Appendix A

Full System State Dependent Matrix Coefficients

$$\dot{x} = \begin{bmatrix} a_{11}(x) & \cdots & a_{115}(x) \\ \vdots & \ddots & \vdots \\ a_{151}(x) & \cdots & a_{1515}(x) \end{bmatrix} x + \begin{bmatrix} b_{11}(x) & \cdots & b_{14}(x) \\ \vdots & \ddots & \vdots \\ b_{151}(x) & \cdots & b_{154}(x) \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_{lon} \\ \theta_{lat} \\ \theta_{0_{TR}} \end{bmatrix}$$

where

$$x = [U_b \; V_b \; W_b \; p \; q \; r \; \phi \; \theta \; \psi \; \dot{\beta}_0 \; \dot{\beta}_{1c} \; \dot{\beta}_{1s} \; \beta_0 \; \beta_{1c} \; \beta_{1s}]^\top$$

$$a_{11}(x) = -\frac{1}{M_{body}} \left[\frac{c_3}{4} + \left(\frac{c_2}{8}\right) \beta_{1c}^2 + \left(\frac{c_4}{2}\right) |U_b| + \left(\frac{c_2}{16} e\right) \beta_{1s}^2 + \left(\frac{3c_2}{16} e\right) \beta_{1s}^2 \right]$$

$$a_{12}(x) = r + \frac{1}{M_{body}} \left[\left(\frac{c_2}{8}\right) \beta_{1s}\beta_{1c} \right], a_{15}(x) = -W_b$$

$$a_{18}(x) = -g\frac{\sin(\theta)}{\theta}, a_{113}(x) = -\frac{1}{M_{body}} \left[\left(\frac{c_1}{8}\right) e \right] \beta_{1s}$$

$$a_{114}(x) = \frac{\lambda_{MR}}{M_{body}} \left[\left(\frac{3c_1}{8}\right) - \left(\frac{5c_1}{8}\right) e^2 - \left(\frac{c_1}{4}\right) e \right] \beta_{1s}$$

$$a_{115}(x) = -\frac{1}{M_{body}}\left[\left(\frac{2c_1}{25}\right)\right]\beta_0$$

$$a_{21}(x) = -\frac{1}{M_{body}}\left(\frac{c_2}{8}\right)\left[\beta_{1s}\beta_{1c} + e\beta_{1s}\beta_{1c} - 6\lambda_{MR}\beta_0\right]$$

$$a_{22}(x) = -\frac{1}{M_{body}}\left[\left(\frac{c_3}{8}\right) + \left(\frac{c_2}{8}\right)\beta_{1s}^2 + \left(\frac{c_8}{8}\right)|V_b|\right]$$

$$a_{24}(x) = W_b, a_{26}(x) = -U_b, a_{27}(x) = -g\cos(\theta)\frac{\sin(\theta)}{\theta}$$

$$a_{214}(x) = -\frac{1}{M_{body}}\left[\left(\frac{2c_1}{25}\right)\beta_0\right]$$

$$\begin{aligned} a_{213}(x) = &\left[\frac{Cl_{TR}R_{TR}^3c_{TR}\lambda_{TR}\Omega_{TR}^2\rho}{2M_{body}}\right]\frac{1}{\beta_0} \\ &- \frac{1}{M_{body}}\left[\left(\frac{c_1}{8}\right)e\right]\beta_{1c} \end{aligned}$$

$$a_{215}(x) = \frac{\lambda_{MR}}{M_{body}}\left(\frac{c_1}{8}\right)\left[5e^2 - 3 - 2e\right]\beta_{1s}$$

$$a_{31}(x) = \frac{1}{M_{body}}\left(\frac{c_2}{4}\right)e\left[e-1\right]\beta_{1c}$$

$$a_{32}(x) = \frac{1}{M_{body}}\left(\frac{c_2}{4}e\right)\beta_{1s}$$

$$a_{33}(x) = -\frac{1}{M_{body}}\left(\frac{c_9}{2}\right)|V_b|, a_{34}(x) = -V_b, a_{35}(x) = U_b$$

$$\begin{aligned} a_{313}(x) = &\frac{1}{M_{body}}\left[\left(\frac{c_1}{4}\right)\lambda_{MR}\right]\frac{1}{\beta_0} + \left[g\cos(\phi)\cos(\theta)\right]\frac{1}{\beta_0} \\ &+ \frac{1}{M_{body}}\left[\left(\frac{c_1}{2}\right)e\lambda_{MR}\right]\frac{1}{\beta_0} - \frac{1}{M_{body}}\left[\left(\frac{3c_1}{4}\right)e^2\lambda_{MR}\right]\frac{1}{\beta_0} \end{aligned}$$

$$\begin{aligned} a_{41}(x) = &\frac{1}{I_{xx}}\left[\left(\frac{3c_2}{4}eR_{Z_{MR}}\lambda_{MR}\right)\beta_0 - \left(\frac{c_2}{4}eR_{Y_{MR}}\right)\beta_{1c}\right. \\ &\left.+ \left(\frac{c_2}{4}e^2R_{Y_{MR}}\right)\beta_{1c} - \frac{c_2}{8}\left(R_{Z_{MR}} - eR_{Z_{MR}}\right)\beta_{1s}\beta_{1c}\right] \end{aligned}$$

$$a_{42}(x) = \frac{1}{I_{xx}}\left[\left(\frac{c_3}{4} + \frac{c_2}{8}\beta_{1s}^2\right)R_{Z_{MR}} + \left(\frac{c_2}{4}eR_{Y_{MR}}\right)\beta_{1s}\right]$$

$$a_{45}(x) = \frac{I_{yy}}{I_{xx}}(r) - \frac{I_{zz}}{I_{xx}}(r)$$

$$\begin{aligned} a_{413}(x) = &\frac{1}{I_{xx}}\left[\frac{c_1}{4}(1+e)R_{Y_{MR}}\lambda_{MR} - \left(\frac{3c_1}{4}e^2R_{Y_{MR}}\lambda_{MR}\right)\right. \\ &\left.- \left(\frac{(Cl_{TR}R_{TR}^3c_{TR}\lambda_{TR}\Omega_{TR}^2\rho R_{Z_{TR}})}{2}\right)\right]\frac{1}{\beta_0} \end{aligned}$$

