Appendix A: FEM verification

The goal of this appendix is to validate the structural performance of a solution selected from the Pareto fronts of Fig. 6 by using the Ansys® software. Indeed, we select the solution that corresponds to minimum reliability in cluster 6 (indicated by a red point in Fig. 6). First, by using Eq. (57), we can deduce the reliability of the two products (Machine A, Machine B). We then consider the second product (Machine B) whose characteristics are $R^{1}_{product}$=0.5027 and $R^{2}_{product}$=0.5 respectively. After that, by using the Ansys® Workbench tool, we verified structural performances for the gear pair, pinion and gearwheel shafts: the dedendum bending strength, surface contact strength and Von Mises stress. These results are compared with those given by the ISO standard and the ASME code.

The FEM based study is performed as follows. In SolidWorks 2015, 3D-CAD models of gear pair, pinon and gearwheel shafts are built based on the geometry data of the selected solution. The involute tooth profile is created by using "Equation Driven Curve" in SolidWorks. The geometric models are then imported directly into Ansys Workbench 16, without losing accuracy, via a bidirectional interface. The gear finite element model includes five teeth for the gearwheel and five teeth for the pinion. The middle pinon tooth is at the beginning of the mesh in the model configuration, it represents the target tooth in static analysis in order to determinate the bending and contact stress (Fig. 7a). The finite element models of the pinion and gearwheel shafts are also built by including keyway dimensions that are designed in accordance to the ISO standard. Due of the model symmetry, only a half part of the shaft is considered in static analysis (Fig. 7b). In Ansys Workbench, we created two static structural analyses: the first evaluates the bending and contact stress of the gear pair (Fig. 8a) and the second evaluates the Von Mises stress of the pinion and gearwheel shafts (Fig. 8b).

The data used in the finite element analysis and the results of two static structural analyses are summarized in Tables 9 and 10. Fig. 9 illustrates some selected FEM results. Globally, we noticed that the numerical results predicted by analytical formulas employed during the optimization process, particularly the ISO standard for gears and the ASME code for shafts, are in good agreement with those given by FEM using the Ansys ® software.

Nomenclature

P Transmitted power (W)
u Gear ratio
α Pressure angle (degree)
β Helix angle (degree)
a Distance center (mm)
m_n Normal module (mm)
D_1 Pinion pitch diameter (mm)
D_2 Gearwheel pitch diameter (mm)
d_{a1} Pinion shaft diameter (mm)
d_{a2} Gearwheel shaft diameter (mm)
F_t Rated tangential tooth force at transverse pitch (N)
N_1 Rotation speed (rmp)
Z_β Helix angle coefficient
Z_ε Contact ratio coefficient
Z_H Nodal field coefficient
Z_N Life coefficient
Z_L Lubricant coefficient
Z_V Velocity coefficient
Z_R Tooth fineness coefficient
Z_E Elastic coefficient
MOOP Multi-Objective Optimization Problem
ASME American Society of Mechanical Engineers
ISO International Organization for Standardization
NSGA-II Non-dominated Sorting Genetic Algorithm
Z_W Work harden coefficient
Z_X Size coefficient
$\sigma_{F\,lim}$ Experimental gear bending fatigue strength(N/m^2)
$\sigma_{H\,lim}$ Experimental gear contact fatigue strength (N/m^2)
σ_{uts} Ultimate tensile strength (N/m^2)
$K_{H\beta}$ Longitudinal load distribution coefficient
$K_{H\alpha}$ Transverse load distribution coefficient
K_V Dynamic load coefficient
K_A Work condition coefficient
$K_{F\beta}$ Longitudinal load distribution coefficient
$K_{F\alpha}$ Transverse load distribution coefficient
Y_β Helix angle coefficient
Y_ε Contact ratio factor
Y_{NT} Life coefficient
$Y_{S\alpha}$ Dedendum stress concentration coefficient
Y_X Size coefficient
$Y_{\delta\, relT}$ Relative sensitive coefficient
$Y_{F\alpha}$ Tooth form factor
Y_{RrelT} Relative surface condition coefficient
Y_{ST} Experimental gear dedendum stress concentration coefficient
DIN Deutsches Institut für Normung
SKF Svenska KullagerFabriken
AGMA American Gear Manufacturers Association

