

Appendix A

Coefficients A_3^*, B_1^*, B_2^* in Eq. (19) are

$$\begin{aligned}
A_3^* &= \frac{b_{24}}{b_{14}} \left(\frac{\delta_n^2}{R_x} + \frac{\beta_m^2}{R_y} \right) - \frac{b_{34}}{b_{14}}, \\
B_1^* &= \frac{b_{13}b_{32} - b_{12}b_{33}}{b_{22}b_{33} - b_{23}b_{32}} \left[\frac{b_{24}}{b_{14}} \left(\frac{\delta_n^2}{R_x} + \frac{\beta_m^2}{R_y} \right) - \frac{b_{34}}{b_{14}} \right] + \frac{b_{32}e_{51}\delta_n - b_{33}e_{41}\beta_m}{b_{22}b_{33} - b_{23}b_{32}} K_s, \\
B_2^* &= \frac{b_{12}b_{23} - b_{13}b_{22}}{b_{22}b_{33} - b_{23}b_{32}} \left[\frac{b_{24}}{b_{14}} \left(\frac{\delta_n^2}{R_x} + \frac{\beta_m^2}{R_y} \right) - \frac{b_{34}}{b_{14}} \right] + \frac{b_{23}e_{41}\beta_m - b_{22}e_{51}\delta_n}{b_{22}b_{33} - b_{23}b_{32}} K_s
\end{aligned} \tag{A1}$$

in which b_{ij} ($i = 1 \div 3, j = 2 \div 4$) have the same form as in the work [67].

The detailed definitions of coefficients a_{k6}, a_{k7} ($k = 1 \div 3$) in Eqs. (20) are

$$\begin{aligned}
a_{16} &= \frac{\bar{c}_1(1 - \bar{c}_2 a_{25}) a_{34} + \bar{c}_1 \bar{c}_2 a_{24} a_{35}}{(1 - \bar{c}_1 a_{14})(1 - \bar{c}_2 a_{25}) - \bar{c}_1 \bar{c}_2 a_{15} a_{24}}, \quad a_{26} = \frac{\bar{c}_1(1 - \bar{c}_2 a_{25}) a_{44} + \bar{c}_1 \bar{c}_2 a_{24} a_{45}}{(1 - \bar{c}_1 a_{14})(1 - \bar{c}_2 a_{25}) - \bar{c}_1 \bar{c}_2 a_{15} a_{24}}, \\
a_{36} &= \frac{\bar{c}_1(1 - \bar{c}_2 a_{25}) a_{54} + \bar{c}_1 \bar{c}_2 a_{24} a_{55}}{(1 - \bar{c}_1 a_{14})(1 - \bar{c}_2 a_{25}) - \bar{c}_1 \bar{c}_2 a_{15} a_{24}}, \quad a_{17} = \frac{(1 - \bar{c}_1 a_{14}) \bar{c}_2 a_{35} + \bar{c}_1 \bar{c}_2 a_{15} a_{34}}{(1 - \bar{c}_1 a_{14})(1 - \bar{c}_2 a_{25}) - \bar{c}_1 \bar{c}_2 a_{15} a_{24}}, \\
a_{27} &= \frac{(1 - \bar{c}_1 a_{14}) \bar{c}_2 a_{45} + \bar{c}_1 \bar{c}_2 a_{15} a_{44}}{(1 - \bar{c}_1 a_{14})(1 - \bar{c}_2 a_{25}) - \bar{c}_1 \bar{c}_2 a_{15} a_{24}}, \quad a_{37} = \frac{(1 - \bar{c}_1 a_{14}) \bar{c}_2 a_{55} + \bar{c}_1 \bar{c}_2 a_{15} a_{54}}{(1 - \bar{c}_1 a_{14})(1 - \bar{c}_2 a_{25}) - \bar{c}_1 \bar{c}_2 a_{15} a_{24}}
\end{aligned} \tag{A2}$$

where

$$\begin{aligned}
(\bar{c}_1, \bar{c}_2) &= \frac{1}{h}(c_1, c_2), \quad a_{14} = -\frac{\bar{e}_{21}}{\bar{a}_{82}}, \quad a_{24} = \frac{\nu_{21} \bar{e}_{11}}{\bar{a}_{82}}, \quad a_{44} = \frac{m^2 \pi^2 B_a^2}{8B_h^2}, \quad a_{54} = -\frac{1}{\bar{a}_{82}} (\bar{e}_{11T} \bar{e}_{21} - \bar{e}_{11} \bar{e}_{21T} \nu_{21}), \\
a_{34} &= -\frac{4\gamma_m \gamma_n}{mn\pi^2 \bar{a}_{82}} \left[\frac{\bar{A}_3^*}{B_h^2} (\nu_{21} \bar{e}_{11} m^2 \pi^2 B_a^2 - \bar{e}_{21} n^2 \pi^2) - \frac{B_a}{B_h} \bar{B}_1^* m \pi (\bar{e}_{11} \bar{e}_{22} \nu_{12} \nu_{21} - \bar{e}_{12} \bar{e}_{21}) \right. \\
&\quad \left. - \frac{n\pi}{B_h} \nu_{21} \bar{B}_2^* (\bar{e}_{11} \bar{e}_{22} - \bar{e}_{12} \bar{e}_{21}) \right] - 4 \frac{\gamma_m \gamma_n B_a R_{ax}}{mn\pi^2 B_h},
\end{aligned}$$

