Supplementary Material

Minimization and Regression Method

As noted in the main text, other parameter estimation approaches may also be appropriate for estimating representative intrinsic period given group illuminance-response curve data. Therefore, we also considered minimization and regression methods that seek to find an intrinsic period value $\hat{\tau}$ that minimizes the difference between functions of both light intensity and the predicted or actual intrinsic period:

$$\min_{\hat{\tau}} |f_{data}(light\ intensity, \tau) - f(light\ intensity, \hat{\tau})|$$

where the light intensities (lux) are the same for both functions and $\hat{\tau}$ is the unknown intrinsic period that is estimated using minimization. For the synthetic data, τ or mean τ are known and may be compared to the estimated representative intrinsic period as well as the MCMC estimate. For the experimental data, we used known phase shift data and compared to the MCMC estimates only. We considered three functions, *f*: (1) a pointwise comparison ($f = phase\ shift(light\ intensity)$), (2) a linear comparison ($f = m * (light\ intensity) + b$), and (3) a log linear comparison ($f = m * (\log_{10}(light\ intensity) + b$). For the second and third functions, the (synthetic or experimental) phase shift data was first fitted with the equation, the model was used to simulate phase shifts, and the resulting phase shifts were fitted to the same equation. The fitted equations were then used to determine the absolute difference between the functions evaluated at τ and $\hat{\tau}$ and, ultimately, find an intrinsic period that minimizes the absolute difference. Minimization was implemented using the built in MATLAB function **fminsearch**.

Generally, the minimization and regression approaches were similar for all functions and robustly estimated representative intrinsic period for the synthetic data for which τ or mean τ were known (**Table S1)**. Slight differences were present in for estimates using the experimental data where more variability was present. All estimates were using the minimization and regression approach were similar to the results obtained using the mean of the twelve MCMC runs (**Table S1**).

Table S1: The estimated intrinsic periods for the synthetic data sets using the three minimization and regression functions and the mean of the twelve MCMC runs of 10,000 iterations with a 5% burn-in are similar across τ values. The mean and standard deviations (std) of the synthetic data sets are also reported.

τ data value or τ data distribution	data mean	data std	point estimate	linear fit	log linear fit	mean of MCMC
23.7	23.7	0	23.6968	23.6966	23.6965	23.6979
N(23.7, 0.2)	23.7017	0.2148	23.7721	23.6849	23.7853	23.6969
24.2	24.2	0	24.2000	24.2000	24.2000	24.1967
N(24.2, 0.2)	24.2158	0.2054	24.3040	24.2183	24.2171	24.2121
N(24.2, 0.4)	24.2202	0.3829	24.1716	24.2473	24.2378	24.2303
24.6	24.6	0	24.6018	24.5995	24.6044	24.5999
N(24.6, 0.2)	24.6146	0.2125	24.6395	24.5916	24.5490	24.6140
24.9	24.9	0	24.8970	24.8956	24.8901	24.8893
Experimental data	--	--	24.3323	24.3093	24.1811	24.2674 (uniform)/24.2642 (normal)

Additional MCMC Results and Metrics

Table S2: MCMC estimates of τ for simulated single τ illuminance-response curves generated from $\tau = 23.7$, 24.2, 24.6, and 24.9 h were similar across initial chain values. Average τ values (±standard deviations) over twelve runs (four runs from each initial chain value $\tau = 23.9$, 24.1, and 24.7 h) are presented. Each MCMC run included 10,000 iterations with a 5% burn-in.

τ value used to generate data	**Estimated τ value with initial chain value τ =23.8 h**	**Estimated τ value with initial chain value τ =24.1 h**	**Estimated τ value with initial chain value τ =24.7 h**	**Average estimated τ value over all runs**
τ =23.7 h	23.6972 (±0.0408) h	23.6984 (±0.0405) h	23.6981 (±0.0409) h	23.6979 (±0.0407) h
τ =24.2 h	24.1967 (±0.0417) h	24.1970 (±0.0420) h	24.1966 (±0.0433) h	24.1967 (±0.0423) h
τ =24.6 h	24.5996 (±0.0433) h	24.6005 (±0.0433) h	24.5996 (±0.0433) h	24.5999 (±0.0433) h
τ =24.9 h	24.8891 (±0.0382) h	24.8891 (±0.0387) h	24.8897 (±0.0381) h	24.8893 (±0.0383) h

Table S3: Average 95% credible intervals of τ were similar across initial chain values and contained the τ value used to generate simulated single τ illuminance-response curves generated from $\tau = 23.7$, 24.2, 24.6, and 24.9 h. Average 95% credible intervals of τ over four runs from each initial chain value $\tau = 23.9$, 24.1, and 24.7 h are reported. Each MCMC run included 10,000 iterations with a 5% burn-in.

