Spectral Analysis of Guided Wave Propagation in Discretized Domains Under Local Interactions

Supplementary material

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1 Coefficient matrices for FE

Material coefficient matrices $\boldsymbol{Q}_i$ in Eqs. (11) and (14) of the manuscript, for the Finite Element method are given as

$$\mathbf{Q}_0 = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_1+\Delta_x^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_1}{3\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_1+\mu_1}{4} \\ \frac{\lambda_1+\mu_1}{4} & \frac{\Delta_x^2\cdot\lambda_1+2\cdot\Delta_x^2\cdot\mu_1+\Delta_y^2\cdot\mu_1}{3\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\mathbf{Q}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{Q}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{Q}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{Q}_4 = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_1-2\cdot\Delta_x^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\mu_1-\lambda_1}{4} \\ \frac{\lambda_1-\mu_1}{4} & -\frac{2\cdot\Delta_x^2\cdot\lambda_1+4\cdot\Delta_x^2\cdot\mu_1-\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\mathbf{Q}_5 = \begin{bmatrix} -\frac{2\cdot\Delta_y^2\cdot\lambda_1-\Delta_x^2\cdot\mu_1+4\cdot\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_1-\mu_1}{4} \\ \frac{\mu_1-\lambda_1}{4} & \frac{\Delta_x^2\cdot\lambda_1+2\cdot\Delta_x^2\cdot\mu_1-2\cdot\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\mathbf{Q}_6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{Q}_7 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{Q}_8 = \begin{bmatrix} -\frac{\Delta_y^2\cdot\lambda_1+\Delta_x^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} & \frac{-\lambda_1-\mu_1}{4} \\ \frac{-\lambda_1-\mu_1}{4} & -\frac{\Delta_x^2\cdot\lambda_1+2\cdot\Delta_x^2\cdot\mu_1+\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_0^{(TOP)} = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_1-2\cdot\Delta_x^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_1-\mu_1}{4} \\ \frac{\mu_1-\lambda_1}{4} & -\frac{2\cdot\Delta_x^2\cdot\lambda_1+4\cdot\Delta_x^2\cdot\mu_1-\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_1^{(TOP)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\hat{\mathbf{Q}}_2^{(TOP)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\hat{\mathbf{Q}}_3^{(TOP)} = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_4-2\cdot\Delta_x^2\cdot\mu_4+2\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\mu_4-\lambda_4}{4} \\ \frac{\lambda_4-\mu_4}{4} & -\frac{2\cdot\Delta_x^2\cdot\lambda_4+4\cdot\Delta_x^2\cdot\mu_4-\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_4^{(TOP)} = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_1+\Delta_y^2\cdot\lambda_4+\Delta_x^2\cdot\mu_1+\Delta_x^2\cdot\mu_4+2\cdot\Delta_y^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_4}{3\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_4-\lambda_1-\mu_1+\mu_4}{4} \\ \frac{\lambda_4-\lambda_1-\mu_1+\mu_4}{4} & \frac{\Delta_x^2\cdot\lambda_1+\Delta_x^2\cdot\lambda_4+2\cdot\Delta_x^2\cdot\mu_1+2\cdot\Delta_x^2\cdot\mu_4+\Delta_y^2\cdot\mu_1+\Delta_y^2\cdot\mu_4}{3\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_5^{(TOP)} = \begin{bmatrix} -\frac{\Delta_y^2\cdot\lambda_1+\Delta_x^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_1+\mu_1}{4} \\ \frac{\lambda_1+\mu_1}{4} & -\frac{\Delta_x^2\cdot\lambda_1+2\cdot\Delta_x^2\cdot\mu_1+\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_6^{(TOP)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\hat{\mathbf{Q}}_7^{(TOP)} = \begin{bmatrix} -\frac{\Delta_y^2\cdot\lambda_4+\Delta_x^2\cdot\mu_4+2\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} & \frac{-\lambda_4-\mu_4}{4} \\ \frac{-\lambda_4-\mu_4}{4} & -\frac{\Delta_x^2\cdot\lambda_4+2\cdot\Delta_x^2\cdot\mu_4+\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_8^{(TOP)} = \begin{bmatrix} -\frac{2\cdot\Delta_y^2\cdot\lambda_1+2\cdot\Delta_y^2\cdot\lambda_4-\Delta_x^2\cdot\mu_1-\Delta_x^2\cdot\mu_4+4\cdot\Delta_y^2\cdot\mu_1+4\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_4-\lambda_1+\mu_1-\mu_4}{4} \\ \frac{\lambda_1-\lambda_4-\mu_1+\mu_4}{4} & \frac{\Delta_x^2\cdot\lambda_1+\Delta_x^2\cdot\lambda_4+2\cdot\Delta_x^2\cdot\mu_1+2\cdot\Delta_x^2\cdot\mu_4-2\cdot\Delta_y^2\cdot\mu_1-2\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_0^{(BOT)} = \begin{bmatrix} -\frac{\Delta_y^2\cdot\lambda_1+\Delta_x^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} & \frac{-\lambda_1-\mu_1}{4} \\ \frac{-\lambda_1-\mu_1}{4} & -\frac{\Delta_x^2\cdot\lambda_1+2\cdot\Delta_x^2\cdot\mu_1+\Delta_y^2\cdot\mu_1}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_1^{(BOT)} = \begin{bmatrix} -\frac{\Delta_y^2\cdot\lambda_2+\Delta_x^2\cdot\mu_2+2\cdot\Delta_y^2\cdot\mu_2}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_2+\mu_2}{4} \\ \frac{\lambda_2+\mu_2}{4} & -\frac{\Delta_x^2\cdot\lambda_2+2\cdot\Delta_x^2\cdot\mu_2+\Delta_y^2\cdot\mu_2}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_2^{(BOT)} = \begin{bmatrix} -\frac{\Delta_y^2\cdot\lambda_3+\Delta_x^2\cdot\mu_3+2\cdot\Delta_y^2\cdot\mu_3}{6\cdot\Delta_x\cdot\Delta_y} & \frac{-\lambda_3-\mu_3}{4} \\ \frac{-\lambda_3-\mu_3}{4} & -\frac{\Delta_x^2\cdot\lambda_3+2\cdot\Delta_x^2\cdot\mu_3+\Delta_y^2\cdot\mu_3}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_3^{(BOT)} = \begin{bmatrix} -\frac{\Delta_y^2\cdot\lambda_4+\Delta_x^2\cdot\mu_4+2\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda 4+\mu_4}{4} \\ \frac{\lambda_4+\mu_4}{4} & -\frac{\Delta_x^2\cdot\lambda_4+2\cdot\Delta_x^2\cdot\mu_4+\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_4^{(BOT)} = \begin{bmatrix} -\frac{2\cdot\Delta_y^2\cdot\lambda_1+2\cdot\Delta_y^2\cdot\lambda_4-\Delta_x^2\cdot\mu_1-\Delta_x^2\cdot\mu_4+4\cdot\Delta_y^2\cdot\mu_1+4\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_1-\lambda_4-\mu_1+\mu_4}{4} \\ \frac{\lambda_4-\lambda_1+\mu_1-\mu_4}{4} & \frac{\Delta_x^2\cdot\lambda_1+\Delta_x^2\cdot\lambda_4+2\cdot\Delta_x^2\cdot\mu_1+2\cdot\Delta_x^2\cdot\mu_4-2\cdot\Delta_y^2\cdot\mu_1-2\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_5^{(BOT)} = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_1+\Delta_y^2\cdot\lambda_2-2\cdot\Delta_x^2\cdot\mu_1-2\cdot\Delta_x^2\cdot\mu_2+2\cdot\Delta_y^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_2}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_2-\lambda_1+\mu_1-\mu_2}{4} \\ \frac{\lambda_1-\lambda_2-\mu_1+\mu_2}{4} & -\frac{2\cdot\Delta_x^2\cdot\lambda_1+2\cdot\Delta_x^2\cdot\lambda_2+4\cdot\Delta_x^2\cdot\mu_1+4\cdot\Delta_x^2\cdot\mu_2-\Delta_y^2\cdot\mu_1-\Delta_y^2\cdot\mu_2}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_6^{(BOT)} = \begin{bmatrix} -\frac{2\cdot\Delta_y^2\cdot\lambda_2+2\cdot\Delta_y^2\cdot\lambda_3-\Delta_x^2\cdot\mu_2-\Delta_x^2\cdot\mu_3+4\cdot\Delta_y^2\cdot\mu_2+4\cdot\Delta_y^2\cdot\mu_3}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_3-\lambda_2+\mu_2-\mu_3}{4} \\ \frac{\lambda_2-\lambda_3-\mu_2+\mu_3}{4} & \frac{\Delta_x^2\cdot\lambda_2+\Delta_x^2\cdot\lambda_3+2\cdot\Delta_x^2\cdot\mu_2+2\cdot\Delta_x^2\cdot\mu_3-2\cdot\Delta_y^2\cdot\mu_2-2\cdot\Delta_y^2\cdot\mu_3}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_7^{(BOT)} = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_3+\Delta_y^2\cdot\lambda_4-2\cdot\Delta_x^2\cdot\mu_3-2\cdot\Delta_x^2\cdot\mu_4+2\cdot\Delta_y^2\cdot\mu_3+2\cdot\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} & \frac{\lambda_4-\lambda_3+\mu_3-\mu_4}{4} \\ \frac{\lambda_3-\lambda_4-\mu_3+\mu_4}{4} & -\frac{2\cdot\Delta_x^2\cdot\lambda_3+2\cdot\Delta_x^2\cdot\lambda_4+4\cdot\Delta_x^2\cdot\mu_3+4\cdot\Delta_x^2\cdot\mu_4-\Delta_y^2\cdot\mu_3-\Delta_y^2\cdot\mu_4}{6\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$