$$a_{15} = \frac{\nu_{12}\bar{e}_{21}}{\bar{a}_{82}}, \quad a_{25} = -\frac{\bar{e}_{11}}{\bar{a}_{82}}, \quad a_{45} = \frac{n^2\pi^2}{8B_h^2}, \quad a_{55} = -\frac{1}{\bar{a}_{82}}(\bar{e}_{11}\bar{e}_{21T} - \nu_{12}\bar{e}_{21}\bar{e}_{11T}), \quad (\text{A3})$$

$$a_{35} = -\frac{4\gamma_m\gamma_n}{mn\pi^2\bar{a}_{82}} \left[\frac{\bar{A}_3^*}{B_h^2} (\bar{e}_{21}\nu_{12}n^2\pi^2 - \bar{e}_{11}m^2\pi^2B_a^2) - \frac{B_a}{B_h}\nu_{12}m\pi\bar{B}_1^*(\bar{e}_{12}\bar{e}_{21} - \bar{e}_{11}\bar{e}_{22}) \right. \\ \left. - \frac{n\pi}{B_h}\bar{B}_2^*(\bar{e}_{12}\bar{e}_{21}\nu_{12}\nu_{21} - \bar{e}_{11}\bar{e}_{22}) \right] - 4\frac{\gamma_m\gamma_n R_{by}}{mn\pi^2 B_h}$$

in which

$$\bar{a}_{82} = (1 - \nu_{12}\nu_{21})\bar{e}_{11}\bar{e}_{21}, \quad (\bar{e}_{11}, \bar{e}_{21}, \bar{e}_{11T}, \bar{e}_{21T}) = \frac{1}{h}(e_{11}, e_{21}, e_{11T}, e_{21T}), \\ (\bar{e}_{12}, \bar{e}_{22}, \bar{A}_3^*) = \frac{1}{h^2}(e_{12}, e_{22}, A_3^*), \quad (\bar{B}_1^*, \bar{B}_2^*) = (B_1^*, B_2^*)h, \quad (\text{A4})$$

$$B_a = b/a, \quad B_h = \frac{b}{h}, \quad R_{ax} = a/R_x, \quad R_{by} = b/R_y, \quad \gamma_i = \frac{1}{2}[1 - (-1)^i] \quad (i = m, n).$$

Appendix B

The coefficients b_l ($l = 1 \div 6$) in the Eqs. (22) are defined as

$$b_1 = \frac{mn\pi^2}{16\gamma_m\gamma_n}a_{13} - \frac{B_a}{B_h}R_{ax}a_{16} - \frac{R_{by}}{B_h}a_{17}, \quad b_2 = \frac{mn\pi^2}{16\gamma_m\gamma_n}a_{23} + \frac{mn\pi^4}{16\gamma_m\gamma_n B_h^2}(m^2B_a^2a_{16} + n^2a_{17}), \\ b_3 = \frac{mn\pi^2}{16\gamma_m\gamma_n}a_{33} - \frac{1}{B_h}(B_aR_{ax}a_{26} + R_{by}a_{27}), \quad b_4 = \frac{mn\pi^2}{16\gamma_m\gamma_n}a_{43} + \frac{mn\pi^4}{16\gamma_m\gamma_n B_h^2}(m^2B_a^2a_{26} + n^2a_{27}), \\ b_5 = \frac{mn\pi^4}{16\gamma_m\gamma_n B_h^2}(m^2B_a^2a_{36} + n^2a_{37}), \quad b_6 = \frac{B_a}{B_h}R_{ax}a_{36} + \frac{R_{by}}{B_h}a_{37} \quad (\text{B1})$$

where

$$a_{13} = -\frac{\pi^3}{B_h^3}(\bar{a}_{11}\bar{B}_1^*m^3B_a^3 + \bar{a}_{21}\bar{B}_1^*mn^2B_a + \bar{a}_{31}\bar{B}_2^*m^2nB_a^2 + \bar{a}_{41}\bar{B}_2^*n^3) - \frac{\pi^4}{B_h^4}\bar{a}_{51}\bar{A}_3^*m^2n^2B_a^2$$

$$\begin{aligned}
& + \frac{\pi^2 B_a}{B_h^3} \bar{A}_3^* (n^2 R_{ax} + m^2 B_a R_{by}) + \frac{E_0^m}{\pi^2 B_h^2} K_1 + \frac{E_0^m}{B_h^2} K_2 (m^2 B_a^2 + n^2), \\
a_{23} &= -\frac{32 B_a^2}{3 B_h^4} \pi^2 \bar{A}_3^* m n \gamma_m \gamma_n, \quad a_{33} = -\frac{2 \gamma_m \gamma_n}{3 m n B_h^3} \left(\frac{m^2 B_a^3}{\bar{a}_{32}} R_{ax} + \frac{n^2 R_{by}}{\bar{a}_{12}} \right), \\
a_{43} &= \frac{m^2 n^2 \pi^4}{16 B_h^4} \left(\frac{n^2}{\bar{a}_{12} m^2} + \frac{m^2 B_a^4}{\bar{a}_{32} n^2} \right)
\end{aligned} \tag{B2}$$

in which

$$\begin{aligned}
(\bar{a}_{11}, \bar{a}_{21}, \bar{a}_{31}, \bar{a}_{41}) &= \frac{1}{h^3} (a_{11}, a_{21}, a_{31}, a_{41}), \quad \bar{a}_{51} = \frac{a_{51}}{h}, \\
(\bar{a}_{12}, \bar{a}_{32}) &= (a_{12}, a_{32}) h, \quad (K_1, K_2) = \frac{b^2}{E_0^m h^3} (k_1 b^2, k_2)
\end{aligned} \tag{B3}$$

and E_0^m is the value of E^m calculated at $T_0 = 300 K$.