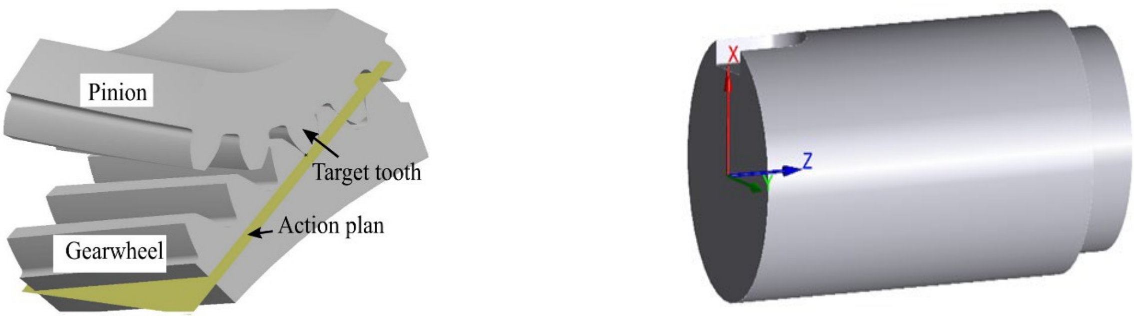
Table 9. Main parameters of the gear pair and the associate FEM study

Material	42CrMo4	
Common geometric parameters	Normal module (mm) =3.5	Width (mm)= 28.021
	Normal pressure angle(deg)=20	Helix angle (deg)= 15
	Contact ratio=2.115	
Pinion	Number of teeth	22
	Speed (rpm)	1150
	Torque (Nm)	124.556
Gearwheel	Number of teeth	77
	Speed (rpm)	328.57
	Torque (Nm)	435.946
Connections	Contact	Five pinion tooth surfaces
	Target	Five wheel tooth surfaces
	Contact Type	Frictionless
	Formulation	Augmented Lagrange
	Penetration Tolerance Value	1E-3mm
	Update Stiffness	Each Iteration
	Interface Treatment	Adjust to Touch
	Joints (Pinion - Gearwheel hubs)	Body-Ground Revolute joint
Mesh	Physic preference	CFD
	Element order	Quadratic

	Method	Sweep
	Sweep Number Division	20
	Pinion and Gearwheel sizing	1mm
	Contact surfaces and root teeth sizing	0.15mm
	Total number of nodes	432962
	Total number of elements	115024
Analysis setting	Auto Time stepping	On
	Initial Substeps	80
	Minimum Substeps	80
	Maximum Substeps	100
	Large Deflection	On
Loads and Supports (see Fig. 8*a*)	Joint- Moment	Joint Revolute - Ground to pinon
	DOF	Rotation Z
	Magnitude	Tabular Data (-124.556Nm)
	Joint- Rotation	Joint Revolute - Ground to Gearwheel
	DOF	Rotation Z
	Magnitude	10 degree
Comparison between FEM and ISO results		
Dedendum bending stress	FEM: 142.37 MPa	ISO: 166.56 MPa
Surface contact stress	FEM: 729.33 MPa	ISO: 706.88 MPa

