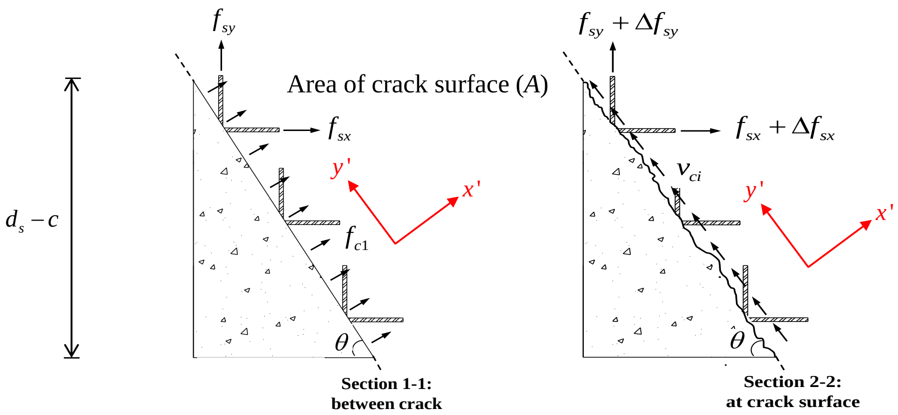
APPENDIX

➢ Detailed calculation procedures for estimating local tensile stress increase (Δf_{sx})



Equilibrium condition (y' direction)

$$f_{sx}A_s\cos\theta + f_{sy}A_v\sin\theta = v_{ci}A - \left(f_{sx}+\Delta f_{sx}\right)A_s\cos\theta + \left(f_{sy}+\Delta f_{sy}\right)A_v\sin\theta$$

$$\rightarrow v_{ci}A = \Delta f_{sx}A_s\cos\theta - \Delta f_{sy}A_v\sin\theta$$

$$\rightarrow v_{ci} = \left(\Delta f_{sx}\frac{A_s}{A\sin\theta} - \Delta f_{sy}\frac{A_v}{A\cos\theta}\right)\sin\theta\cos\theta$$

$$= \left(\Delta f_{sx}\frac{A_s}{b_w\left(d-c\right)} - \Delta f_{sv}\rho_{sv}\right)\sin\theta\cos\theta$$

$$= \left(\Delta f_{sx}\rho_{sx,eff} - \Delta f_{sv}\rho_{sv}\right)\sin\theta\cos\theta$$

$$= \boxed{\left(\rho_{sx,eff}\Delta f_{sx} - \rho_{sv}f_{vy}\right)\sin\theta\cos\theta}$$

Fig. A1 – Stress distribution between adjacent cracks and corresponding local stresses at crack

Fig. A1 shows a detailed derivation process of Eq. (2), and the similar results of the equilibrium conditions can be found in previous researches (Vecchio and Collins, 1986; Vecchio, 2010; Pang and Hsu, 1996; Hsu and Mo, 2010). In addition, as shown in Fig. A2, the local tensile stress increase in steel reinforcement (Δf_{sx}) should be equilibrated by the bond stress, as follows:

$$\Delta f_{sx} = f_{sx,\max} - f_{sx,\min} = 2\tau_x S_{mx}/d_b \quad \text{(A1)}$$

where τ_x is the bond stress, S_{mx} is the flexural crack spacing, d_b is the diameter of the longitudinal reinforcement.

