Appendix

Let $\hat{\alpha}_{2p}$ denotes $\hat{\alpha}_2$ at percentile p.

$$\hat{\alpha}_{2p} = \tilde{\alpha}_{2p,L} + \tilde{\alpha}_{2p,S} \tag{A.1}$$

where $E(\tilde{\alpha}_{2p,L}) = \alpha_{2p,L}$ and $E(\tilde{\alpha}_{2p,S}) = \alpha_{2p,S}$, with $\alpha_{2p,L}$ and $\alpha_{2p,S}$ denote the true parameter of agglomeration and the natural-selection-induced bias, respectively, and assume that

$$\tilde{\alpha}_{2p,L} \geq 0, \text{ and} \tag{A.2}$$

$$\tilde{\alpha}_{2p,S} \geq 0.\text{[1]} \tag{A.3}$$

While $\tilde{\alpha}_{2p,L}$ and $\tilde{\alpha}_{2p,S}$ are unobservable, $\hat{\alpha}_{2p}$ can be estimated from equation (1). Thus, we try to bound $\tilde{\alpha}_{2p,L}$ and $\tilde{\alpha}_{2p,S}$ with $\hat{\alpha}_{2p}$.

According to the prediction of the theory, we have

$$\tilde{\alpha}_{2p,L} \leq \tilde{\alpha}_{2p',L} \text{ and} \tag{A.4}$$

$$\tilde{\alpha}_{2p,S} \geq \tilde{\alpha}_{2p',S}, \text{ for } p < p' \tag{A.5}$$

Let $\hat{\alpha}_{2p*}$ denotes the minimum nonzero[2] $\hat{\alpha}_{2p}$ across all p's. By (A.1) and (A.3),

$$\hat{\alpha}_{2p*} = \tilde{\alpha}_{2p*,L} + \tilde{\alpha}_{2p*,S} \tag{A.6}$$

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[1] This is not a restrictive assumption for this paper per se, as I find almost none significantly negative effect of agglomeration on innovation at any percentile in any specification, but it does impose a restriction on data when applied elsewhere.

[2] $\hat{\alpha}_{2p}$ may be zero at the left-tail percentiles, as the least innovative establishments in or out of agglomerations may be non-innovators. These estimates are uninformative and therefore dismissed throughout this paper.

$\tilde{\alpha}_{2p*,S} \geq 0$ (A.7)

By (A.6) and (A.7),

$\tilde{\alpha}_{2p*,L} \leq \hat{\alpha}_{2p*}$ (A.8)

By (A.8) and (A.2),

$0 \leq \tilde{\alpha}_{2p*,L} \leq \hat{\alpha}_{2p*}$ (A.9)

By (A.4) and (A.9),

$\tilde{\alpha}_{2p,L} \leq \tilde{\alpha}_{2p*,L} \leq \hat{\alpha}_{2p*}$ for $p < p^*$ (A.10)

By (A.1) and (A.10),

$\tilde{\alpha}_{2p,S} \geq \hat{\alpha}_{2p} - \hat{\alpha}_{2p*}$ for $p < p^*$ (A.11)

Also, by (A.1)-(A.3), (A.10) and (A.11), we obtain

$0 \leq \tilde{\alpha}_{2p,L} \leq \hat{\alpha}_{2p*}$, and (A.12)

$\hat{\alpha}_{2p} - \hat{\alpha}_{2p*} \leq \tilde{\alpha}_{2p,S} \leq \hat{\alpha}_{2p}$ for $p < p^*$ (A.13)

These two equations bound the natural-selection-induced bias and the effect of agglomeration at percentiles smaller than p*.

By (A.2), $\tilde{\alpha}_{2p*,L} \geq 0$ (A.14)

By (A.6) and (A.14),

$\tilde{\alpha}_{2p*,S} \leq \hat{\alpha}_{2p*}$ (A.15)

By (A.5) and (A.15), $0 \leq \tilde{\alpha}_{2p*,S} \leq \hat{\alpha}_{2p*}$ (A.16)

By (A.5),

$\tilde{\alpha}_{2p,S} \leq \tilde{\alpha}_{2p*,S} \leq \hat{\alpha}_{2p*}$ for $p > p^*$ (A.17)

By (A.1) and (A.17),

$\tilde{\alpha}_{2p,L} \geq \hat{\alpha}_{2p} - \hat{\alpha}_{2p*}$ for $p > p^*$ (A.18)

Also, by (A.1)-(A.3), (A.17) and (A.18), we obtain

$0 \leq \tilde{\alpha}_{2p,S} \leq \hat{\alpha}_{2p*}$, and (A.19)

$\hat{\alpha}_{2p} - \hat{\alpha}_{2p*} \leq \tilde{\alpha}_{2p,L} \leq \hat{\alpha}_{2p}$ for $p > p^*$ (A.20)

These two equations bound the selection-induced bias and the agglomeration effect at percentiles greater than p*.

Summarizing (A.9), (A.12), (A.13), (A.16), (A.19) and (A.20), we obtain

$$\begin{cases} p = p^* \begin{cases} 0 \leq \tilde{\alpha}_{2p,L} \leq \hat{\alpha}_{2p*} \\ 0 \leq \tilde{\alpha}_{2p,s} \leq \hat{\alpha}_{2p*} \end{cases} \\ p < p^* \begin{cases} 0 \leq \tilde{\alpha}_{2p,L} \leq \hat{\alpha}_{2p*} \\ \hat{\alpha}_{2p} - \hat{\alpha}_{2p*} \leq \tilde{\alpha}_{2p,S} \leq \hat{\alpha}_{2p} \end{cases} \\ p > p^* \begin{cases} \hat{\alpha}_{2p} - \hat{\alpha}_{2p*} \leq \tilde{\alpha}_{2p,L} \leq \hat{\alpha}_{2p} \\ 0 \leq \tilde{\alpha}_{2p,S} \leq \hat{\alpha}_{2p*} \end{cases} \end{cases}$$

These inequalities finally bound the natural-selection-induced biases and the true effect of agglomeration at every percentile with estimated coefficients.