Online Supplement

Below is the commented OpenBUGS code used to fit the empirical data to the BRB IRT model as demonstrated in empirical study 2. The first code block presents the model in Equation 4 with covariances between factors and a gender covariate. The second code block presents the model in Equation 5 with covariances between factors and the effect of gender on each factor individually. Table 1A provides a summary of the data structures used in OpenBUGS to fit each model.

Table 1A
Data Structure Used in OpenBUGS

Data	Summary
Nitem	The number of pairwise comparisons, C, to be made
Nblock	The number of blocks, D
Dim	The number of dimensions
x	N x C matrix of response data
PersonData	An N x 2 matrix, where the first column contains an ID variable and the second column contains the gender variable
ItemData	An N x 3 matrix, where the first column contains the item number, the second column contains the dimension the item loads onto, and the third column contains the direction which the item loads onto the dimension ($\pm$ 1)
Omega	A Dim x Dim identity matrix used in the definition of the hyperprior for the covariances between factors

BRB IRT Model (Equation 4)

```
#Defining priors for person thetas of each dimension, defining
priors for the person random effect within all blocks
for (j in 1:Nperson) {
     Theta[j,1:Dim] ~ dmnorm(mu.theta[],prec.sigma[,])
     RandomBlockTheta[j] ~ dnorm(0,prec.tau2)
}
#Hyper priors for thetas
for (f in 1:Dim){
     mu.theta[f] ~ dnorm(0,1)
}
Sigma[1:Dim,1:Dim] <- inverse(prec.sigma[,])
#Including covariances between thetas for each dimension
prec.sigma[1:Dim,1:Dim] ~ dwish(Omega[,],Dim)
#Hyper prior for random effect
prec.tau2 ~ dgamma(1,1)
```

```
#Defining priors for item parameters
for (i in 1: Nitem){
     a[i] ~ dlnorm(0,.25)
     d[i] ~ dnorm(0,1)
}

#Defining prior for gender effect
GenderEffect ~ dnorm(0,1)

#The BRB IRT model including random block and gender effects
for (j in 1:Nperson) {
     for(b in 1: Nblock){
#Probability of endorsing item 1 over 2
     p12T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-2),3]*
          a[((3*b)-2)])*Theta[j, ItemData[((3*b)-2),2]])-
          ((ItemData[((3*b)-1),3]*a[((3*b)-1)])*Theta[j,
          ItemData[((3*b)-1),2]]) -
          GenderEffect*PersonData[j,2]) - RandomBlockTheta[j] -
          d[((3*b)-2)])))
#Probability of endorsing item 2 over 1
     p21T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-1),3]*
          a[((3*b)-1)])*Theta[j, ItemData[((3*b)-1),2]])-
          ((ItemData[((3*b)-2),3]*a[((3*b)-2)])*Theta[j,
          ItemData[((3*b)-2),2]]) +
          (GenderEffect*PersonData[j,2]) + RandomBlockTheta[j] +
          d[((3*b)-2)])))
#Response based on probabilities
     response1[j,b] <- (p12T[j,b]*(1-p21T[j,b]))/((p12T[j,b]*
          (1-p21T[j,b]))+((1-p12T[j,b])*p21T[j,b]))
     x[j,((3*b)-2)] ~ dbern(response1[j,b])
     mx[j,((3*b)-2)] <- 1-x[j,((3*b)-2)]

#Probability of endorsing item 1 over 3
     p13T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-2),3]*
          a[((3*b)-2)])*Theta[j, ItemData[((3*b)-2),2]])-
          ((ItemData[((3*b)),3]*a[((3*b))])*Theta[j,
          ItemData[((3*b)),2]]) - (GenderEffect*PersonData[j,2])
          - RandomBlockTheta[j] - d[((3*b)-1)])))
#Probability of endorsing item 3 over 1
     p31T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)),3]*
          a[((3*b))])*Theta[j, ItemData[((3*b)),2]])-
          ((ItemData[((3*b)-2),3]*a[((3*b)-2)])*Theta[j,
          ItemData[((3*b)-2),2]]) +
          (GenderEffect*PersonData[j,2]) + RandomBlockTheta[j] +
          d[((3*b)-1)])))
#Response based on probabilities
     response2[j,b] <- (p13T[j,b]*(1-p31T[j,b]))/((p13T[j,b]*
```

```
          (1-p31T[j,b]))+((1-p13T[j,b])*p31T[j,b]))
     x[j,((3*b)-1)] ~ dbern(response2[j,b])
     mx[j,((3*b)-1)] <- 1-x[j,((3*b)-1)]

#Probability of endorsing item 2 over 3
     p23T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-1),3]*
          a[((3*b)-1)])*Theta[j,ItemData[((3*b)-1),2]])-
          ((ItemData[((3*b)),3]*a[((3*b))])*Theta[j,
          ItemData[((3*b)),2]]) - (GenderEffect*PersonData[j,2])
          + RandomBlockTheta[j] - d[(3*b)])))
#Probability of endorsing item 3 over 2
     p32T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)),3]*
          a[((3*b))])*Theta[j,ItemData[((3*b)),2]])-
          ((ItemData[((3*b)-1),3]*a[((3*b)-1)])*Theta[j,
          ItemData[((3*b)-1),2]]) +
          (GenderEffect*PersonData[j,2]) - RandomBlockTheta[j] +
          d[(3*b)])))
#Response based on probabilities
     response3[j,b] <- (p23T[j,b]*(1-p32T[j,b]))/((p23T[j,b]*
          (1-p32T[j,b]))+((1-p23T[j,b])*p32T[j,b]))
     x[j,(3*b)] ~ dbern(response3[j,b])
     mx[j,(3*b)] <- 1-x[j,(3*b)]
     }
}
```

