ballot for the *lower-appeal* candidate within their party with probability one. The rest of the electorate (share $1 - 2\lambda$) is made up by independent voters, whose behavior and and preferences over candidate appeal is the same as the voters in our baseline model: we denote by V_J the value of voting for one's preferred candidate form party J. An independent voter i votes for party B iff

$$V_B + \xi + \sigma_i \geq V_A.$$

Independent voters' *within party* preferences are entirely driven by appeal: with probability one, they vote for the higher-appeal candidate.

These assumptions require a more precise restatement of the notion of appeal q as a candidate's general ability to advance *local issues*; this is consistent with the idea that some politicians specialize in pursuing a party's programmatic policy goals, while others are more appealing owing to broader *personal vote-earning attributes* Åsa von Schoultz and Shugart (2018). As a result, we can interpret loyalists as programmatic-oriented voters and independents as less programmatic-oriented voters.

Let v_A the total vote share of party A. We have that

$$v_A = \lambda + (1-2\lambda)\left(\frac{1}{2} + \phi(V_A - V_B - \xi)\right)$$

We also assume that when independent voters equally divide their vote across parties, both high-quality candidates get elected: for all $\Delta \in [-1, 1]$

$$\lambda < (1-2\lambda)\left(\frac{1}{2} + \phi\Delta\right).$$

This assumption also implies that when a party wins both seats, the high-appeal candidates must obtain more preference votes (i.e. $\lambda < 1/4$). We also assume that λ is large enough so that loyal voters *can* determine the intra-party allocation of seats: for all $\Delta \in [-1, 1]$

$$\lambda > (1-2\lambda)\left(\frac{1}{2} + \phi\Delta - \frac{\phi}{2\psi}\right).$$

As before, there are two realizations of the aggregate shock ξ_1^{OL} and ξ_2^{OL} where a voter's vote is pivotal for the allocation of party A's first and second seat. However, there are two additional thresholds ξ_A^{OL} and ξ_B^{OL} so that when $\xi = \xi_J^{OL}$, a voter is pivotal for allocation of the seat between the higher-appeal candidate ($q = 2$) and the lower-appeal candidate ($q = 1$).

Under the assumptions, we have that $\xi_2^{OL} < \xi_B^{OL} < \xi_A^{OL} < \xi_1^{OL}$. As before, we assume that an independent voter's payoff is the average appeal among elected representatives, and that a high-appeal politician yields a payoff of 2 while a low-appeal politicians yields a payoff of 1.

We now proceed to derive the values $V_A(l_A, l_B)$ and $V_B(l_A, l_B)$. Let $\Pr(pivot) = \Pr(\xi \in \{\xi_2^{OL}, \xi_B^{OL}, \xi_A^{OL}, \xi_1^{OL}\})$

$$V_A = \frac{\Pr(\xi = \xi_2^{OL})}{\Pr(pivot)}\frac{3}{2} + \frac{\Pr(\xi = \xi_B^{OL})}{\Pr(pivot)}\frac{3}{2} + \frac{\Pr(\xi = \xi_A^{OL})}{\Pr(pivot)}\frac{4}{2} + \frac{\Pr(\xi = \xi_1^{OL})}{\Pr(pivot)}\frac{3}{2}.$$

In words: voting for A affects the quality of representation only when (i) party B gets more votes than A, but (ii) enough independent preference votes are cast so that voting for the high-appeal candidate of party A is decisive for the outcome of the intra-party contest for the single seat obtained by party A.

Similarly, we have

$$V_B = \frac{\Pr(\xi = \xi_2^{OL})}{\Pr(pivot)}\frac{3}{2} + \frac{\Pr(\xi = \xi_B^{OL})}{\Pr(pivot)}\frac{4}{2} + \frac{\Pr(\xi = \xi_A^{OL})}{\Pr(pivot)}\frac{3}{2} + \frac{\Pr(\xi = \xi_1^{OL})}{\Pr(pivot)}\frac{3}{2}.$$

We then obtain

$$\begin{aligned} \Delta \equiv V_A - V_B &= -\frac{\Pr(\xi = \xi_B^{OL})}{\Pr(pivot)}\frac{1}{2} + \frac{\Pr(\xi = \xi_A^{OL})}{\Pr(pivot)}\frac{1}{2} \\ &= \frac{\theta\tilde{f}(\Delta + \tilde{\lambda}) - \theta\tilde{f}(\Delta - \tilde{\lambda})}{4\psi(1-\theta) + \theta[\tilde{f}(\Delta - \tilde{\pi}) + \tilde{f}(\Delta - \tilde{\lambda}) + \tilde{f}(\Delta + \tilde{\lambda})\tilde{f}(\Delta + \tilde{\pi})]}, \end{aligned}$$

where:

$$\begin{aligned} \xi_2^{OL} &\equiv \Delta - \frac{\pi^* - \frac{1}{2}}{\phi(1-2\lambda)} \equiv \Delta - \tilde{\pi} \\ \xi_B^{OL} &\equiv \Delta - \frac{\frac{\lambda}{1-2\lambda} - \frac{1}{2}}{\phi} \equiv \Delta - \tilde{\lambda} \\ \xi_A^{OL} &\equiv \Delta + \frac{\frac{\lambda}{1-2\lambda} - \frac{1}{2}}{\phi} \equiv \Delta + \tilde{\lambda} \\ \xi_1^{OL} &\equiv \Delta + \frac{\pi^* - \frac{1}{2}}{\phi(1-2\lambda)} \equiv \Delta + \tilde{\pi}. \end{aligned}$$

This implies that $\Delta \in (0, \overline{\xi})$ (that $\Delta \leq \overline{\xi}$ has to be true, because otherwise $\Delta > \overline{\xi} \Rightarrow \Delta < 0$) and voters pay relatively more attention to the situation in which they are pivotal for the election of the lower-appeal candidate ($q = 1$) of party A than the lower-appeal candidate of party B.

References

Åsa von Schoultz and Matthew S. Shugart. 2018. "Modeling intraparty competition under OLPR: Contextualizing the personal vote." unpublished.