τ value used to generate data	**Initial Chain Value: τ =23.8 h**	**Initial Chain Value: τ =24.1 h**	**Initial Chain Value: τ =24.7 h**	**Average of all runs**
τ =23.7 h	[23.6163, 23.7771]	[23.6214, 23.7785]	[23.6193, 23.7785]	[23.6190, 23.7780]
τ =24.2 h	[24.1157, 24.2797]	[24.1140, 24.2785]	[24.1110, 24.2815]	[24.1136, 24.2799]
τ =24.6 h	[24.5146, 24.6847]	[24.5156, 24.6851]	[24.5145, 24.6828]	[24.5149, 24.6842]
τ =24.9 h	[24.8090, 24.9542]	[24.8089, 24.9562]	[24.8109, 24.9559]	[24.8096, 24.9554]

Table S4: MCMC estimates of τ for simulated multi-τ illuminance-response curves approximated average τ values and were similar across initial chain values. Simulated multi-τ illuminance-response curves were generated from τs drawn from $N(23.7, 0.2^2)$, $N(24.2, 0.2^2)$, $N(24.2, 0.4^2)$, and $N(24.6, 0.2^2)$. Average τ values (±standard deviations) from four runs from each initial chain value (τ = 23.9, 24.1, and 24.7 h) are presented. Each MCMC run included 10,000 iterations with a 5% burn-in.

τ distribution data was drawn from	**Initial Chain Value: τ =23.8 h**	**Initial Chain Value: τ =24.1 h**	**Initial Chain Value: τ =24.7 h**	**Average of all runs**
$N(23.7, 0.2^2)$	23.6970 (±0.0412) h	23.6974 (±0.0409) h	23.6963 (±0.0399) h	23.6969 (±0.0407) h
$N(24.2, 0.2^2)$	24.2122 (±0.0425) h	24.2127 (±0.0421) h	24.2114 (±0.0422) h	24.2121 (±0.0423) h
$N(24.2, 0.4^2)$	24.2310 (±0.0426) h	24.2301 (±0.0421) h	24.2298 (±0.0421) h	24.2303 (±0.0423) h
$N(24.6, 0.2^2)$	24.6140 (±0.0433) h	24.6141 (±0.0435) h	24.6138 (±0.0438) h	24.6140 (±0.0435) h

Table S5: Average 95% credible intervals of τ contained mean τ value used to generate simulated multi-τ illuminance-response curves and were similar across initial chain values. Average 95% credible intervals of τ were computed from four runs from each initial chain value ($\tau =$ 23.9, 24.1, and 24.7 h). Simulated multi-τ illuminance-response curves were generated from τs drawn from $N(23.7, 0.2^2)$, $N(24.2, 0.2^2)$, $N(24.2, 0.4^2)$, and $N(24.6, 0.2^2)$. Each MCMC run included 10,000 iterations with a 5% burn-in.

τ value data was generated from	**Initial Chain Value: τ =23.8 h**	**Initial Chain Value: τ =24.1 h**	**Initial Chain Value: τ =24.7 h**	**Average of all runs**
$N(23.7, 0.2^2)$	[23.6171, 23.7779]	[23.6164, 23.7781]	[23.6190, 23.7757]	[23.6175, 23.7772]
$N(24.2, 0.2^2)$	[24.1286, 24.2947]	[24.1314, 24.2966]	[24.1267, 24.2930]	[24.1289, 24.2948]
$N(24.2, 0.4^2)$	[24.1486, 24.3157]	[24.1486, 24.3137]	[24.1475, 24.3137]	[24.1482, 24.3144]
$N(24.6, 0.2^2)$	[24.5286, 24.6986]	[24.5297, 24.6995]	[24.5277, 24.6990]	[24.5286, 24.6990]

Table S6: MCMC estimates of τ for experimental illuminance-response curve data with a uniform ($U(23.5,\ 25)$) and a normal ($N(24.2, 0.2^2)$) prior were similar across initial chain values. Average τ values (±standard deviations) from four runs from each initial chain value ($\tau =$ 23.9, 24.1, and 24.7 h) are presented. Each MCMC run included 10,000 iterations with a 5% burn-in.

Prior distribution	**Initial Chain Value: τ =23.8 h**	**Initial Chain Value: τ =24.1 h**	**Initial Chain Value: τ =24.7 h**	**Average of all runs**
$U(23.5, 25)$	24.2668 (±0.0427) h	24.2673 (±0.0420) h	24.2680 (±0.0430) h	24.2674 (±0.0425) h
$N(24.2, 0.2^2)$	24.2636 (±0.0410) h	24.2648 (±0.0418) h	24.2643 (±0.0413) h	24.2642 (±0.0414) h

Table S7: Average 95% credible intervals of τ from four runs to estimate τ for experimental illuminance-response curve data with a uniform ($U(23.5,\ 25)$) and a normal ($N(24.2, 0.2^2)$) prior contains $\tau =$ 24.2h, the average estimated intrinsic period for healthy adults. Results were similar across initial chain values ($\tau =$ 23.9, 24.1, and 24.7 h). Each MCMC run included 10,000 iterations with a 5% burn-in.

Prior distribution	**Initial Chain Value: τ =23.8 h**	**Initial Chain Value: τ =24.1 h**	**Initial Chain Value: τ =24.7 h**	**Average of all runs**
$U(23.5, 25)$	[24.1823, 24.3506]	[24.1832, 24.3490]	[24.1845, 24.3521]	[24.1833, 24.3506]
$N(24.2, 0.2^2)$	[24.1833, 24.3447]	[24.1845, 24.3469]	[24.1828, 24.3458]	[24.1835, 24.3458]