“What are the Effects of Entry of New Extremist Parties on the Policy Platforms of Mainstream Parties?”

PROPOSITION 1: In a model consisting of only the two mainstream parties, assume that they are policy seeking based on the parliamentary mean, $q_2 \le \bar{x}$ and $q_3 \ge \bar{x}$. Assume further that the voter density, f, is symmetric and $f(0) > 0$. Then a unique Nash equilibrium occurs for $x_2 = -1/(2f(0))$ and $x_3 = +1/(2f(0))$.[1]

PROOF: First we show that if a Nash equilibrium exists for which x_2 and x_3 are symmetric about 0, then it is of the form specified. Note that
$U_3 = p_2 x_2 + p_3 x_3 - q_3 = F(m_{23})x_2 + [1 - F(m_{23})]x_3 - q_3$, so that

$$\begin{aligned} \frac{\partial U_3}{\partial x_3} &= x_2 f(m_{23})/2 + [1 - F(m_{23})] - x_3 f(m_{23})/2 \\ &= -f(m_{23})(x_3 - x_2)/2 - F(m_{23}) + 1 = 0, \end{aligned} \tag{A1}$$

because x_2 and x_3 constitute a Nash equilibrium. By the symmetry assumption above, $m_{23} = 0$, so that $F(m_{23}) = 1/2$. We conclude that
$f(0)(x_3 - x_2)/2 = 1/2$, so that

$$x_3 = 1/(2f(0)), \text{ and similarly, } x_2 = -1/(2f(0)). \tag{A2}$$

[1] Brams and Merrill (1991) obtain (and Adams and Merrill, 2006, cite) a similar-looking formula for a two-party equilibrium in which parties seek to maximize the probability that they obtain the vote of the median voter. In their case, however, the density function f represents the parties' uncertainty about the location of the median voter, whereas in the present case f represents the distribution of voters.

Conversely, we show that $x_2 = -1/(2f(0))$ and $x_3 = +1/(2f(0))$ constitute a Nash equilibrium. If x_2 is fixed at $-1/(2f(0))$, then

$$\left.\frac{\partial U_3}{\partial x_3}\right|_{x_3=1/(2(f(0))} = -f(0)(1/f(0))/2 - F(0) + 1 = 0$$ by equation (A1) above. Furthermore,

$$\frac{\partial^2 U_3}{\partial x_3^2} = -f(m_{23}) - (1/4)f'(m_{23})(x_3 - x_2)$$. Thus,

$$\left.\frac{\partial^2 U_3}{\partial x_3^2}\right|_{x_3=1/(2f(0))} = -f(0) - (1/4)f'(0)(1/f(0)) = -f(0) < 0$$ because $f'(0) = 0$ (by symmetry of the voter density) and $f(0) > 0$. It follows that U_3 has a maximum at $1/(2f(0))$ (and a similar result holds for U_2 with x_3 fixed), so that these formulas in equation (A2) define a Nash equilibrium.

Next we show that if x_2 and x_3 are not symmetrically located, i.e., that $m_{23} \neq 0$, then they do not constitute a Nash equilibrium. Suppose by way of contradiction, that $\bar{x}_2$ and $\bar{x}_3$ do constitute a Nash equilibrium for which $\bar{m}_{23} = \dfrac{\bar{x}_2 + \bar{x}_3}{2} \neq 0$. Without loss of generality, assume that $\bar{m}_{23} < 0$. Then $\bar{x}_3$ is a solution of the equation

$$\frac{\partial U_3}{\partial x_3} = -f(\bar{m}_{23})(x_3 - x_2)/2 - F(\bar{m}_{23}) + 1 = 0$$, so that

$$\bar{x}_3 = \bar{m}_{23} + \frac{1 - F(\bar{m}_{23})}{f(\bar{m}_{23})} \text{ and } \bar{x}_2 = \bar{m}_{23} - \frac{1 - F(\bar{m}_{23})}{f(\bar{m}_{23})}.$$

Since $\dfrac{\partial U_2}{\partial x_2} = f(m_{23})(x_3 - m_{23}) - F(m_{23})$, we have

$$\begin{aligned}\left.\frac{\partial U_2}{\partial x_2}\right|_{x_2=\bar{x}_2} &= f(\bar{m}_{23})(\bar{x}_3 - \bar{m}_{23}) - F(\bar{m}_{23}) = f(\bar{m}_{23})\left[\frac{1 - F(\bar{m}_{23})}{f(\bar{m}_{23})}\right] - F(\bar{m}_{23}) \\ &= 1 - 2F(\bar{m}_{23}) > 0\end{aligned}.$$

Hence U_2 is strictly increasing at $\bar{x}_2$, so that $\bar{x}_2$ and $\bar{x}_3$ do not constitute a Nash equilibrium. We conclude that the Nash equilibrium specified in the Proposition is unique. q.e.d.