$$\hat{\mathbf{Q}}_8^{(BOT)} = \begin{bmatrix} \frac{\Delta_y^2\cdot\lambda_1+\Delta_y^2\cdot\lambda_2+\Delta_y^2\cdot\lambda_3+\Delta_y^2\cdot\lambda_4+\Delta_x^2\cdot\mu_1+\Delta_x^2\cdot\mu_2+\Delta_x^2\cdot\mu_3+\Delta_x^2\cdot\mu_4+2\cdot\Delta_y^2\cdot\mu_1+2\cdot\Delta_y^2\cdot\mu_2+2\cdot\Delta_y^2\cdot\mu_3+2\cdot\Delta_y^2\cdot\mu_4}{3\cdot\Delta_x\cdot\Delta_y} \\ \frac{\lambda_1-\lambda_2+\lambda_3-\lambda_4+\mu_1-\mu_2+\mu_3-\mu_4}{4}; \\ \frac{\lambda_1-\lambda_2+\lambda_3-\lambda_4+\mu_1-\mu_2+\mu_3-\mu_4}{4} \\ \frac{\Delta_x^2\cdot\lambda_1+\Delta_x^2\cdot\lambda_2+\Delta_x^2\cdot\lambda_3+\Delta_x^2\cdot\lambda_4+2\cdot\Delta_x^2\cdot\mu_1+2\cdot\Delta_x^2\cdot\mu_2+2\cdot\Delta_x^2\cdot\mu_3+2\cdot\Delta_x^2\cdot\mu_4+\Delta_y^2\cdot\mu_1+\Delta_y^2\cdot\mu_2+\Delta_y^2\cdot\mu_3+\Delta_y^2\cdot\mu_4}{3\cdot\Delta_x\cdot\Delta_y} \end{bmatrix}$$