# Appendix 5: Recommendations for numerical integration

Many of the distributions frequently encountered in health economic modelling do not have a closed form, or convergent series representations, and so it is necessary to employ numerical integration techniques (or to make structural changes to the model so that the distribution can be approximated by an exponential, Erlang, Coxian or phase-type distribution, as described by van Rosmalen et al.1). Well-established numerical methods can be employed, in particular Gaussian quadrature methods.

Gauss–Laguerre quadrature with a simple substitution of gives:

Where and are the nodes and weights for -node Gaussian quadrature.

However, when is small this leads to evaluation of at large values (where it is typically close to zero), and ultimately very poor numerical performance.

Gauss–Laguerre quadrature can still be appropriate if instead the exponential term is extracted:

Gauss–Legendre quadrature can also be used with two different approaches. The first uses the substitution :

The second approach is based on the quantile function and the substitution :

|  |  |
| --- | --- |
|  | (A5:1) |

Of all the approaches, Equation (A5:1) appears to have the most desirable numerical qualities, since it avoids excessive exploration of very low density areas. However, the quantile function may not be readily available for all distributions in all settings.

In some cases it may be advantageous to split an integral in the following manner:

Where is selected such that covers the majority of the behaviour of , and may be informed by properties of the underlying random variables (e.g., ).

Gaussian quadrature schemes have the advantage that they can be readily implemented in spreadsheet software, since the weights and quadrature points can be hard-coded (provided a constant number of nodes is used).

Users are recommended to check that satisfactory convergence has been achieved, noting that errors will accumulate with arithmetic operations, and that convergence may depend on the values of parameters (e.g., in a probabilistic sensitivity analysis).

## References

1. van Rosmalen J, Toy M, O’Mahony JF. A Mathematical Approach for Evaluating Markov Models in Continuous Time without Discrete-Event Simulation. Medical Decision Making. 2013;33(6):767-79. doi: 10.1177/0272989x13487947