LEMMA 1: Assume that the two mainstream parties are policy seeking based on the parliamentary mean, $q_2 \le \bar{x}$ and $q_3 \ge \bar{x}$. If there is one extreme party x_4 on the right (the three-party model), then the rate of change of utility of each mainstream party with respect to that party's location is given by

$$\begin{aligned}\frac{\partial U_3}{\partial x_3} &= \int_{m_{23}}^{m_{34}} f(x)dx - \left[f(m_{23})(x_3 - m_{23}) + f(m_{34})(m_{34} - x_3) \right] \\ \frac{\partial U_2}{\partial x_2} &= -\int_{0}^{m_{23}} f(x)dx + f(m_{23})(m_{23} - x_2)\end{aligned} \quad . \tag{A3a}$$

If there are two extreme parties x_1 and x_4, the first on the left and the latter on the right (the four-party model), then the rate of change of utility of each mainstream party with respect to that party's location is given by

$$\begin{aligned}\frac{\partial U_3}{\partial x_3} &= \int_{m_{23}}^{m_{34}} f(x)dx - \left[f(m_{23})(x_3 - m_{23}) + f(m_{34})(m_{34} - x_3) \right] \\ \frac{\partial U_2}{\partial x_2} &= -\int_{m_{12}}^{m_{23}} f(x)dx + \left[f(m_{12})(x_2 - m_{12}) + f(m_{23})(m_{23} - x_2) \right]\end{aligned} \quad . \tag{A3b}$$

Assume further that the voter density, f, is symmetric and unimodal. If $\partial U_3 / \partial x_3$ is continuous, then, for any fixed value of x_2, U_3, reaches a maximum at some point x_3' on the interval from 0 to x_4 and this point satisfies the recursive equation

$$x_3 = \frac{p_3 + m_{23} f(m_{23}) - m_{34} f(m_{34})}{f(m_{23}) - f(m_{34})} \, . \tag{A4}$$

Similar recursive equations for maximum points hold for x_2 and x_3 in the 3-party and 4-party scenarios.

PROOF: We derive the first formula in (A3a). The other derivations for (A3a) and (A3b) are similar. Utility for party 3 is:

$$U_3 = -|q_3 - \bar{x}| = -\left|q_3 - \sum_{i=2}^{4} p_i x_i\right|$$
$$= F(m_{23})x_2 + F(m_{34})x_3 - F(m_{23})x_3 + x_4 - F(m_{34})x_4 - q_3$$
$$= x_4 - F(m_{23})(x_3 - x_2) - F(m_{34})(x_4 - x_3) - q_3$$

where m_{ik} is the midpoint between x_i and x_k. It follows that

$$\frac{\partial U_3}{\partial x_3} = -F(m_{23}) - \frac{1}{2} f(m_{23})(x_3 - x_2) + F(m_{34}) - \frac{1}{2} f(m_{34})(x_4 - x_3)$$
$$= p_3 - \frac{1}{2}\left[f(m_{23})(x_3 - x_2) + f(m_{34})(x_4 - x_3)\right]$$
$$= \int_{m_{23}}^{m_{34}} f(x)dx - \left[f(m_{23})(x_3 - m_{23}) + f(m_{34})(m_{34} - x_3)\right].$$

To prove the second part of the proposition, recall that the voter density is assumed symmetric and unimodal. Note that, for $x_3 = 0$, $\partial U_3/\partial x_3 \geq 0$ by applying equation (3a), (because, by unimodality of f, $f(x) \geq f(m_{23})$ *for* $m_{23} \leq x \leq x_3$ and $f(x) \geq f(m_{34})$ *for* $x_3 \leq x \leq m_{34}$; see Figure A1, part a), whereas for $x_3 = x_4$, $\partial U_3/\partial x_3 \leq 0$, (again, by unimodality of f; see Figure A1, part b). Thus since $\partial U_3/\partial x_3$ is continuous, then $\partial U_3/\partial x_3 = 0$ for some x_3' between 0 and x_4. Since $\partial U_3/\partial x_3\big|_{x_3=0} \geq 0$ and $\partial U_3/\partial x_3\big|_{x_3=x_4} \leq 0$, U_3 must have a maximum at one such point x_3' between 0 and x_4 that is a solution of $\partial U_3/\partial x_3 = 0$. Solving the first equation in (A3a) for x_3, we obtain the three-party recursive equation in (A4). Derivation of the four-party recursive equation is similar. q.e.d.

<<< Figure A1 about here >>>

PROPOSITION 2 (Effects of Extreme Party in a 3-Party Scenario): Assume the voter distribution is symmetric and unimodal, $f(0) > 0$, and the two mainstream parties are policy seeking based on the parliamentary mean, and that $q_2 \leq \bar{x}$ and $q_3 \geq \bar{x}$. If a right wing extreme party is added to this two party scenario, both the mainstream

parties move to the left. Similarly, if the extreme party is left wing, the mainstream parties move to the right.