$$a_{414}(x) = \frac{1}{I_{xx}}\left[\left(\frac{2c_1}{25}R_{Z_{MR}}\right)\beta_0 + \left(\frac{c_1}{8}eR_{Z_{MR}}\right)\beta_0\right]$$

$$\begin{aligned} a_{415}(x) = &\frac{1}{I_{xx}}\left[\left(\frac{3c_1}{8}R_{Z_{MR}}\lambda_{MR}\right) - \left(\frac{5c_1}{8}e^2R_{Z_{MR}}\lambda_{MR}\right)\right. \\ &\left.+ \left(\frac{c_1}{4}eR_{Z_{MR}}\lambda_{MR}\right)\right] \end{aligned}$$

$$\begin{aligned} a_{51}(x) = &\frac{1}{I_{yy}}\left[\left(\frac{c_2}{4}eR_{X_{MR}}\right)\beta_{1c} - \frac{c_3}{4}R_{Z_{MR}} - \left(\frac{c_2}{8}R_{Z_{MR}}\right)\beta_{1c}^2\right. \\ &\left.- \left(\frac{c_2}{16}eR_{Z_{MR}}\right)\beta_{1s}^2 - \left(\frac{c_2}{4}e^2R_{X_{MR}}\right)\beta_{1c} - \left(\frac{3c_2}{16}eR_{Z_{MR}}\right)\beta_{1c}^2\right] \end{aligned}$$

$$a_{52}(x) = \frac{1}{I_{yy}}\left[\left(\frac{c_2}{8}R_{Z_{MR}}\right)\beta_{1c}\beta_{1s} - \left(\frac{c_2}{4}eR_{X_{MR}}\right)\beta_{1s}\right]$$

$$a_{54}(x) = -\frac{I_{xx}}{I_{yy}}(r), a_{56}(x) = \frac{I_{zz}}{I_{yy}}(p)$$

$$a_{513}(x) = \frac{1}{I_{yy}}\left[\left(\frac{3c_1}{4}e^2 - \frac{c_1}{2}e - \frac{1}{4}\right)R_{X_{MR}}\lambda_{MR}\right]\frac{1}{\beta_0}$$

$$a_{514}(x) = \frac{1}{I_{yy}}\left[\left(-\frac{5c_1}{8}e^2 + \frac{c_1}{4}e + \frac{3c_1}{8}\right)R_{Z_{MR}}\lambda_{MR}\right]$$

$$a_{515}(x) = -\frac{1}{I_{yy}}\left[\left(\frac{c_1}{8}e + \frac{2c_1}{25}\right)R_{Z_{MR}}\beta_0\right]$$

$$\begin{aligned} a_{61}(x) = &\frac{1}{I_{zz}}\left[\left(\frac{c_3}{4}R_{Y_{MR}}\right) + \left(\frac{3c_{12}}{32}U_b\right)\beta_{1c}^2 + \left(\frac{c_2}{8}R_{Y_{MR}}\right)\beta_{1c}^2\right. \\ &- \left(\frac{c_{13}}{4}R_{Y_{MR}}\lambda_{MR}\right)\beta_{1c} - \left(\frac{3c_2}{4}R_{X_{MR}}\lambda_{MR}\right)\beta_0 + \left(\frac{c_2}{16}eR_{Y_{MR}}\right) \\ &\beta_{1s}^2 + \left(\frac{3c_2}{16}eR_{Y_{MR}}\right)\beta_{1c}^2 + \left(\frac{3c_2}{16}eR_{Y_{MR}}\right) + \left(\frac{4c_{13}}{25}\right)\beta_0\beta_{1s} \\ &\left.+ \left(\frac{c_2}{8}R_{X_{MR}}\right)\beta_{1s}\beta_{1c} + \left(\frac{c_2}{8}eR_{X_{MR}}\right)\beta_{1s}\beta_{1c} - \left(\frac{c_{13}}{2}e\lambda_{MR}\right)\beta_{1c}\right] \end{aligned}$$

$$\begin{aligned} a_{62}(x) = &\frac{1}{I_{zz}}\left[\left(\frac{c_{13}}{4}\lambda_{MR}\right)\beta_{1s} - \left(\frac{c_3}{4} - \frac{c_2}{8}\beta_{1s}^2\right)R_{X_{MR}}\right. \\ &\left.- \left(\frac{c_2}{8}R_{Y_{MR}}\right)\beta_{1s}\beta_{1c}\right] \end{aligned}$$

$$a_{64}(x) = -\frac{I_{yy}}{I_{zz}}(q), a_{65}(x) = \frac{I_{xx}}{I_{zz}}(p)$$

$$\begin{aligned} a_{613}(x) = &\frac{1}{I_{zz}\beta_0}\left[\left(\frac{c_{10}}{4}\lambda_{MR}^2\right) - \left(\frac{c_{11}}{8}\right) + \left(\frac{c_{10}}{2}e\lambda_{MR}^2\right)\right. \\ &\left.- \left(\frac{c_{11}}{2}e\right) + \left(\frac{Cl_{TR}R_{TR}^3c_{TR}\lambda_{TR}\Omega_{TR}^2\rho R_{X_{TR}}}{2}\right)\right] \end{aligned}$$

$$\begin{aligned} a_{614}(x) = &\frac{1}{I_{zz}}\left[\left(\frac{c_{10}}{16}\right)\beta_{1c} - \left(\frac{2c_1}{25}R_{X_{MR}}\right)\beta_0 + \left(\frac{2c_{10}}{25}e\right)\beta_{1c}\right. \\ &- \left(\frac{3c_1}{8}\lambda_{MR}R_{Y_{MR}}\right) + \left(\frac{5c_1}{8}e^2\lambda_{MR}R_{Y_{MR}}\right) \\ &\left.- \left(\frac{c_1}{8}eR_{X_{MR}}\right)\beta_0 - \left(\frac{c_1}{4}eR_{Y_{MR}}\lambda_{MR}\right)\right] \end{aligned}$$

