SUPPLEMENTARY MATERIAL

Insights into traumatic brain injury from MRI of harmonic brain motion

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Mathematical details of the data analysis methods to estimate propagation direction and strain are included here for completeness.

S1. Amplitude-weighted propagation direction

To identify prominent directions of propagation for harmonic waves with frequency ω, the following steps are implemented.

1. Each scalar component $u(\boldsymbol{X},t)$ is Fourier transformed in time to extract the Fourier coefficient field, $U(\boldsymbol{X})$, where $u(\boldsymbol{X},t) = \mathrm{Re}\,[U(\boldsymbol{X})\,\exp(i\omega t)]$ (the physical displacement is the real part of the complex expression).

2. The $U(\boldsymbol{X})$ field is further decomposed into harmonic functions of space, each with a different 3D wavenumber vector, $\boldsymbol{k}$, as :

$$U(\boldsymbol{X}) = \sum_{\mathrm{m}=1}^{\mathrm{M}} \sum_{\mathrm{n}=1}^{\mathrm{N}} \sum_{\mathrm{p}=1}^{\mathrm{P}} A_{mnp} \exp\left(-i\boldsymbol{k}_{mnp} \cdot \boldsymbol{X}\right) \quad \text{(S1.1)}$$

   or, alternatively

$$U(\boldsymbol{X}) = \sum_{\mathrm{m}=1}^{\mathrm{M}} \sum_{\mathrm{n}=1}^{\mathrm{N}} \sum_{\mathrm{p}=1}^{\mathrm{P}} A_{mnp} \exp\left(-i(k_m x + k_n y + k_p z)\right). \quad \text{(S1.2)}$$

   The coefficients A_{mnp} (the "k-space" components of the image) are obtained by spatial Fourier transform of the $U(\boldsymbol{X})$ field.

3. Then, a directional spatial filter is used to isolate plane wave components inside a conical sector centered around the vector, $\boldsymbol{n}_q$. First the angle of every wavenumber vector, $\boldsymbol{k}_{mnp}$, relative to this direction is defined as $\theta_{nmpq} = \mathrm{acos}(\boldsymbol{n}_q \cdot \boldsymbol{n}_{mnp})$ where $\boldsymbol{n}_{mnp} = \frac{\boldsymbol{k}_{mnp}}{|\boldsymbol{k}_{mnp}|}$. The directional spatial filter is defined by,

$$f(\theta_{nmpq}) = \begin{cases} \cos^2 4\theta_{nmpq}, & |\theta_{nmpq}| \le \pi/8 \\ 0 & |\theta_{nmpq}| > \pi/8 \end{cases} \quad \text{(S1.3)}$$

4. The directionally-filtered scalar field

$$U_q(\boldsymbol{X}) = \sum_{\mathrm{m}=1}^{\mathrm{M}} \sum_{\mathrm{n}=1}^{\mathrm{N}} \sum_{\mathrm{p}=1}^{\mathrm{P}} f\left(\theta_{nmpq}\right) A_{mnp} \exp\left(-i\boldsymbol{k}_{mnp} \cdot \boldsymbol{X}\right) \quad \text{(S1.4)}$$

   is the part of the original field explained by propagating waves with wavenumber vectors within the conical sector centered on $\boldsymbol{n}_q$. It is obtained by inverse Fourier transformation of the filtered k-space field.

5. Finally, the amplitude-weighted propagation direction vector field, $\boldsymbol{N}^{(\boldsymbol{u})}(\boldsymbol{X})$, is defined by from the vector sum of multiple unit vectors, $\boldsymbol{n}_q$, for $q = 1,2,3,..Q$, evenly distributed on the unit sphere, each weighted by the amplitude of the field filtered about that direction

$$\boldsymbol{N}^{(\boldsymbol{u})}(\boldsymbol{X}) = \sum_{\boldsymbol{q}=\mathbf{1}}^{\boldsymbol{Q}} \boldsymbol{n}_q |U_q(\boldsymbol{X})| \tag{S1.5}$$

S2. Strain tensor, octahedral shear strain and axonal strain

Displacement fields $(\boldsymbol{u}(\boldsymbol{X},t) = [u_{LR}(x,y,z,t), u_{AP}(x,y,z,t), u_{SI}(x,y,z,t)]$, were differentiated with respect to spatial coordinates (x,y,z) by analytically calculating the derivatives of polynomial functions fitted to the displacement data (26). A matrix (3x3) representation of the strain tensor in Voigt notation (27) was constructed at each voxel from the appropriate derivatives at that voxel. Using the shorthand notation: $u = u_{LR}$, $v = u_{AP}$, and $w = u_{SI}$, we write the strain tensor as:

$$\boldsymbol{\epsilon}(x,y,z) = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{xy} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{xz} & \epsilon_{yz} & \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)/2 & \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)/2 \\ \text{(sym.)} & \frac{\partial v}{\partial y} & \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)/2 \\ \text{(sym.)} & \text{(sym.)} & \frac{\partial w}{\partial z} \end{bmatrix} \tag{S2.1}$$

Octahedral shear strain (OSS) is then defined as:

$$\epsilon_{oss} = \frac{2}{3}\sqrt{\left(\epsilon_{xx} - \epsilon_{yy}\right)^2 + \left(\epsilon_{yy} - \epsilon_{zz}\right)^2 + (\epsilon_{zz} - \epsilon_{xx})^2 + 6(\epsilon_{xy}^2 + \epsilon_{yz}^2 + \epsilon_{xz}^2)} \tag{S2.3}$$

Strain, ϵ_a, in the axonal fiber direction was estimated from the strain tensor and the unit vector in the fiber direction. The unit vector in the fiber direction, $\boldsymbol{a} = [a_x, a_y, a_z]$, was estimated as the direction of maximal diffusivity obtained from diffusion tensor imaging (DTI). The fiber stretch is then defined by the expression:

$$\epsilon_a = \boldsymbol{a}^T \cdot \boldsymbol{\epsilon} \cdot \boldsymbol{a} = a_x\epsilon_{xx}^2 + a_y\epsilon_{yy}^2 + a_z\epsilon_{zz}^2 + 2a_x\epsilon_{xy}\epsilon_{xz} + 2a_y\epsilon_{xy}\epsilon_{yz} + 2a_z\epsilon_{xz}\epsilon_{yz} \tag{S2.4}$$