How Should Firms Manage Excessive Product Use? A Continuous-Time Demand Model to Test Reward Schedules, Notifications, and Time Limits

WEB APPENDICIES

Appendix A. Parameter Recovery: Monte Carlo Results

We tested three scenarios: (1) the correctly specified model, i.e. the researcher uses the model that generated the data for estimation (data-generation process is similar to the one described in the paper); (2) the data generation process includes the process of habit formation and evolution, but the model used for estimation ignores that, which means the model is incorrectly specified; and (3) a data generation process does not include the habit process, but the researcher allows for it in his empirical model. Parameter true values, estimates for 50 Monte Carlo samples, and standard deviations are displayed in table below.

In the first case, with a well-specified model, we find that the parameters are all correctly identified. In the second setting, in which the habit influences consumer decisions and the model used in estimation ignores it, we observe that several parameters–especially those related to flow utility–are now biased. This is because the habit state picks up patterns of increased frequency and length of active sessions over time. If not accounted for, these effects will then be imputed to flow utility and overestimate the respective parameters. This setting highlights the importance of accounting for habit as an unobserved state that influences usage decisions. Finally, when the data do not contain the effect of habit but a researcher specifies a habit process in his model, we find that the model recovers the parameters well. Estimates show that there is no difference between habitual and non-habitual decision arrival rates, and that parameters that pick up the additional effect of habit on flow utility and the probability of changing the habit state are not significantly different from zero. This shows that a richer model, i.e. the one allowing for the habit process, is able to correctly identify a case when habit does not impact consumer product usage decisions.

Table 1: Parameter recovery: Monte Carlo simulation results.