$$\begin{aligned} a_{615}(x) = &\frac{1}{I_{zz}}\left[\left(\frac{c_{10}}{16}\right)\beta_{1s} - \left(\frac{2c_1}{25}R_{Y_{MR}}\right)\beta_0 + \left(\frac{2c_{10}}{25}e\right)\beta_{1s}\right. \\ &- \left(\frac{3c_1}{8}\lambda_{MR}R_{X_{MR}}\right) + \left(\frac{5c_1}{8}e^2\lambda_{MR}R_{X_{MR}}\right) \\ &\left.+ \left(\frac{c_1}{8}eR_{Y_{MR}}\right)\beta_0 - \left(\frac{c_1}{4}eR_{X_{MR}}\lambda_{MR}\right)\right] \end{aligned}$$

$$\begin{aligned} a_{36}(x) &= a_{37}(x) = a_{38}(x) = a_{39}(x) = a_{310}(x) = a_{311}(x) \\ &= a_{312}(x) = a_{314}(x) = a_{315}(x) = 0 \end{aligned}$$

$$a_{13}(x) = a_{14}(x) = a_{16}(x) = a_{17}(x) = a_{19}(x) = a_{110}(x) = a_{111}(x) = a_{112}(x) = 0$$

$$a_{23}(x) = a_{25}(x) = a_{28}(x) = a_{29}(x) = a_{210}(x) = a_{211}(x) = a_{212}(x) = 0$$

$$a_{43}(x) = a_{44}(x) = a_{46}(x) = a_{47}(x) = a_{48}(x) = a_{49}(x) = a_{410}(x) = a_{411}(x) = a_{412}(x) = 0$$

$$a_{53}(x) = a_{55}(x) = a_{57}(x) = a_{58}(x) = a_{59}(x) = a_{510}(x) = a_{511}(x) = a_{512}(x) = 0$$

$$a_{63}(x) = a_{66}(x) = a_{67}(x) = a_{68}(x) = a_{69}(x) = a_{610}(x) = a_{611}(x) = a_{612}(x) = 0$$

$$a_{71}(x) = a_{72}(x) = a_{73}(x) = a_{77}(x) = a_{78}(x) = a_{79}(x) = a_{710}(x) = a_{711}(x) = a_{712}(x) = a_{713}(x) = a_{714}(x) = a_{715}(x) = 0$$

$$a_{81}(x) = a_{82}(x) = a_{83}(x) = a_{84}(x) = a_{87}(x) = a_{88}(x) = a_{89}(x) = a_{810}(x) = a_{811}(x) = a_{812}(x) = a_{813}(x) = a_{814}(x) = a_{815}(x) = 0$$

$$a_{91}(x) = a_{92}(x) = a_{93}(x) = a_{94}(x) = a_{97}(x) = a_{98}(x) = a_{99}(x) = a_{910}(x) = a_{911}(x) = a_{912}(x) = a_{913}(x) = a_{914}(x) = a_{915}(x) = 0$$

$$a_{103}(x) = a_{104}(x) = a_{105}(x) = a_{106}(x) = a_{107}(x) = a_{108}(x) = a_{109}(x) = a_{1014}(x) = a_{1015}(x) = 0$$

$$a_{113}(x) = a_{114}(x) = a_{115}(x) = a_{116}(x) = a_{117}(x) = a_{118}(x) = a_{119}(x) = a_{1110}(x) = a_{1113}(x) = 0$$

$$a_{123}(x) = a_{124}(x) = a_{125}(x) = a_{126}(x) = a_{127}(x) = a_{128}(x) = a_{129}(x) = a_{1210}(x) = a_{1213}(x) = 0$$

$$a_{131}(x) = a_{132}(x) = a_{133}(x) = a_{134}(x) = a_{135}(x) = a_{136}(x) = a_{137}(x) = a_{138}(x) = a_{139}(x) = a_{1311}(x) = a_{1312}(x) = a_{1313}(x) = a_{1314}(x) = a_{1315}(x) = 0$$

$$a_{141}(x) = a_{142}(x) = a_{143}(x) = a_{144}(x) = a_{145}(x) = a_{146}(x) = a_{147}(x) = a_{148}(x) = a_{149}(x) = a_{1310}(x) = a_{1412}(x) = a_{1413}(x) = a_{1414}(x) = a_{1415}(x) = 0$$

$$a_{151}(x) = a_{152}(x) = a_{153}(x) = a_{154}(x) = a_{155}(x) = a_{156}(x) = a_{157}(x) = a_{158}(x) = a_{159}(x) = a_{1510}(x) = a_{1511}(x) = a_{1513}(x) = a_{1514}(x) = a_{1515}(x) = 0$$

$$a_{75} = \sin(\psi)\tan(\theta), a_{76}(x) = \cos(\psi)\tan(\theta), a_{85}(x) = \cos(\psi)$$

$$a_{86}(x) = \sin(\psi), a_{95}(x) = \frac{\sin(\psi)}{\cos(\theta)}, a_{96}(x) = \frac{\cos(\phi)}{\cos(\theta)}$$

$$a_{101}(x) = \left(\frac{e\gamma_{MR}\Omega_{MR}}{8R_{MR}}\right)\beta_{1c}, a_{102}(x) = -\left(\frac{e\gamma_{MR}\Omega_{MR}}{8R_{MR}}\right)\beta_{1s}$$

$$a_{1010}(x) = -\left(\frac{\gamma_{MR}\Omega_{MR}}{8} + \frac{416e\gamma_{MR}\Omega_{MR}}{2497}\right)$$