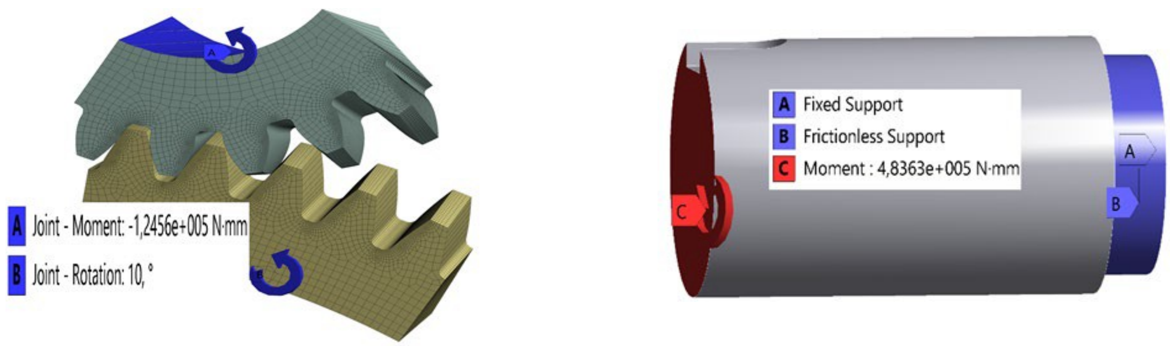
Table 10. Main parameters of the shafts and the associate FEM study

Material	25CrMo4	
Pinion shaft	Diameter	46.32 mm
	Length	200 mm
	Speed	1150 rpm
	Torque	124.556 Nm
Gearwheel shaft	Diameter	51.07 mm
	Length	200 mm
	Speed	328.57 rpm
	Torque	435.946 Nm
Mesh	Method	Hex Dominant
	Body sizing	2mm
	Fillet keyway sizing	0.2 mm
	Total number of nodes (half pinion)	509182
	Total number of nodes (half shaft)	1028104
	Total number of elements (half pinion)	148973
	Total number of elements (half shaft)	300775
Loads and Supports (see Fig. 8*b*)	Fixed supports	Frictionless Support
	Pinion shaft moment	X component = -74.933 Nm Y component = 198.860 Nm Z component = 124.556 Nm
	Gearwheel shaft moment	X component= -74.933 Nm Y component = 198.860 Nm Z component = 435.946 Nm
Comparison between FEM and ASME results		
Von Mises stress - pinon shaft	FEM: 82.35 MPa	ASME: 93.574 MPa
Von Mises stress - gearwheel shaft	FEM: 126.69 MPa	ASME: 132.58 MPa



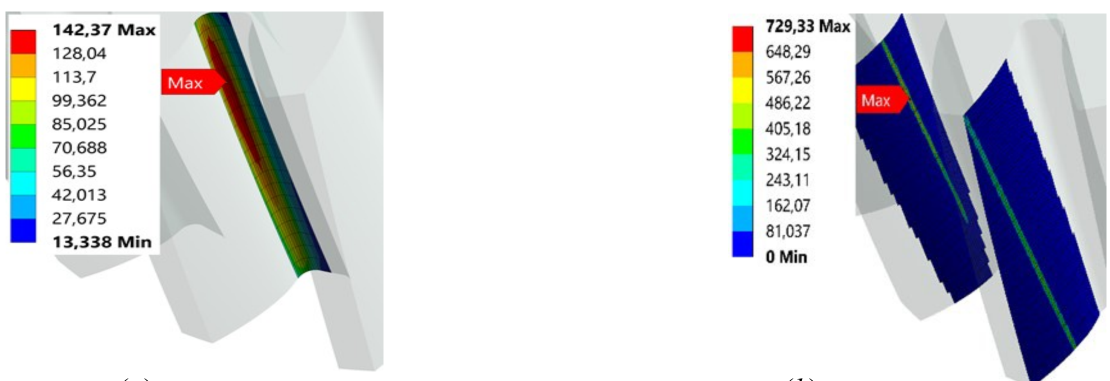
(*a*) (*b*)

Figure 7. Geometry model of: (*a*) helical gear pair, (*b*) half shaft.



(*a*) (*b*)

Figure 8. FEM loads and supports: (*a*) two joints applied on hubs, (*b*) boundary conditions.



(*a*) (*b*)

(*c*) (*d*)

Figure 9. FEM Results: (*a*) bending stress, (*b*) contact stress, (*c*) gearwheel shaft (*d*) pinion shaft.