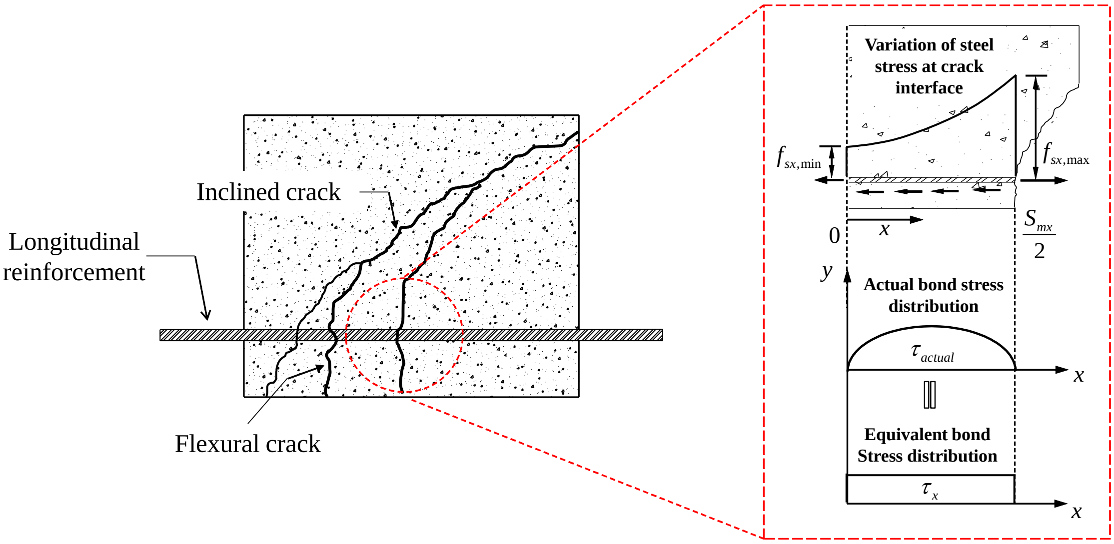


Fig. A2 – Bond stress distribution between adjacent cracks

If the concrete compressive strain at the extreme top fiber of the section (ε_t) is selected, the tensile stress ($f_{sx,\max}$) at the crack surface can be obtained by performing a non-linear flexural analysis at the considered critical section, as shown in Fig. A3. In the flexural analysis, Collins model shown in Fig. A4(a) was used for the stress-strain relationship of concrete in compression, and the elastic-linear work-hardening model shown in Fig. A4(b) was used for the constitutive laws of steel reinforcement. For the bond-slip relationship between the steel bar and surrounding concrete, the CEB-FIP model code was adopted, as shown in Fig. A4(c).

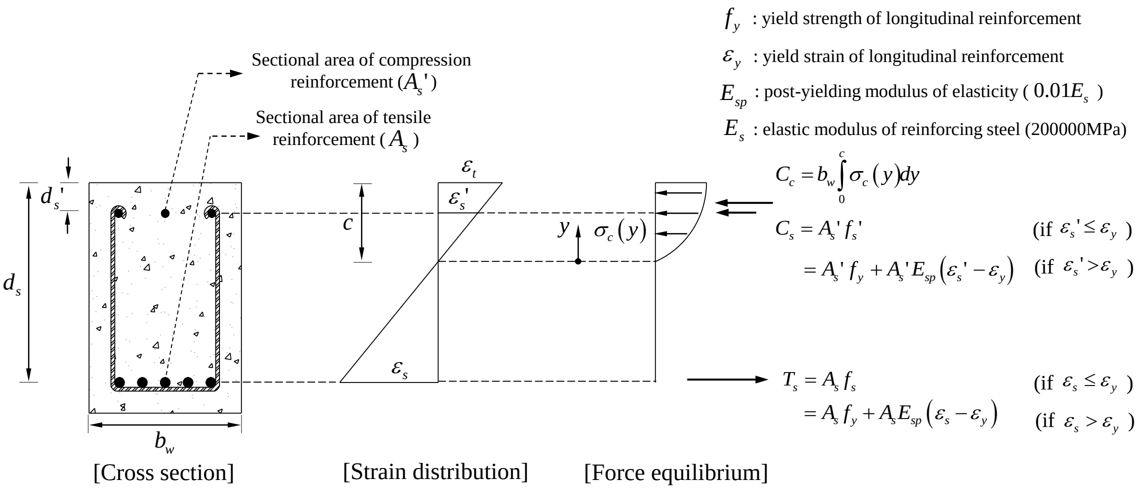
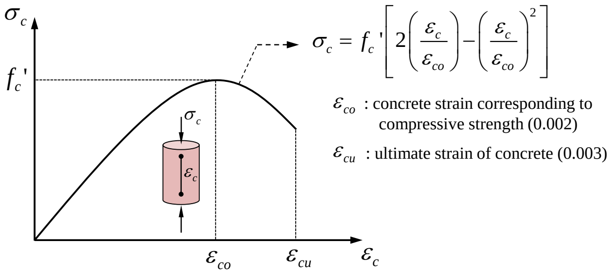
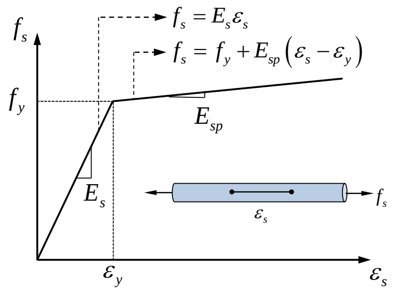