BRB IRT Model (Equation 5)

```
#Defining priors for person thetas of each dimension, defining
priors for the person random effect within all blocks
for (j in 1:Nperson) {
     Theta[j,1:Dim] ~ dmnorm(mu.theta[],prec.sigma[,])
     RandomBlockTheta[j] ~ dnorm(0,prec.tau2)
}
#Hyper priors for thetas
for (f in 1:Dim){
     mu.theta[f] ~ dnorm(0,1)
}
Sigma[1:Dim,1:Dim] <- inverse(prec.sigma[,])
#Including covariances between thetas for each dimension
prec.sigma[1:Dim,1:Dim] ~ dwish(Omega[,],Dim)
#Hyper prior for random effect
prec.tau2 ~ dgamma(1,1)

#Defining priors for item parameters
for (i in 1: Nitem){
     a[i] ~ dlnorm(0,.25)
     d[i] ~ dnorm(0,1)
}
```

```
#Defining prior for gender effect
GenderEffect ~ dnorm(0,1)

#The BRB IRT model including random block and 5 gender effects
for (j in 1:Nperson) {
     for(b in 1: Nblock){
#Probability of endorsing item 1 over 2
     p12T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-2),3]*
          a[((3*b)-2)])*Theta[j, ItemData[((3*b)-2),2]])-
          ((ItemData[((3*b)-1),3]*a[((3*b)-1)])*Theta[j,
          ItemData[((3*b)-1),2]]) - ((ItemData[((3*b)-
          1),3]*GenderEffect[ItemData[((3*b)-1),2]]) *
          Gender[j,2]) + ((ItemData[((3*b)-2),3] *
          GenderEffect[ItemData[((3*b)-2),2]]) * Gender[j,2]) -
          RandomBlockTheta[j] - d[((3*b)-2)])))
#Probability of endorsing item 2 over 1
     p21T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-1),3]*
          a[((3*b)-1)])*Theta[j, ItemData[((3*b)-1),2]])-
          ((ItemData[((3*b)-2),3]*a[((3*b)-2)])*Theta[j,
          ItemData[((3*b)-2),2]]) + ((ItemData[((3*b)-1),3] *
          GenderEffect[ItemData[((3*b)-1),2]])*Gender[j,2]) -
          ((ItemData[((3*b)-2),3]* GenderEffect[ItemData[((3*b)-
          2),2]]) * Gender[j,2]) + RandomBlockTheta[j] +
          d[((3*b)-2)])))
#Response based on probabilities
     response1[j,b] <- (p12T[j,b]*(1-p21T[j,b]))/((p12T[j,b]*
          (1-p21T[j,b]))+((1-p12T[j,b])*p21T[j,b]))
     x[j,((3*b)-2)] ~ dbern(response1[j,b])
     mx[j,((3*b)-2)] <- 1-x[j,((3*b)-2)]

#Probability of endorsing item 1 over 3
     p13T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-2),3]*a[((3*b)-
          2)])*Theta[j, ItemData[((3*b)-2),2]])-