PROOF: For a *2*-party scenario, by Proposition 1, $x_2 = -1/(2f(0))$ and $x_3 = +1/(2f(0))$ constitute a unique Nash equilibrium. Denote these equilibrium locations by $x_2(2)$ and $x_3(2)$, so that, in particular, $x_2(2) = -1/(2f(0))$ and $x_3(2) = +1/(2f(0))$; in turn, $m_{23} = 0$ in this 2-party scenario. Suppose that x_3 is set at $x_3(2)$ and a right wing extreme party is added to the scenario at x_4. As before, the voter distribution has cumulative distribution function F and density function f. Then, $\partial U_3/\partial x_3 = -A + B$, where $A = -\int_{m_{23}}^{x_3} f(x)dx + f(m_{23})(x_3 - m_{23})$ and $B = \int_{x_3}^{m_{34}} f(x)dx - f(m_{34})(m_{34} - x_3)$ are both non-negative (because of unimodality of f), (see equations (5a) and Figure 3, both in the main text), so we have

$$A = f(0) * x_3(2) - [F(x_3(2)) - 0.5]$$

$$= f(0) * \frac{1}{2f(0)} - F(x_3(2)) + 0.5$$

$$= 1 - F(x_3(2)) = \int_{x_3(2)}^{\infty} f(x)\, dx \ ,$$

and

$$B = \int_{x_3(2)}^{m_{34}} f(x)dx - f(m_{34})(m_{34} - x_3) \le \int_{x_3(2)}^{\infty} f(x)\, dx .$$

Thus,

$$A \ge B .$$

Because $\frac{\partial U_3}{\partial x_3} = B - A$, it follows that

$$\frac{\partial U_3}{\partial x_3} \le 0 \text{ at } x_3 = x_3(2) ,$$

so party 3 gains utility (or does not lose utility) by moving to the left of $x_3(2)$.[2]

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[2] This inequality is strict unless party 4 is so far to the right that the midpoint between $x_3(2)$ and x_4 lies to the right of the entire voter distribution, a highly atypical situation.

By equations (5b) (see also Figure 3), both in the main text, $\frac{\partial U_2}{\partial x_2} = A' - B'$. As long as x_2 and x_3 are in their 2-party-scenario equilibrium positions, $m_{23} = 0$ and $\frac{\partial U_2}{\partial x_2} = A' - B' = 0$. But if x_3 moves to the left, so does m_{23} and the quantity A' declines (because f is increasing on the left of 0), so that at $x_2(2)$, $\frac{\partial U_2}{\partial x_2} = A' - B' \leq 0$. Thus party 2 gains utility (or does not lose utility) by moving to the left of $x_2(2)$. q.e.d.

PROPOSITION 3 (Effects of Extreme Parties in a 4-Party Scenario): Assume the voter distribution is symmetric and unimodal, $f(0) > 0$, and the two mainstream parties are policy seeking based on the parliamentary mean, and that $q_2 \leq \bar{x}$ and $q_3 \geq \bar{x}$. If we add to this two party situation two extreme parties at fixed positions, one to the left and one to the right, then both mainstream parties move inward, i.e., become less extreme.

PROOF: By Proposition 1, given two (mainstream) parties, equilibrium occurs for $x_2 = -1/(2f(0))$ and $x_3 = +1/(2f(0))$. Denote these values by $x_2(2)$ and $x_3(2)$, to indicate that they are equilibrium locations for a *2*-party scenario, so that, in particular, $x_2(2) = -1/(2f(0))$ and $x_3(2) = +1/(2f(0))$. With the addition of extreme parties 1 and 4, located equidistant from 0 and on either side, we can assume that the new equilibrium positions of parties 2 and 3, to be denoted by $x_2(4)$ and $x_3(4)$, are also equidistant from 0 and on either side of 0, so that $m_{23} = 0$ (for either the 2-party or 4-party scenario). As before, the voter distribution has cumulative distribution function F and density function f. If x_3 is set as $x_3(2)$ then (see Figure 3 in the main text, noting that the following equation is not affected by the presence of party 1),

$$A = f(0) * x_3(2) - \left[F(x_3(2)) - 0.5\right]$$

$$= f(0) * \frac{1}{2 f(0)} - F(x_3(2)) + 0.5$$

$$= 1 - F(x_3(2)) = \int_{x_3(2)}^{\infty} f(x)\, dx \ ,$$

and

$$B \le \int_{x_3(2)}^{\infty} f(x)\, dx \, .$$

Thus,

$$A \ge B \, .$$

Because $\frac{\partial U_3}{\partial x_3} = B - A$, it follows that

$$\frac{\partial U_3}{\partial x_3} \le 0 \ \text{ at } \ x_3 = x_3(2) \, ,$$

so party 3 gains utility (or does not lose utility) by moving to the left of $x_3(2)$. Because of the symmetry, a similar argument applies to party 2, so that $\frac{\partial U_2}{\partial x_2} \ge 0$ at $x_2 = x_2(2)$. Thus, party 2 gains utility (or does not lose utility) by moving to the right of $x_2(2)$. Finally, note that inequality $\frac{\partial U_3}{\partial x_3} \le 0$ is strict unless party 4 is so far to the right that the midpoint between $x_3(2)$ and x_4 lies to the right of the entire voter distribution, a highly atypical situation. A similar statement holds concerning $\frac{\partial U_2}{\partial x_2} \ge 0$. q.e.d.