<table>
<tr><th></th><th></th><th colspan="3">Correct Specification</th><th colspan="3">Habit in data, not in model</th><th colspan="3">Habit in model, not in data</th></tr>
<tr><th>Parameter</th><th></th><th>True</th><th>Estim.</th><th>St. Dev.</th><th>True</th><th>Estim.</th><th>St. Dev.</th><th>True</th><th>Estim.</th><th>St. Dev.</th></tr>
<tr><td>Flow gaming utility–intercept</td><td>α_1</td><td>1.00</td><td>1.02</td><td>0.29</td><td>1.00</td><td>0.92</td><td>0.21</td><td>1.00</td><td>0.96</td><td>0.13</td></tr>
<tr><td>Flow utility of gaming–additional per each level</td><td>α_2</td><td>0.70</td><td>0.68</td><td>0.16</td><td>0.70</td><td>2.11</td><td>0.19</td><td>0.70</td><td>0.71</td><td>0.12</td></tr>
<tr><td>Decision arrival rate in idle period, no habit state</td><td>λ_0^0</td><td>2.00</td><td>1.96</td><td>0.09</td><td>2.00</td><td rowspan="2">2.61</td><td rowspan="2">0.05</td><td rowspan="2">2.00</td><td>2.06</td><td>0.44</td></tr>
<tr><td>Decision arrival rate in idle period, habit state</td><td>λ_1^0</td><td>3.00</td><td>2.95</td><td>0.15</td><td>3.00</td><td>1.97</td><td>0.18</td></tr>
<tr><td>Decision arrival rate in active period, no habit state</td><td>λ_0^1</td><td>5.00</td><td>4.96</td><td>0.54</td><td>5.00</td><td rowspan="2">2.67</td><td rowspan="2">0.18</td><td rowspan="2">3.00</td><td>2.99</td><td>0.19</td></tr>
<tr><td>Decision arrival rate in active period, habit state</td><td>λ_1^1</td><td>4.00</td><td>3.97</td><td>0.52</td><td>4.00</td><td>2.96</td><td>0.32</td></tr>
<tr><td>Additional flow utility from gaming if in habit state</td><td>α_3</td><td>2.00</td><td>2.00</td><td>0.09</td><td>2.00</td><td>-</td><td>-</td><td>-</td><td>-0.02</td><td>0.11</td></tr>
<tr><td>Probability of going from habit to no-habit state</td><td>φ^h</td><td>0.70</td><td>0.71</td><td>0.08</td><td>0.70</td><td>-</td><td>-</td><td>-</td><td>0.30</td><td>0.22</td></tr>
<tr><td>Probability of getting satiated</td><td>φ^b</td><td>0.40</td><td>0.41</td><td>0.07</td><td>0.40</td><td>0.25</td><td>0.04</td><td>0.40</td><td>0.40</td><td>0.06</td></tr>
<tr><td>Flow utility of staying idle if got satiated during a gaming session</td><td>ω</td><td>1.00</td><td>1.00</td><td>0.06</td><td>1.00</td><td>1.83</td><td>0.06</td><td>1.00</td><td>1.02</td><td>0.07</td></tr>
<tr><td>Instantaneous extrinsic utility of leveling</td><td>β_2</td><td>1.00</td><td>0.96</td><td>0.26</td><td>1.00</td><td>1.44</td><td>0.33</td><td>1.00</td><td>0.99</td><td>0.17</td></tr>
<tr><td>Instantaneous intrinsic utility of leveling</td><td>β_1</td><td>0.70</td><td>0.74</td><td>0.27</td><td>0.70</td><td>1.05</td><td>0.37</td><td>0.70</td><td>0.73</td><td>0.18</td></tr>
<tr><td>Costs of starting a gaming session</td><td>γ_{setup}</td><td>-2.00</td><td>-1.95</td><td>0.19</td><td>-2.00</td><td>-0.80</td><td>0.14</td><td>-2.00</td><td>-1.93</td><td>0.17</td></tr>
<tr><td>Leveling arrival rates</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>to level 2</td><td>λ_1^{exp}</td><td>1.48</td><td>1.48</td><td>0.03</td><td>1.48</td><td>1.49</td><td>0.03</td><td>1.48</td><td>1.48</td><td>0.03</td></tr>
<tr><td>to level 3</td><td>λ_2^{exp}</td><td>1.46</td><td>1.47</td><td>0.04</td><td>1.46</td><td>1.45</td><td>0.04</td><td>1.46</td><td>1.46</td><td>0.03</td></tr>
<tr><td>to level 4</td><td>λ_3^{exp}</td><td>1.44</td><td>1.44</td><td>0.03</td><td>1.44</td><td>1.44</td><td>0.03</td><td>1.44</td><td>1.44</td><td>0.03</td></tr>
<tr><td>to level 5</td><td>λ_4^{exp}</td><td>1.42</td><td>1.42</td><td>0.03</td><td>1.42</td><td>1.41</td><td>0.03</td><td>1.42</td><td>1.42</td><td>0.04</td></tr>
<tr><td>to level 6</td><td>λ_5^{exp}</td><td>1.40</td><td>1.41</td><td>0.03</td><td>1.40</td><td>1.40</td><td>0.03</td><td>1.40</td><td>1.41</td><td>0.04</td></tr>
</table>