$$a_{1011}(x) = -\left(\frac{208\gamma_{MR}}{2497R_{MR}}\right)V_b, a_{1012}(x) = -\left(\frac{208\gamma_{MR}}{2497R_{MR}}\right)U_b$$

$$a_{1013}(x) = -K_s - \Omega_{MR}^2 - \left(\frac{416\gamma\lambda_{MR}\Omega_{MR}^2}{2497}\right)\frac{1}{\beta_{1c}} - \left(\frac{e\gamma\lambda_{MR}\Omega_{MR}^2}{4}\right)\frac{1}{\beta_0}$$

$$a_{111}(x) = \frac{\gamma_{MR}\Omega_{MR}}{R}\left[\left(\frac{416}{2497} + \frac{e}{4}\right)\beta_0\right] + \frac{\gamma_{MR}}{R_{MR}^2}\left[\left(\frac{1}{16}\right)U_b\beta_{1s} + \left(\frac{1}{8}\right)V_b\beta_{1c}\right]$$

$$a_{112}(x) = -\frac{416\gamma_{MR}}{2497}\beta_0 + \frac{\gamma_{MR}}{16R_{MR}^2}V_b\beta_{1s} - \frac{\gamma_{MR}\lambda_{MR}\Omega_{MR}}{4R_{MR}}$$

$$a_{1111}(x) = -\frac{\lambda_{MR}\Omega_{MR}}{8}\left(1 + \frac{e}{2}\right), a_{1112}(x) = 2\Omega_{MR}$$

$$a_{1114}(x) = -K_s, a_{1115}(x) = \frac{\lambda_{MR}\Omega_{MR}}{8}\left(1 + \frac{e}{2}\right)$$

$$a_{121}(x) = \frac{\lambda_{MR}}{R_{MR}}\left[\frac{\lambda_{MR}\Omega_{MR}}{4} - \frac{1}{16}\dot{\beta}_0\right] + \frac{\lambda_{MR}}{R_{MR}^2}\left[\frac{416}{2497}U_b\beta_{1c}\right]$$

$$a_{122}(x) = \frac{\lambda_{MR}}{R_{MR}^2}\left[\left(\frac{1}{8}\right)U_b\beta_{1c} - \left(\frac{416}{2497}+\right)V_b\beta_{1c}\right] + \frac{\lambda_{MR}\Omega_{MR}}{R_{MR}}\left[-\left(\frac{\Omega_{MR}}{16} + \frac{e}{4}\right)\beta_0\right]$$

$$a_{1211}(x) = -2\Omega_{MR}, a_{1212}(x) = -\frac{\lambda_{MR}\Omega_{MR}}{8}\left(1 + \frac{e}{2}\right)$$

$$a_{1214}(x) = -\frac{\lambda_{MR}\Omega_{MR}^2}{8}\left(1 + \frac{e}{2}\right), a_{1215}(x) = -K_s$$

$$a_{74}(x) = a_{1310}(x) = a_{1411}(x) = a_{1512}(x) = 1$$

$B(x)$ state dependent coefficient matrix

$$b_{11}(x) = -\frac{c_1}{M_{body}}\left[\left(\frac{1}{16} + \frac{3e}{8} + \frac{e^2}{4}\right)\beta_{1c}\right] - \frac{c_2\lambda_{MR}}{4M_{body}}U_b$$

$$b_{13}(x) = -\frac{c_1}{M_{body}}\left[\left(\frac{208}{2497} + \frac{e}{4}\right)\beta_0\right] - \left(\frac{c_2}{4M_{body}}\right)V_b\beta_{1c}$$

$$b_{12}(x) = \frac{c_1}{M_{body}}\left[\frac{\lambda_{MR}}{8} + \frac{e\lambda_{MR}}{4} - \frac{e^2\lambda_{MR}}{8}\right] + \frac{c_2}{16M_{body}}[2U_b\beta_{1c} + 2V_b\beta_{1s} + eU_b\beta_{1c}]$$

$$b_{21}(x) = \frac{c_1}{M_{body}}\left[\left(\frac{e^2}{4}\right)\beta_{1s} + \left(\frac{e}{8} + \frac{416}{2497}\right)\beta_{1s}\right] + \frac{c_7U_b^2\beta_{1s}}{4M_{body}} + \frac{c_2}{M_{body}}\left[\left(\frac{3e^2}{8}\right)U_b\beta_0 - \left(\frac{\lambda_{MR}}{4}\right)V_b\right]$$

$$b_{22}(x) = -\frac{c_1}{M_{body}}\left[\left(\frac{e}{4}+\frac{208}{2497}\right)\beta_0\right] + \frac{c_2}{M_{body}}\left[\left(\frac{11e^2}{16}+\frac{1}{4} -\frac{7e}{16}\right)U_b\beta_{1s}\right]$$

$$b_{23}(x) = \frac{c_1}{M_{body}}\left[\frac{\lambda_{MR}}{8}(1-2e+3e^2)\right] + \frac{c_2}{M_{body}}\left[\frac{1}{8}V_b\beta_{1s} + \left(\frac{1}{8}+\frac{5e}{16}\right)U_b\beta_{1c}\right]$$

$$b_{24}(x) = -\left[\frac{Cl_{TR}R_{TR}c_{TR}\rho}{2M_{body}}\right]U_b^2 - \left[\frac{Cl_{TR}R_{TR}c_{TR}\rho}{2M_{body}}\right]W_b^2 - \frac{33}{100}\left[\frac{Cl_{TR}R_{TR}^3c_{TR}\Omega_{TR}^2\rho}{M_{body}}\right]$$

$$b_{31}(x) = -\frac{c_1}{M_{body}}\left(\frac{4}{25}+\frac{1}{2}e+\frac{1}{2}e^2-\frac{29}{25}e^3\right) - \frac{c_7}{4M_{body}}U_b^2$$

$$b_{32}(x) = \frac{c_2U_b}{M_{body}}\left(\frac{1}{4}+\frac{e}{2}-\frac{3e^2}{4}\right), b_{33}(x) = -\frac{c_2V_b}{M_{body}}\left(\frac{1}{4}+\frac{e}{2}\right)$$

