**Appendix A. is harmonic load**

When the arbitrary mechanical load is a harmonic load, we assumed the expression of is:

|  |  |
| --- | --- |
|  | (A.1) |

here and are the amplitude and angular frequency of .

It is noticed that the can do both integral and derivative operations during the analysis. To illustrate the application of Taylor series expansion in the analysis, the theoretical solutions of the composite under can be obtained in the following analysis.

Substituting Equation (A.1) into Equations (65) and (70), can be expressed as:

|  |  |
| --- | --- |
|  | (A.2) |

where

|  |  |
| --- | --- |
|  | (A.3) |

By use of Equations (68)-(70) and (A.3), we can obtain the expression of :

|  |  |
| --- | --- |
|  | (A.4) |

With displacement as an example, the theoretical solutions of the composite under harmonic load can be obtained as follows based on Equations (55), (71) and (A.4):

|  |  |
| --- | --- |
|  | (A.5) |

where is shown in Equation (69). Then the analytical solutions of the mechanical quantities of the 2-2 multi-layered cement-based piezoelectric composite under harmonic load can be obtained by using Equations (1)-(4) and (A.5).

Meanwhile, in order to facilitate the analysis of the magnification factors of the composite, we also obtained the analytical solution of at the free end of the composite by using Duhamel integral. We have:

|  |  |
| --- | --- |
|  | (A.6) |

Equation (A.6) show that contains two distinct vibration components; the term is the free vibration, giving an oscillation at the natural frequency of the composite; the term is the forced vibration, giving an oscillation at the forcing frequency. Noting that the free vibration will decay with time due to damping in practical engineering, and only the forced vibration is left (Craig, 1981; Clough and Penzien, 1993). Therefore, the forced vibration can be expressed as follows by substituting into the term of Equation (A.6):

|  |  |
| --- | --- |
|  | (A.7) |

Moreover, the static deformation of the composite under the amplitude of harmonic load is:

|  |  |
| --- | --- |
|  | (A.8) |

where is the stress. Then, the displacement magnification factor at the free end of the composite under harmonic load can be obtained as:

|  |  |
| --- | --- |
|  | (A.9) |

here is the angular frequency of the composite.

Similarly, the stress magnification factor at the fixed end of the composite, and the total electric potential amplification factor of each piezoelectric layer can be obtained as:

|  |  |
| --- | --- |
|  | (A.10) |
|  | (A.11) |

It can be found from Equations (A.9), (A.10) and (A.11) that , and of 2-2 multi-layered cement-based piezoelectric composite are mainly related to the angular frequency and material parameters of the composite; and is determined by external harmonic load.

**Appendix B. is piecewise functional load**

When the arbitrary mechanical load is a piecewise functional load, we assumed the expression of is:

|  |  |
| --- | --- |
|  | (B.1) |

here (*i*=1,2,3) are the arbitrary known functions; and are the known times, respectively.

The theoretical solutions of the composite under can be obtained in the following analysis.

Firstly, based on Duhamel integral and using Equations (63) and (B.1), can be expressed as below for :

|  |  |
| --- | --- |
|  | (B.2) |

Simultaneously, by using the piecewise Duhamel integral, we obtained and as follows, respectively, for and :

|  |  |
| --- | --- |
|  | (B.3) |
|  | (B.4) |

Then take displacement as an example, the theoretical solutions of the composite under piecewise functional load can be obtained as follows:

|  |  |
| --- | --- |
|  | (B.5) |
|  | (B.6) |

Then the analytical solutions of the mechanical quantities of the 2-2 multi-layered cement-based piezoelectric composite under piecewise functional load can be obtained by using Equations (1)-(4), (B.5) and (B.6).

**Appendix C. is impact load**

When the arbitrary mechanical load is an impact load, we assumed the expression of is:

|  |  |
| --- | --- |
|  | (C.1) |

here satisfies . The physical meaning of is that an impact force acts at and a finite impulse is generated in an infinitesimal time.

It should be mentioned that is easier to perform integral calculations than derivative calculations during the analysis. Therefore, the theoretical solutions of 2-2 multi-layered cement-based piezoelectric composite under can be obtained by using Duhamel integral and basic equations.

Firstly, based on Duhamel integral and using Equations (63) and (C.1), can be expressed as below for :

|  |  |
| --- | --- |
|  | (C.2) |

Then, by using the piecewise Duhamel integral, we obtained as follows for :

|  |  |
| --- | --- |
|  | (C.3) |

With displacement as an example, the theoretical solutions of the composite under impact load can be obtained as:

|  |  |
| --- | --- |
|  | (C.4) |
|  | (C.5) |

Then the analytical solutions of the mechanical quantities of the 2-2 multi-layered cement-based piezoelectric composite under impact load can be obtained by using Equations (1)-(4), (C.4) and (C.5).

**References**

Clough RW and Penzien J (1993) *Dynamics of Structures (second edition)*. New York: McGraw-Hill.

Craig RR (1981) *Structural Dynamics: An Introduction to Computer Method*. New York: John Wiley & Sons.