          ((ItemData[((3*b)),3]*a[((3*b))])*Theta[j,
          ItemData[((3*b)),2]]) -(ItemData[((3*b)),3] *
          GenderEffect[ItemData[((3*b)),2]])*Gender[j,2]) +
          ((ItemData[((3*b)-2),3]*GenderEffect[ItemData[((3*b)-
          2),2]]) * Gender[j,2]) - RandomBlockTheta[j] -
          d[((3*b)-1)])))
#Probability of endorsing item 3 over 1
     p31T[j,b] <- 1/(1+exp(-
          (((ItemData[((3*b)),3]*a[((3*b))])*Theta[j,
          ItemData[((3*b)),2]])-((ItemData[((3*b)-2),3]*
          a[((3*b)-2)])*Theta[j, ItemData[((3*b)-2),2]]) +
          ((ItemData[((3*b)),3] *
          GenderEffect[ItemData[((3*b)),2]]) * Gender[j,2]) -
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          ((ItemData[((3*b)-2),3]*GenderEffect[ItemData[((3*b)-
          2),2]]) * Gender[j,2]) + RandomBlockTheta[j] +
          d[((3*b)-1)])))
#Response based on probabilities
     response2[j,b] <- (p13T[j,b]*(1-p31T[j,b]))/((p13T[j,b]*
          (1-p31T[j,b]))+((1-p13T[j,b])*p31T[j,b]))
     x[j,((3*b)-1)] ~ dbern(response2[j,b])
     mx[j,((3*b)-1)] <- 1-x[j,((3*b)-1)]
#Probability of endorsing item 2 over 3
     p23T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)-1),3]*
          a[((3*b)-1)])*Theta[j,ItemData[((3*b)-1),2]])-
          ((ItemData[((3*b)),3]*a[((3*b))])*Theta[j,
          ItemData[((3*b)),2]]) - ItemData[((3*b)),3] *
          GenderEffect[ItemData[((3*b)),2]])*Gender[j,2]) +
          ((ItemData[((3*b)-1),3]*GenderEffect[ItemData[((3*b)-
          1),2]]) * Gender[j,2])  + RandomBlockTheta[j] -
          d[(3*b)])))
#Probability of endorsing item 3 over 2
     p32T[j,b] <- 1/(1+exp(-(((ItemData[((3*b)),3]*
          a[((3*b))])*Theta[j,ItemData[((3*b)),2]])-
          ((ItemData[((3*b)-1),3]*a[((3*b)-1)]) *
          Theta[j,ItemData[((3*b)-1),2]]) +
          ((ItemData[((3*b)),3]*GenderEffect[ItemData[((3*b)),2]
          ]) * Gender[j,2]) - ((ItemData[((3*b)-
          1),3]*GenderEffect[ItemData[((3*b)-1),2]]) *
          Gender[j,2]) - RandomBlockTheta[j] + d[(3*b)])))
#Response based on probabilities
     response3[j,b] <- (p23T[j,b]*(1-p32T[j,b]))/((p23T[j,b]*
          (1-p32T[j,b]))+((1-p23T[j,b])*p32T[j,b]))
     x[j,(3*b)] ~ dbern(response3[j,b])
     mx[j,(3*b)] <- 1-x[j,(3*b)]
     }
}
```