$$b_{41}(x) = \frac{c_1}{I_{xx}}\left\{\left(-\frac{4}{25}-\frac{e}{2}-\frac{e^2}{2}+\frac{29e^3}{25}\right)R_{Y_{MR}} - \left[\left(\frac{e^2}{4}+\frac{3e}{8}+\frac{4}{25}\right)R_{Z_{MR}}\right]\beta_{1s}\right\} + \frac{c_2}{I_{xx}}\left\{R_{Z_{MR}}\left[\left(-\frac{3}{8}\right)U_b\beta_0 + (\lambda_{MR}4)V_b\right]\right\} - \frac{c_7}{4I_{xx}}\left\{(R_{Z_{MR}})U_b^2\beta_{1s} + (R_{Y_{MR}})U_b^2\right\}$$

$$b_{42}(x) = \frac{c_1}{I_{xx}}\left\{\left(-\frac{e}{2}-\frac{e^2}{2}+\frac{29e^3}{25}R_{Y_{MR}}\right)+\left[\left(\frac{e}{4}+\frac{208}{2497}\right)R_{Z_{MR}}\right]\beta_0\right\} + \frac{c_2}{I_{xx}}\left\{R_{Z_{MR}}\left[\left(\frac{11e^2}{16}+\frac{7e}{16}+\frac{1}{4}\right)U_b\beta_{1s}\right] + R_{Y_{MR}}\left[\left(-\frac{3e^2}{4}+\frac{e}{2}+\frac{1}{4}\right)U_b\right]\right\}$$

$$b_{43}(x) = \frac{c_1}{I_{xx}}\left[\frac{\lambda_{MR}}{8}(1+2e-3e^2)R_{Z_{MR}}\right] - \frac{c_2}{I_{xx}}\left\{R_{Z_{MR}}\left[\left(\left(\frac{5e}{16}\right)U_b\beta_{1c}+\left(\frac{1}{8}\right)V_b\beta_{1s}+\left(\frac{1}{8}\right)U_b\beta_0\right)\right] + R_{Y_{MR}}\left[\left(\frac{e}{2}+\frac{1}{4}\right)V_b\right]\right\}$$

$$b_{44}(x) = \frac{33}{100}\left[\frac{Cl_{TR}R_{TR}^3c_{TR}\omega_{TR}^2\rho}{I_{xx}}R_{Z_{TR}}\right] + \left[\frac{Cl_{TR}R_{TR}c_{TR}\rho}{2I_{xx}}R_{Z_{TR}}\right]U_b^2 + \left[\frac{Cl_{TR}R_{TR}c_{TR}\rho}{2I_{xx}}R_{Z_{TR}}\right]W_b^2$$

$$b_{51}(x) = \frac{c_1}{I_{yy}}\left\{\left(\frac{4}{25}+\frac{e}{2}+\frac{e^2}{2}+\frac{29e^3}{25}\right)R_{X_{MR}} - \left[\left(\frac{e^2}{4}+\frac{3e}{8}+\frac{4}{25}\right)R_{Z_{MR}}\right]\beta_{1c}\right\} + \frac{c_7}{4I_{yy}}[(R_{X_{MR}})U_b^2]$$

$$b_{52}(x) = \frac{1}{I_{yy}}\left[\left(\frac{c_1}{8}R_{Z_{MR}}\lambda_{MR}\right) - \left(\frac{c_2}{4}R_{X_{MR}}\right)U_b + \left(\frac{3c_2}{4}R_{X_{MR}}e^2\right)U_b + \left(\frac{c_1}{4}R_{Z_{MR}}e\lambda_{MR}\right) + \left(\frac{c_2}{8}R_{Z_{MR}}\right)U_b\beta_{1c} + \left(\frac{c_2}{8}R_{Z_{MR}}\right)V_b\beta_{1s} - \left(\frac{c_2}{2}eR_{X_{MR}}\right)U_b - \left(\frac{3c_1}{8}R_{Z_{MR}}e^2\lambda_{MR}\right) + \left(\frac{c_2}{16}R_{Z_{MR}}e\right)U_b\beta_{1c}\right]$$

$$b_{53}(x) = \frac{1}{I_{yy}}\left[\left(\frac{c_2}{4}R_{X_{MR}}\right)V_b - \left(\frac{2c_1}{25}-\frac{c_1}{4}e\right)R_{Z_{MR}}\beta_0 - \left(\frac{c_2}{4}R_{Z_{MR}}\right)V_b\beta_{1c} + \left(\frac{c_2}{2}eR_{X_{MR}}\right)V_b\right]$$

$$b_{61}(x) = \frac{1}{I_{zz}}\left[\left(\frac{4c_1}{25}R_{X_{MR}}\right)\beta_{1s} + \left(\frac{4c_1}{25}R_{Y_{MR}}\right)\beta_{1c} - \left(\frac{4c_{10}}{25}\lambda_{MR}\right) + \left(\frac{3c_1}{8}eR_{X_{MR}}\right)\beta_{1s} + \left(\frac{3c_1}{8}eR_{Y_{MR}}\right)\beta_{1c} - \left(\frac{c_{10}}{2}e\lambda_{MR}\right) + \left(\frac{3c_2}{8}R_{X_{MR}}\right)U_b\beta_0 + \left(\frac{c_2}{4}R_{Y_{MR}}\lambda_{MR}\right)U_b - \left(\frac{c_2}{4}R_{X_{MR}}\lambda_{MR}\right)V_b + \left(\frac{c_1}{4}R_{X_{MR}}e^2\right)\beta_{1s} + \left(\frac{c_1}{4}R_{Y_{MR}}e^2\right)\beta_{1c} + \left(\frac{c_7}{4}R_{X_{MR}}\right)\beta_{1s}U_b^2 + \left(\frac{c_13}{8}e\right)\beta_{1c}U_b\right]$$

