Appendix 1

Notation

A	blade passage area
b	hub-to-shroud passage width
C	absolute velocity
C_f	skin friction coefficient
D_{eq}	equivalent diffusion factor
d	diameter
d_H	hydraulic diameter
F	shape factor
I	work input coefficient
L	blade length
$\dot{m}$	mass flow
P^*	total pressure
Re	Reynolds number
R_n	rotate speed
r	radius
s	pitch
T^*	total temperature
t	blade thickness
U	blade tangential velocity
W	relative velocity
Z	main blade number
β	blade angle with respect to tangent
γ	meridional streamline slope angle
Δq	adiabatic head loss coefficient
δ	tip clearance
ζ	dimensionless splitter vane length
η	adiabatic efficiency
λ_b	tip distortion factor
${\pi_r}^*$	total pressure ratio
ρ	gas density
σ	slip factor
τ	quasi blade solidity
ϕ	flow coefficient
ω	rotate angular speed

Subscripts

B	main blade parameter
bl	blade loading
const	constant
cl	clearance gap parameter
hs	hub-to-shroud loading
inc	incidence
m	meridional, mean
maxeff	maximum efficiency point

mix	wake mixing
out	outlet
s	splitter vane (inlet)
sep	free stream fluid in wake
sf	skin friction
th	throat
total	total parameter
u	tangential
z	axial
1	impeller inlet
2	impeller outlet

Appendix 2

Some formulas of the related parameters in the modified loss models are shown below. More details about the original 1D models can be found in Reference [5].

Shape factor in slip factor model of Qiu et al. [22]:

$$F = 1 - \sin\frac{\pi}{Z}\sin\left(\frac{\pi}{Z}+\beta\right)\cos\beta\sin\gamma - \frac{s}{t\cos\beta} \tag{A1}$$

Tip distortion factor [6]:

$$\lambda_b = 1 - 0.003Z/\cos\beta \tag{A2}$$

Blade length estimation:

$$L_B \approx \frac{\pi}{4}(r_2 - r_1 + L_z)\left(\frac{2}{\frac{\cos\beta_{t1}+\cos\beta_{h1}}{2}+\cos\beta_2}\right) \tag{A3}$$

where β_t and β_h are tip and hub blade angles.

Ratio of mean hydraulic-to-exit diameter [4]:

$$\frac{d_{H_{1-s}}}{d_s} = \frac{\cos\beta_s}{\frac{Z}{\pi}+\frac{d_s\cos\beta_s}{b_s}} + \frac{\frac{d_{1t}}{d_s}}{\frac{2}{1-\lambda_1}+\frac{2Z}{\pi(1+\lambda_1)}\sqrt{1+\frac{1+{\lambda_1}^2}{2}\tan^2\beta_{t1}}} \tag{A4}$$

$$\frac{d_{H_{s-2}}}{d_2} = \frac{\cos\beta_2}{\frac{2Z}{\pi}+\frac{d_2\cos\beta_2}{b_2}} + \frac{\frac{d_{s_t}}{d_2}}{\frac{2}{1-\lambda_s}+\frac{4Z}{\pi(1+\lambda_s)}\sqrt{1+\frac{1+{\lambda_s}^2}{2}\tan^2\beta_{t_s}}} \tag{A5}$$

where λ is the inducer tip-exit diameter ratio.

Skin friction coefficient [6]:

$$C_f = 0.0046/Re^{0.2} \tag{A6}$$

Square of average blade channel velocity:

$$\bar{W}_{1-s}{}^2 = \left(W_1{}^2 + W_s{}^2\right)/2 \quad \text{(A7)}$$

$$\bar{W}_{s-2}{}^2 = \left(W_s{}^2 + W_2{}^2\right)/2 \quad \text{(A8)}$$

Average blade velocity difference:

$$\Delta W = 2\pi d_2 U_2 I_B / Z L_B \quad \text{(A9)}$$

Average pressure difference across the clearance:

$$\Delta P_{cl} = \frac{\dot{m} I_B}{Z \frac{r_1 + r_2}{2} \frac{b_1 + b_2}{2} L_m \omega} \quad \text{(A10)}$$

Equivalent diffusion factor:

$$D_{eq} = W_{max}/W_2 \quad \text{(A11)}$$

$$W_{max} = (W_1 + W_2 + \Delta W)/2 \quad \text{(A12)}$$

Free stream fluid velocity and outlet fluid velocity in the wake:

$$W_{sep} = \begin{cases} \frac{W_2 D_{eq}}{2} & ; \ D_{eq} > 2 \\ W_2 & ; \ D_{eq} \le 2 \end{cases} \quad \text{(A13)}$$

$$W_{out} = \sqrt{(C_{m2} A_2 / \pi d_2 b_2)^2 + W_u{}^2} \quad \text{(A14)}$$

Previous empirical formulas for picking blade number:

NASA [4]/ Pfleiderer [7]:

$$Z = k_z \frac{d_1 + d_2}{d_2 - d_1} \cos\left(\frac{\beta_1 + \beta_2}{2}\right) \quad \text{(A15)}$$

where $k_z = 6.5$ in Reference [4] and $k_z = 6.5$ to 8.0 for compressors in Reference [7].

Eckert [8]:

$$Z = \frac{2\pi \sin \beta_m}{0.4 \ln(d_2/d_1)} \quad \text{(A16)}$$

Rodgers [6]:

$$Z = 25 \sin \beta_2 / N_s \quad \text{(A17)}$$

where N_s is the specific speed.

Xu [9]:

$$Z = 25 \sin \beta_2 / N_s \quad \text{(A18)}$$