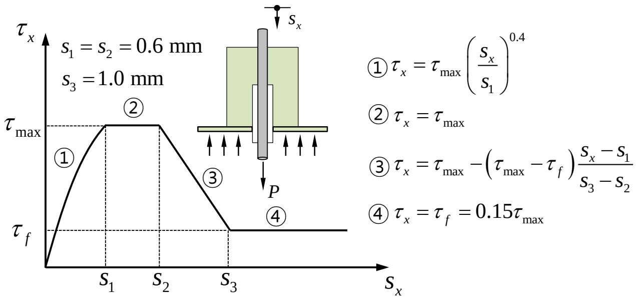
Fig. A3 – Non-linear flexural analysis model



(a) Concrete



(b) Steel reinforcement



(c) Bond between steel reinforcement and concrete

Fig. A4 – Constitutive models used in flexural analysis

From Eq. (3) and Fig. A2, the elongation of steel reinforcement (e_s) can be calculated, as follows:

$$e_s = \frac{S_{mx}}{E_s}\left(f_{sx,\max} - \frac{\tau_x S_{mx}}{d_b}\right) \quad \text{(when } f_{sx,\max} \le f_y\text{)} \qquad \text{(A2-a)}$$

$$e_s = \frac{S_{mx}}{E_s}\left(f_{sx,\max} - \frac{\tau_x S_{mx}}{d_b}\right) + \frac{S_{mx}}{E_{sp}}\left(f_{sx,\max} - f_y\right) \quad \text{(when } f_{sx,\max} > f_y\text{)} \qquad \text{(A2-b)}$$

If the slip (s_x) is assumed, the bond stresses τ_{x1} and τ_{x2} can be obtained from the bond stress-slip relationship shown in Fig. A4 and Eqs. (3) and (A2), respectively. Consequently, the local tensile stress increase in steel reinforcement (Δf_{sx}) can be estimated by iterating previous calculation process mentioned above until the τ_{x1} and τ_{x2} are converged. In Fig. A5, the analysis flow to estimate Δf_{sx} is summarized.

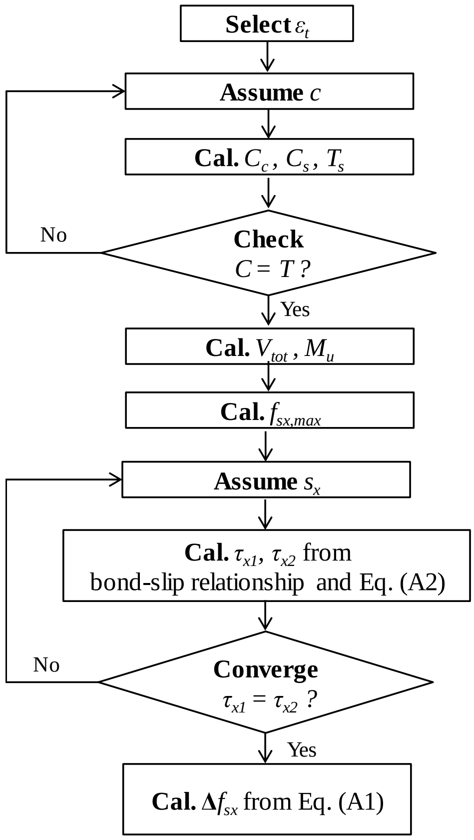


Fig. A5 – Analysis flow chart for estimating Δf_{sx}