PROPOSITION 4: Divergence of Equilibria for Bimodal Voter Distributions. Assume that the voter density f is symmetric and bimodal, and that c denotes the least value greater than the right-hand mode for which $f(c) = 0$ (see Figure A2). In the four-party model, if x_3 and $x_2 (= -x_3)$ constitute a Nash equilibrium, and $x_1 < x_2$, $x_4 > x_3$, then

$$x_3 \ge c + \frac{h}{f(0)}, \tag{A5}$$

where $h = \int_0^c [f(x) - f(0)]\,dx$ is the area of (either) hump.

PROOF: Note first that U_3 may have a critical point between 0 and c, but such a critical point defines a minimum, not a maximum for U_3. For U_3 to have a critical point maximum at a point x_3 greater than c, requires that

$$\frac{\partial U_3}{\partial x_3} = \int_0^{x_3} f(x)\,dx - f(0)[x_3 - 0] + \int_{x_3}^{m_{34}} f(x)\,dx - f(m_{34})[m_{34} - x_3] = 0, \qquad \text{(A6)}$$

since $m_{23} = 0$. We will construct a value $\bar{x}$ that is greater than or equal to $c + \dfrac{h}{f(0)}$ and such that x_3 must be greater than $\bar{x}$ for equation (A6) to hold. That will show that $x_3 \ge c + \dfrac{h}{f(0)}$. To do this, because f is decreasing to the right of c, we can choose $\bar{x}$ greater than c so that

$$\int_0^{\bar{x}} f\,dx = f(0)\bar{x}. \qquad \text{(A7)}$$

We first show that $x_3 \ge \bar{x}$. Again, because f is decreasing to the right of c, the term $\int_{x_3}^{m_{34}} f(x)\,dx - f(m_{34})[m_{34} - x_3] \ge 0$, which implies that for $\dfrac{\partial U_3}{\partial x_3} = 0$ (that is, for equation (A6) to hold), the term $\int_0^{x_3} f(x)\,dx - f(0)x_3$ must be non-positive. But the function g, defined by $g(x) = \int_0^x f(t)\,dt - f(0)x$, is decreasing in x and $g(x_3) \le 0 = g(\bar{x})$ by equation (A7), so that $x_3 \ge \bar{x}$. Finally, equation (A7) can be rephrased as $\int_0^c [f(x) - f(0)]\,dx = \int_c^{\bar{x}} [f(0) - f(x)]\,dx$, or equivalently, using the definition of h, as $h = \int_c^{\bar{x}} [f(0) - f(x)]\,dx$, which in turn becomes $f(0)(\bar{x} - c) = h + \int_c^{\bar{x}} f(x)\,dx$, so that, dropping the non-negative term $\int_c^{\bar{x}} f(x)\,dx$, we obtain

$$\bar{x} \ge c + \frac{h}{f(0)}, \text{ so } x_3 \ge c + \frac{h}{f(0)}. \text{ q.e.d.}$$

<<< Figure A2 about here >>>

PROPOSITION 5 (Policy-seeking Nash Equilibrium): Assume a four-party model for which the voter density, f, is symmetric and unimodal and that $\partial U_3/\partial x_3$ is continuous as a function of x_2 and x_3. Furthermore, if recursive application of equation (A4) leads to sequences of values of x_3' that converge to a limit x_3* (and x_2* is defined as $-x_3*$), then (x_2*, x_3*) constitute a policy-seeking Nash equilibrium.[3]

PROOF: To be specific, define $x_2^{(0)} = x_3^{(0)} = 0$, define $x_3^{(1)}$ by equation (A4) and specify $x_2^{(1)} = -x_3^{(1)}$. Recursively, define $x_3^{(n)}$ by equation (A4) and $x_2^{(n)} = -x_3^{(n)}$. The hypothesis states that $x_3^{(n)}$ converges to a limit, x_3*, and $x_2* = -x_3*$. By equation (A4), $\partial U_3/\partial x_3 = 0$ when $x_2 = x_2^{(n-1)}$ and $x_3 = x_3^{(n)}$, for all positive n. Thus, by Lemma 1, U_3 has a maximum at $x_3 = x_3^{(n)}$ for given $x_2 = x_2^{(n-1)}$. By the continuity of $\partial U_3/\partial x_3$,

$$\partial U_3/\partial x_3 \Big|_{|x_2 = x_2*, x_3 = x_3*} = \lim_{n \to \infty} \partial U_3/\partial x_3 \Big|_{|x_2 = x_2^{(n-1)}, x_3 = x_3^{(n)}} = 0 .$$

Thus, U_3 has a maximum at $x_3 = x_3*$, given $x_2 = x_2*$ (and for $x_2 = x_2*$, given $x_3 = x_3*$). q.e.d.