$$b_{62}(x) = \frac{1}{I_{zz}}\left[\left(\frac{c_{10}}{16}\right)\beta_{1c} - \left(\frac{2c_1}{25}R_{X_{MR}}\right)\beta_0 - \left(\frac{c_1}{8}R_{Y_{MR}}\lambda_{MR}\right) + \left(\frac{4c_{10}}{25}e\right)\beta_{1c} - \left(\frac{c_1}{4}R_{X_{MR}}e\right)\beta_0 - \left(\frac{c_1}{4}R_{Y_{MR}}e\lambda_{MR}\right) - \left(\frac{c_2}{4}R_{X_{MR}}\right)U_b\beta_{1s} - \left(\frac{c_2}{8}R_{Y_{MR}}\right)U_b\beta_{1c} - \left(\frac{c_2}{8}R_{Y_{MR}}\right)V_b\beta_{1s} + \left(\frac{c_13}{8}\lambda_{MR}\right)U_b + \left(\frac{c_10}{8}e^2\right)\beta_{1c} + \left(\frac{3c_1}{8}R_{Y_{MR}}e^2\lambda_{MR}\right) - \left(\frac{c_{12}}{32}\right)\beta_{1c}U_b^2 - \left(\frac{7c_2}{16}R_{X_{MR}}e\right)U_b\beta_{1s} - \left(\frac{c_2}{16}R_{Y_{MR}}e\right)\beta_{1c}U_b + \left(\frac{c_{13}}{4}e\lambda_{MR}\right)U_b + \left(\frac{11c_2}{16}R_{X_{MR}}e^2\right)\beta_{1s}U_b\right]$$

$$b_{63}(x) = \frac{1}{I_{zz}}\left[\left(\frac{c_{10}}{16}\right)\beta_{1s} + \left(\frac{2c_1}{25}R_{Y_{MR}}\right)\beta_0 - \left(\frac{c_1}{8}R_{X_{MR}}\lambda_{MR}\right) + \left(\frac{4c_{10}}{25}e\right)\beta_{1s} + \left(\frac{c_1}{4}R_{Y_{MR}}e\right)\beta_0 - \left(\frac{c_1}{4}R_{X_{MR}}e\lambda_{MR}\right) + \left(\frac{c_2}{8}R_{X_{MR}}\right)U_b\beta_{1c} + \left(\frac{c_2}{8}R_{X_{MR}}\right)V_b\beta_{1s} + \left(\frac{c_2}{4}R_{Y_{MR}}\right)V_b\beta_{1c} + \left(\frac{c_{10}}{8}e^2\right)\beta_{1s} + \left(\frac{3c_1}{8}R_{X_{MR}}e^2\lambda_{MR}\right) + \left(\frac{5c_2}{16}R_{X_{MR}}e\right)\beta_{1c}U_b\right]$$

$$b_{64}(x) = -\frac{Cl_{TR}R_{TR}R_{X_{TR}}c_{TR}\rho}{2I_{zz}}\left(U_b^2+W_b^2+\frac{3R_{TR}^2\Omega_{TR}^2}{5}\right)$$

$$b_{101}(x) = \left[\frac{\gamma_{MR}\Omega_{MR}^2}{8} + \frac{\gamma_{MR}}{8R_{MR}^2}\left(U_b^2+V_b^2\right) + \frac{3}{10}e\gamma_{MR}\Omega_{MR}^2\right]$$

$$b_{102}(x) = -\left(\frac{4\gamma_{MR}\Omega_{MR}}{25R_{MR}} + \frac{e\gamma_{MR}\Omega_{MR}}{4R}\right)U_b$$

$$b_{103}(x) = \left(\frac{4\gamma_{MR}\Omega_{MR}}{25R_{MR}} + \frac{e\gamma_{MR}\Omega_{MR}}{4R_{MR}}\right)V_b$$

$$b_{111}(x) = \left(\frac{3\gamma_{MR}\Omega_{MR}}{10R} + \frac{e\gamma_{MR}\Omega_{MR}}{2R}\right)V_b$$

$$b_{112}(x) = -\left(\frac{\gamma_{MR}}{8R_{MR}^2}\right)U_bV_b$$

$$b_{113}(x) = \left[\frac{\gamma_{MR}\Omega_{MR}^2}{8} + \left(\frac{\gamma_{MR}}{16R_{MR}^2}\right)U_b^2 + \left(\frac{3\gamma_{MR}}{16R_{MR}^2}\right)V_b^2 + \frac{3}{10}e\gamma_{MR}\Omega_{MR}^2\right]$$

$$b_{121}(x) = \left(\frac{3\gamma_{MR}\Omega_{MR}}{10R_{MR}} + \frac{e\gamma_{MR}\Omega_{MR}}{2R_{MR}}\right)U_b$$

$$b_{122}(x) = \left[\frac{\gamma_{MR}\Omega_{MR}^2}{8} + \left(\frac{3\gamma_{MR}}{16R_{MR}^2}\right)U_b^2 + \left(\frac{\gamma_{MR}}{16R_{MR}^2}\right)V_b^2 + \frac{3}{10}e\gamma_{MR}\Omega_{MR}^2\right]$$

$$b_{123}(x) = \left(\frac{\gamma_{MR}}{8R_{MR}^2}\right)U_bV_b$$

$$\begin{aligned} b_{34}(x) &= b_{14}(x) = b_{54}(x) = b_{71}(x) = b_{72}(x) = b_{73}(x) \\ &= b_{74}(x) = b_{81}(x) = b_{82}(x) = b_{83}(x) = b_{84}(x) \\ &= b_{91}(x) = b_{92}(x) = b_{93}(x) = b_{94}(x) = b_{104}(x) \\ &= b_{114}(x) = b_{124}(x) = b_{131}(x) = b_{132}(x) = b_{133}(x) \\ &= b_{134}(x) = b_{141}(x) = b_{142}(x) = b_{143}(x) = b_{144}(x) \\ &= b_{151}(x) = b_{152}(x) = b_{153}(x) = b_{154}(x) = 0 \end{aligned}$$