Contrast between the Calvert (1985) model and the parliamentary-mean model.

Calvert (1985) obtains convergence to the median under assumptions that are quite different from the parliamentary-mean model. He defines the goal of each party as maximizing its utility for the winning platform, for example, for Party 3, that is minimizing the quantity $|q_3 - y|$, where y is the location of the (single) party who wins a majority. Under the parliamentary-mean model, by contrast, Party 3 seeks to minimize

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[3] For a uniform voter distribution, $\partial U_3/\partial x_3$ is constant, so all locations for x_2 and x_3 that are more moderate than the mainstream parties' ideal points are trivially Nash equilibria.

$\left|q_3 - \bar{x}\right|$, where the parliamentary mean $\bar{x}$ weights the vote share (and hence the seat share) of both parties.

In the Calvert model, if, say, the center-left party is to the left of the median, the center-right party can move closer to the median and win with a platform to the right of the median. In the same situation under the parliamentary-mean model, movement by the center-right party closer to the median would move the parliamentary mean into negative territory as long as the two parties are not too far from the median.

Detailed Effects of Alternative Assumptions on the Locations of Nash Equilibria

1. What if the mainstream parties evaluate utility from the locations of only the mainstream parties?

As noted earlier, it may make sense, at least for some countries to explore an alternative utility function in which the mainstream parties evaluate their utility of the election outcome entirely in terms of the locations of the mainstream parties. This might make sense if the mainstream parties consider the extreme parties as pariahs that could not be considered for a coalition government under any circumstances. Accordingly, utility for party j in the four-party model is given by

$$U_j'' = -\left| q_j - \frac{(p_2 x_2 + p_3 x_3)}{(p_2 + p_3)} \right|.$$

Under this alternative utility function, given a normal voter distribution with mean 0 and standard deviation 0.5, we investigated locations of the extreme right party anywhere between 0.75 and 1.5 (and the extreme left party between -0.75 and -1.5). Throughout this range, numerically evaluated Nash equilibria for the mainstream parties range from -0.639 to -0.648 for x_2 and 0.639 to 0.648 for x_3. These locations for the mainstream parties are comparable to their equilibrium strategies when no extreme parties are present and indicate none of the moderation that is projected when extreme parties do exist and are accounted for in mainstream party utilities. But under the present assumption, neither is the mainstream response to extreme party entry more extreme. Intuitively, when, say, a center-right party that discounts the contribution of an extreme right party to policy formation moderates, no value is provided to compensate for the mainstream party's loss of vote share to that extreme party. This reduces the motivation to moderate relative to a center-right party that does assign utility to the position of the party on its flank. If the mainstream parties only partly take into account the locations of extreme parties in assessing their utility, then equilibrium positions are intermediate between those for U and U''.[4]

2. What role might abstention play?

So far we have assumed that citizens' decisions whether to cast a vote are not affected by the locations of available parties. Suppose instead that citizens located at relatively extreme locations abstain if no party is located near their position. What qualitative effects would this assumption have on our results? Consider a scenario in which only the two mainstream parties are in the competition and they face the threat of abstention by extreme members of the potential electorate. To be specific, suppose that all citizens abstain if located at least a distance d more extreme than the mainstream parties. We can show that at equilibrium the location of, say, the mainstream party on the right satisfies the recursive equation $x_3 = [2F(x_3 + d) - 1]/[2 * f(0)]$.[5] Thus, for a normal distribution with mean 0 and standard deviation 0.5, if there were no abstention, $x_3 = 0.627$, but with abstention, $x_3 = 0.626, 0.610, \mathit{or}\ 0.561$, if $d = 1, 0.5, \mathit{or}\ 0.25$, respectively. This example suggests that plausible levels of abstention do not greatly affect the mainstream strategies when no extreme parties are present. These strategies are still more extreme than the equilibrium strategies (± 0.427; see Table 1) that would occur if extreme parties do enter and attract the votes of citizens who would otherwise have abstained.

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[4] If a lottery over competing parties, rather than the parliamentary mean, is used to specify utility, then the effect of using only the mainstream parties to define utility is the same as when the parliamentary mean specification is employed, as long as the ideal points of the mainstream parties are at least as extreme as their declared positions.

[5] Modifying slightly the proof of Lemma 1, we conclude that $x_3 f(0) = \int_0^{x_3+d} f(x)\,dx$, so that $x_3 = [2F(x_3 + d) - 1]/[2 * f(0)]$.

Thus, even with the threat of abstention when extreme parties are not available, it appears likely that mainstream parties would be motivated to moderate their strategies when extreme parties do materialize, just as when there was no abstention. Accordingly, accounting for abstention does not change our results qualitatively.

3. What if the extreme parties are themselves mobile?

Next, we return to the utility function U based on the parliamentary mean and on all parties, to investigate what happens when the extreme parties can also change location (while as before, we allow the mainstream parties to change location). Even when constraints on movement by the extreme parties are relaxed, the differences between this more general scenario and what we obtained when we held the positions of extremist parties fixed tend to be minor -- as shown in Table A.1. Intuitively, given policy-seeking incentives, the additional vote share gained from moderation by mobile extreme parties generally forfeits too much utility in moving away from the party's most preferred policy position to make substantial movement in a centrist direction a viable option for extremist parties.[6]

<<< Table A.1 about here >>>

4. What if each mainstream party values its vote share per se in addition to its contribution to the parliamentary mean?

Although a party's vote share is a significant constituent of its policy-seeking utility, we can imagine that a party may value its vote share per se in addition to its effect

[6] However, when a previously extreme party commits to a clearly centrist move, i.e., actually moderates its ideal point, it may be because its new leadership has made a conscious decision to reject a veto by its more extreme elements in the confidence that the new electorate gained by a more centrist move will provide a new and offsetting cadre of supporters. This may be the mechanism underlying Marine Le Pen's decision to "evict" her father from the party that he founded (BBC, 2015)!

on its policy-seeking utility. Valuation of vote share per se could be interpreted as an office-seeking motivation. Although it is difficult to determine a common scale on which to precisely compare these two effects, adding an independent vote-share component exerts a strong moderating effect on optimal strategies of mainstream parties, drawing them toward the center point of the scale, whether or not extreme parties impinge. However, mainstream parties are drawn to complete convergence with a smaller admixture of vote share when extreme parties are present than when they are not.

5. What is the effect of alternative locations for the fixed positions of the extreme party(s)? Calculation of Nash equilibria for the 3-party and 4-party models with normal voter distributions.

For a parliamentary mean model with $q_2 \le \bar{x}$ and $q_3 \ge \bar{x}$, for the two-party, three-party, and four-party models and normal voter distributions (with standard deviation = 0.5), Table A.2 depicts Nash equilibria, for a range of possible fixed positions of the extreme parties, for both the parliamentary-mean and dominant-party models. As the fixed locations for the extreme parties become more extreme (ranging from locations one standard deviation from the mean to three standard deviations from the mean), the Nash equilibria for the mainstream parties are correspondingly more extreme, and concomitantly, their vote shares are substantially larger. Note that the patterns of movement by mainstream parties upon entry by extreme parties are qualitatively similar either when the ideal points of the mainstream parties are as extreme as the locations of the extreme parties or when these ideal points are less extreme. The degree of movement of mainstream positions in response to extreme-party entrance, however, is dampened in the latter case.[7] Finally, Table A.2 reports Nash equilibrium positions for the dominant party model with $q_2 > x_1$ and $q_3 < x_4$.

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[7] Note, as before, that under the assumptions, the equilibrium location for x_3 is the same for any $q_3 \ge \bar{x}$, because $\partial U_3 / \partial x_3$ is independent of the location of q_3 as long as $q_3 \ge \bar{x}$.

<<< Table A.2 about here >>>

6. What if, in the dominant-party model, seat share is super-proportional, relative to vote share.

Suppose, in the dominant-party model, party influence on governmental policy is proportional not to vote share, but to seat share, i.e., relative to smaller parties, assume that larger parties have more influence on policy relative to smaller parties than their vote proportions would indicate.[8] A simple model for this effect is to assume that seat share is proportional not to p_i, but rather to p_i^2.[9] Under this assumption, for a normally distributed voter distribution with mean 0 and standard deviation 0.5, equilibrium strategies for the mainstream parties in the two-party, dominant party scenario are constricted by fifty percent, from ± 0.627 to ± 0.314, and a bit less in the four-party model, from ± 0.427 to ± 0.235. This movement toward the median is to be expected because super-proportional party influence gives greater value to vote shares relative to position.

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[8] If we interpret the dominant-party model in a plurality-based setting, the super-proportional assumption suggests that larger parties have a disproportionate probability of selection as the winner, a not-unreasonable assumption.

[9] Thus, the weights representing party influence are given by $p_j / \sum_i p_i^2$, so, for example, if vote shares in a four-party model are (0.1, 0.4, 0.4, 0.1), then the weights are approximately (0.03, 0.47, 0.47, 0.03).