SAS Loop State Dependent Matrix Coefficients

$$\begin{aligned} a_{11}(x_{SAS}) &= a_{22}(x_{SAS}) = a_{33}(x_{SAS}) = a_{14}(x_{SAS}) \\ &= a_{15}(x_{SAS}) = a_{16}(x_{SAS}) = a_{24}(x_{SAS}) \\ &= a_{25}(x_{SAS}) = a_{26}(x_{SAS}) = a_{34}(x_{SAS}) \\ &= a_{35}(x_{SAS}) = a_{42}(x_{SAS}) = a_{43}(x_{SAS}) \\ &= a_{44}(x_{SAS}) = a_{45}(x_{SAS}) = a_{46}(x_{SAS}) = 0 \end{aligned}$$

$$\begin{aligned} a_{51}(x_{SAS}) &= a_{53}(x_{SAS}) = a_{54}(x_{SAS}) = a_{55}(x_{SAS}) \\ &= a_{56}(x_{SAS}) = a_{61}(x_{SAS}) = a_{62}(x_{SAS}) \\ &= a_{64}(x_{SAS}) = a_{65}(x_{SAS}) = a_{66}(x_{SAS}) = 0 \end{aligned}$$

$$a_{41}(x_{SAS}) = a_{52}(x_{SAS}) = a_{63}(x_{SAS}) = 1$$

$$a_{12}(x_{SAS}) = \left(\frac{I_{yy}}{I_{xx}}\right)r, a_{13}(x_{SAS}) = -\left(\frac{I_{zz}}{I_{xx}}\right)q$$

$$a_{21}(x_{SAS}) = -\left(\frac{I_{xx}}{I_{yy}}\right)r, a_{23}(x_{SAS}) = \left(\frac{I_{zz}}{I_{yy}}\right)p$$

$$a_{31}(x_{SAS}) = \frac{I_{xx}}{I_{yy}}, a_{32}(x_{SAS}) = -\left(\frac{I_{yy}}{I_{zz}}\right)p$$

$$b_{11}(x) = \frac{c_1}{I_{xx}}\left\{\left[\left(\frac{2}{25} + \frac{e}{4}\right)R_{Z_{MR}}\right]\beta_0\right\} + \frac{c_2}{I_{xx}}\left\{\left[\left(\frac{1}{4} + \frac{e}{2} - \frac{3e^2}{4}\right)R_{Y_{MR}}\right]U_b + \left[\left(\frac{1}{4} + \frac{7e}{16} - \frac{11e^2}{16}\right)R_{Z_{MR}}\right]\beta_{1s}U_b\right\}$$

$$b_{12}(x) = \frac{c_1}{I_{xx}}\left\{\left[\left(\frac{1}{8} + \frac{e}{4} - \frac{3e^2}{8}\right)\lambda_{MR}R_{Z_{MR}}\right]\right\} - \frac{c_2}{I_{xx}}\left\{\left[\left(\frac{1}{4} + \frac{e}{2}\right)R_{Y_{MR}}\right]V_b + \left[\left(\frac{1}{8} + \frac{5e}{16}\right)R_{Z_{MR}}\right]\beta_{1c}U_b + \left[\left(\frac{1}{8}\right)R_{Z_{MR}}\right]\beta_{1s}V_b\right\}$$

$$b_{13}(x) = \frac{Cl_{TR}R_{TR}c_{TR}\rho R_{Z_{TR}}}{I_{xx}}\left(\frac{33R_{TR}^2\Omega_{TR}^2}{100} + \frac{U_b^2}{2} + \frac{W_b^2}{2}\right)$$

$$b_{21}(x) = \frac{c_1}{I_{yy}}\left\{\left[\left(\frac{1}{8} + \frac{e}{4} - \frac{3e^2}{8}\right)\lambda_{MR}R_{Z_{MR}}\right]\right\} - \frac{c_2}{I_{yy}}\left\{\left[\left(\frac{1}{4} + \frac{e}{2} - \frac{3e^2}{4}\right)R_{X_{MR}}\right]U_b + \left[\left(-\frac{1}{8} - \frac{e}{16}\right)R_{Z_{MR}}\right]\beta_{1c}U_b + \left[\left(-\frac{1}{8}\right)R_{Z_{MR}}\right]\beta_{1s}V_b\right\}$$

$$b_{22}(x) = -\frac{c_1}{I_{yy}}\left\{\left[\left(\frac{2}{25} + \frac{e}{4}\right)R_{Z_{MR}}\right]\beta_0\right\} + \frac{c_2}{I_{yy}}\left\{\left[\left(\frac{1}{4} + \frac{e}{2}\right)R_{X_{MR}}\right]V_b - \left[\left(\frac{1}{4}\right)R_{Z_{MR}}\right]\beta_{1c}V_b\right\}$$

$$b_{23} = 0$$

$$b_{31}(x) = \frac{c_1}{I_{zz}}\left\{\left[\left(\frac{1}{16} + \frac{4e}{25} + \frac{e^2}{8}\right)R_{MR}\right]\beta_{1c} - \left[\left(\frac{2}{25} + \frac{e}{4}\right)R_{X_{MR}}\right]\beta_0 - \left[\left(\frac{1}{8} + \frac{e}{4} - \frac{3e^2}{8}\right)\lambda_{MR}R_{Y_{MR}}\right]\right\} - \frac{c_2}{I_{zz}}\left\{\left[\left(\frac{1}{4} + \frac{7e}{16} - \frac{11e^2}{16}\right)R_{X_{MR}}\right]\beta_{1s}U_b + \left[\left(\frac{1}{8} - \frac{e}{16}\right)R_{Y_{MR}}\right]\beta_{1c}U_b + \left[\left(\frac{1}{8}\right)R_{Y_{MR}}\right]\beta_{1s}V_b + \left[\left(-\frac{1}{8} + \frac{e}{4}\right)R_{MR}\lambda_{MR}\right]U_b\right\} - \left(\frac{c_7}{32I_{zz}}R_{MR}\right)\beta_{1c}U_b^2$$