Table A.1. Policy-seeking Nash equilibrium assuming <u>all parties are mobile</u>, with two-, three-, and four-party contests, assuming a normal electorate.

	x_1	x_2	x_3	x_4
2 parties		**-0.627** (50.0%)	**0.627** (50.0%)	
3 parties		**-0.777** (38.2%)	**0.476** (54.8%)	**0.998** (7.0%)
4 parties	**-0.96** (8.5%)	**-0.415** (41.5%)	**0.415** (41.5%)	**0.96** (8.5%)

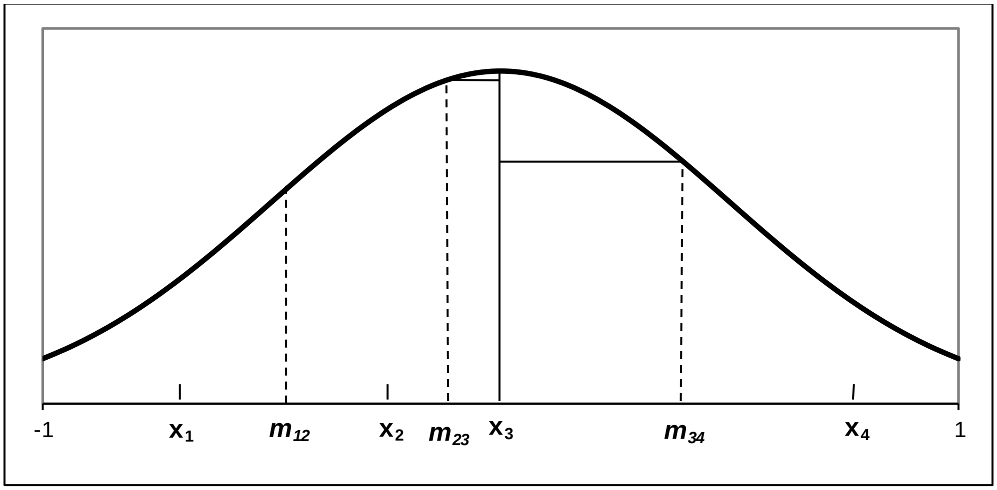
Note: Voters are normally distributed, with mean 0 and standard deviation 0.5. Values in parentheses are the vote shares at equilibrium. A parliamentary mean model with $q_2 \le \bar{x} \le q_3$ *or* dominant party model with $q_2 \le x_1$ and $q_3 \ge x_4$ is assumed.

Table A.2. Policy-seeking Nash equilibria for mainstream parties in a four-party model, for a range of extreme party positions; voters are normally distributed, with standard deviation = 0.5.

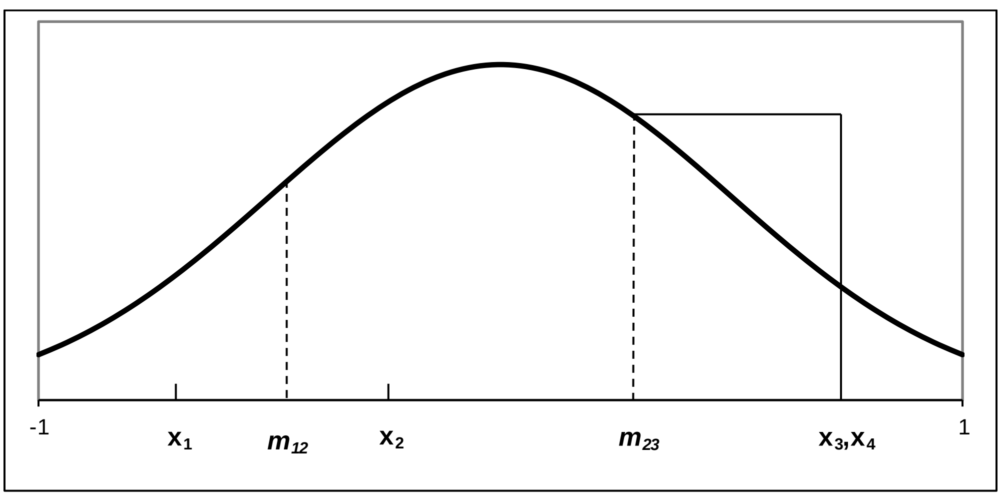
Extremity of extreme parties		x_1	x_2	x_3	x_4
Parliamentary mean model with $q_2 \le \bar{x} \le q_3$ *or* **dominant party model with** $q_2 \le x_1$ and $q_3 \ge x_4$					
$\pm$ **0.5** **(1 s.d.)**		**0.5** (23.0%)	**-0.240** (27.0%)	**0.240** (27.0%)	**0.5** (23.0%)
$\pm$ **1.0** **(2 s.d.)**		**1.0** (7.7%)	**-0.427** (42.3%)	**0.427** (42.3%)	**1.0** (7.7%)
$\pm$ **1.5** **(3 s.d.)**		**1.5** (2.1%)	**-0.541** (47.9%)	**0.541** (47.9%)	**1.5** (2.1%)
Dominant party model with $q_2 > x_1$ and $q_3 < x_4$					
$\pm$ **0.5** **(1 s.d.)**	$q_2, q_3 = \pm$ **0.429**	**0.5** (17.7%)	**-0.429** (32.3%)	**0.429** (32.3%)	**0.5** (17.7%)
$\pm$ **1.0** **(2 s.d.)**	$q_2, q_3 = \pm$ **0.8**	**1.0** (6.2%)	**-0.540** (43.8%)	**0.540** (43.8%)	**1.0** (6.2%)
$\pm$ **1.5** **(3 s.d.)**	$q_2, q_3 = \pm$ **1.2**	**1.5** (1.9%)	**-0.586** (48.1%)	**0.586** (48.1%)	**1.5** (1.9%)

Notes: Table entries are locations of the mainstream parties x_2, x_3 at Nash equilibrium; these parties are assumed to locate in symmetrical positions at equilibrium. Values in parentheses are vote shares for each party at Nash equilibrium.

Figure A1. Values of $\partial U_3/\partial x_3$ as x_3 changes (pictured for the four-party scenario)

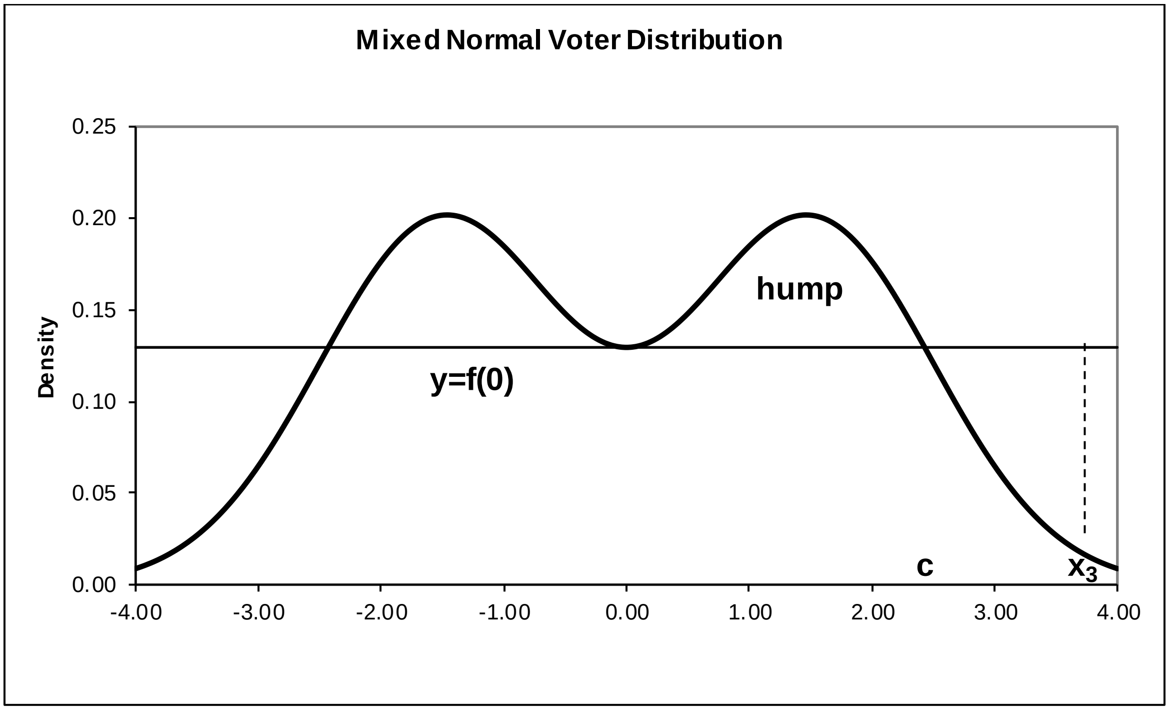


a. If $x_3 = 0$, then $\partial U_3/\partial x_3 \geq 0$.



b. If $x_3 = x_4$, then $\partial U_3/\partial x_3 \leq 0$.

Figure A2. Mixed normal voter distribution with extreme Nash equilibria



Note: The mixed-normal voter distribution is specified by $f(x) = 0.5 * f_1(x) + 0.5 * f_2(x)$, where f_1 is normal with mean $-\mu$ and standard deviation σ and f_2 is normal with mean $+\mu$ and standard deviation σ. In this example $\sigma = 1$ and $\mu = 1.5$. If in the four-party scenario, $x_2 \cong -3.80$ and $x_3 \cong 3.80$ (and provided that x_1 and x_4 are more extreme than these mainstream parties), then a Nash equilibrium occurs, but not for more moderate locations for the mainstream parties.