$$b_{32}(x) = \frac{c_1}{I_{zz}}\left\{\left[\left(\frac{1}{16} + \frac{4e}{25} + \frac{e^2}{8}\right)R_{MR}\right]\beta_{1s} - \left[\left(\frac{1}{8} + \frac{e}{4} + \frac{3e^2}{8}\right)\lambda_{MR}R_{X_{MR}}\right] + \left[\left(\frac{2}{25} + \frac{e}{4} - \frac{3e^2}{8}\right)R_{Y_{MR}}\right]\beta_0\right\} + \frac{c_2}{I_{zz}}\left\{\left[\left(\frac{1}{8} + \frac{5e}{16}\right)R_{X_{MR}}\right]\beta_{1c}U_b + \left[\left(\frac{1}{8}\right)R_{X_{MR}}\right]\beta_{1s}V_b + \left[\left(\frac{1}{4} - \frac{e}{16}\right)R_{Y_{MR}}\right]\beta_{1c}V_b\right\}$$

$$b_{33}(x) = \frac{Cl_{TR}R_{TR}c_{TR}\rho R_{X_{TR}}}{I_{zz}}\left(-\frac{33R_{TR}^2\Omega_{TR}^2}{100} + \frac{U_b^2}{2} + \frac{W_b^2}{2}\right)$$

$$\begin{aligned} b_{41}(x) &= b_{42}(x) = b_{43}(x) = b_{51}(x) = b_{52}(x) = b_{53}(x) = b_{61}(x) \\ &= b_{62}(x) = b_{63}(x) = 0 \end{aligned}$$

$$c_1 = C_{l_{\alpha_{MR}}}N_bR_{MR}^3c_{MR}\Omega_{MR}^2\rho$$

$$c_2 = C_{l_{\alpha_{MR}}}N_bR_{MR}^2c_{MR}\Omega_{MR}\rho$$

$$c_7 = C_{l_{\alpha_{MR}}}N_bR_{MR}c_{MR}\rho$$

Attitude Loop State Dependent Matrix Coefficients

$$b_{11}(x) = 1, b_{12}(x) = \sin(\phi)\tan(\theta), b_{13}(x) = \cos(\phi)\tan(\theta)$$

$$b_{32}(x) = \sin(\phi)/\cos(\theta), b_{33}(x) = \cos(\phi)/\cos(\theta)$$

$$\begin{aligned} b_{21}(x) &= b_{31}(x) = b_{41}(x) = b_{42}(x) = b_{43}(x) = b_{51}(x) = b_{52}(x) \\ &= b_{53}(x) = b_{61}(x) = b_{62}(x) = b_{63}(x) = 0 \end{aligned}$$

$$A(x) = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} \\ I_{3\times3} & 0_{3\times3} \end{bmatrix}$$

Speed Loop State Dependent Matrix Coefficients

$$a_{11}(x_{SPD}) = -\frac{c_2}{m}\left\{\left(\frac{1}{8}+\frac{3e}{16}\right)\beta_{1c}^2 + \left(\frac{e}{16}\right)\beta_{1s}^2\right\} - \frac{c_3}{4m} - \frac{c_4}{2m}|U_b|$$

$$a_{12}(x_{SPD}) = r + z\frac{c_2}{8m}\beta_{1s}\beta_{1c}$$

$$a_{21}(x_{SPD}) = \frac{c_2}{8m}\left[(1+e)\beta_{1s}\beta_{1c} - \left(\frac{3\gamma_{MR}}{4}\right)\beta_0\right] - r$$

$$a_{22}(x_{SPD}) = -\left(\frac{c_2}{8m}\beta_{1s}^2 + \frac{c_3}{4m} + \frac{c_8}{2m}|V_b|\right)$$

$$\begin{aligned} a_{13}(x_{SPD}) &= a_{14}(x_{SPD}) = a_{23}(x_{SPD}) = a_{24}(x_{SPD}) \\ &= a_{32}(x_{SPD}) = a_{33}(x_{SPD}) = a_{34}(x_{SPD}) \\ &= a_{41}(x_{SPD}) = a_{43}(x_{SPD}) = a_{44}(x_{SPD}) = 0 \end{aligned}$$

$$a_{31}(x_{SPD}) = a_{42}(x_{SPD}) = 1$$

$$b_{12} = -gL_1, b_{21} = g\cos(\theta)L_2$$

$$b_{11}(x) = b_{22}(x) = b_{31}(x) = b_{32}(x) = b_{41}(x) = b_{42}(x) = 0$$

$$\begin{aligned} &L_1 = \text{sinc}(\theta), L_2 = \text{sinc}(\phi) \\ &c_2 = C_{l_{\alpha_{MR}}} N_b R_{MR}^2 c_{MR}\Omega_{MR}\rho, c_3 = C_d N_b R_{MR}^2 c_{MR}\Omega_{MR}\rho \\ &c_4 = A_x D_{x_{fus}}\rho, c_8 = A_y D_{y_{fus}}\rho \end{aligned}$$

Vertical Speed Loop State Dependent Matrix Coefficients

$$a_{11}(x_{VS}) = -\frac{1}{m}\left[\left(\frac{c_9}{2}\right)|W_b|\right], a_{21}(x_{VS}) = 1$$

$$a_{12}(x_{VS}) = a_{22}(x_{VS}) = 0$$

$$b_1(x) = -\frac{c_1}{m}\left(\frac{4}{25} + \frac{1}{2}e + \frac{1}{2}e^2 - \frac{29}{25}e^3\right) - \frac{c_7}{4m}U_b^2, b_2(x) = 0$$

$$c_1 = C_{l_{\alpha_{MR}}} N_b R^3 c_{MR}\Omega_{MR}^2\rho, c_7 = C_{l_{\alpha_{MR}}} N_b R_{MR} c_{MR}\rho$$

$$c_9 = A_z D_{z_{fus}}\rho$$