Appendix B. The Intensity Matrix *R*

			z_1=(g=0,l=1)				z_2=(g=0,l=2)				z_3=(g=1,l=1)				z_4=(g=1,l=2)				s_{churn}
s=(z,k)			k_1	k_2	k_3	k_4	k_1	k_2	k_3	k_4	k_1	k_2	k_3	k_4	k_1	k_2	k_3	k_4	
			s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}	s_{16}	
z_1	k_1=(h=0,b=0)	s_1	$-\mu(s_1)$	0	0	0	0	0	0	0	$\mu_1(s_1)$	0	0	0	0	0	0	0	0
	k_2=(h=1,b=0)	s_2	$\mu_0(s_2)$	$-\mu(s_2)$	0	0	0	0	0	0	0	$\mu_1(s_2)$	0	0	0	0	0	0	0
	k_3=(h=0,b=1)	s_3	0	0	$-\mu(s_3)$	0	0	0	0	0	$\mu_1(s_3)$	0	0	0	0	0	0	0	0
	k_4=(h=1,b=1)	s_4	0	0	$\mu_0(s_4)$	$-\mu(s_4)$	0	0	0	0	0	$\mu_1(s_4)$	0	0	0	0	0	0	0
z_2	k_1=(h=0,b=0)	s_5	0	0	0	0	$-\mu(s_5)$	0	0	0	0	0	0	0	$\mu_1(s_5)$	0	0	0	0
	k_2=(h=1,b=0)	s_6	0	0	0	0	$\mu_0(s_6)$	$-\mu(s_6)$	0	0	0	0	0	0	0	$\mu_1(s_6)$	0	0	0
	k_3=(h=0,b=1)	s_7	0	0	0	0	0	0	$-\mu(s_7)$	0	0	0	0	0	$\mu_1(s_7)$	0	0	0	0
	k_4=(h=1,b=1)	s_8	0	0	0	0	0	0	$\mu_0(s_8)$	$-\mu(s_8)$	0	0	0	0	0	$\mu_1(s_8)$	0	0	0
z_3	k_1=(h=0,b=0)	s_9	0	$\eta_0(s_9)$	0	0	0	0	0	0	$-\eta(s_9)$	0	$\eta_1(s_9)$	0	λ_1^{exp}	0	0	0	$\eta_2(s_9)$
	k_2=(h=1,b=0)	s_{10}	0	$\eta_0(s_{10})$	0	0	0	0	0	0	0	$-\eta(s_{10})$	0	$\eta_1(s_{10})$	0	λ_1^{exp}	0	0	$\eta_2(s_{10})$
	k_3=(h=0,b=1)	s_{11}	0	0	0	$\eta_0(s_{11})$	0	0	0	0	0	0	$-\eta(s_{11})$	0	0	0	λ_1^{exp}	0	$\eta_2(s_{11})$
	k_4=(h=1,b=1)	s_{12}	0	0	0	$\eta_0(s_{12})$	0	0	0	0	0	0	0	$-\eta\ (s_{12})$	0	0	0	λ_1^{exp}	$\eta_2(s_{12})$
z_4	k_1=(h=0,b=0)	s_{13}	0	$\eta_0(s_{13})$	0	0	0	0	0	0	0	0	0	0	$-\eta(s_{13})$	0	$\eta_1(s_{13})$	0	$\eta_2(s_{13})$
	k_2 =(h=1,b=0)	s_{14}	0	$\eta_0(s_{14})$	0	0	0	0	0	0	0	0	0	0	0	$-\eta(s_{14})$	0	$\eta_1(s_{14})$	$\eta_2(s_{14})$
	k_3=(h=0,b=1)	s_{15}	0	0	0	$\eta_0(s_{15})$	0	0	0	0	0	0	0	0	0	0	$-\eta(s_{15})$	0	$\eta_2(s_{15})$
	k_4=(h=1,b=1)	s_{16}	0	0	0	$\eta_0(s_{16})$	0	0	0	0	0	0	0	0	0	0	0	$-\eta(s_{16})$	$\eta_2(s_{16})$
s_{churn}			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note: We show the intensity matrix *R* for underlying processes *H* with states *h = 0,1* and *B* with states *b = 0,1* and observable processes *L* with states *l = 1,2* and *G* with states *g = 0,1,2*.

Appendix C. Details on Estimation Algorithm

The estimation combines two expectation-maximization (EM) algorithms. The first, outer algorithm, handles the discrete representation of unobserved consumer heterogeneity; the second, inner algorithm, handles the unobserved state transitions of the continuous-time Markov chain $S(t)$:

1) Set initial values for the vector of structural parameters Θ for each consumer segment, and each element p_{ij} $(i = 1, \ldots, N, j = 1, \ldots, J)$ in the matrix p containing probabilities of consumers' association with latent segments.

2) Outer EM algorithm: Repeat the Steps (a) and (b) until $p_{ij}^{k+1} - p_{ij}^{k} < \varepsilon_{out}$ $(\mathrm{i} = 1, \ldots, \mathrm{N}, \mathrm{j} = 1, \ldots, \mathrm{J})$, where k is the outer loop iterator, ε_{out} is a stopping parameter:

   a) Inner EM algorithm:

      Repeat Steps (i) and (ii) until $\Theta_{m+1} - \Theta_m < \varepsilon_{inner}$, where m is the inner loop iterator and ε_{inner} is a stopping parameter:

      i. Given vector Θ and matrix p, for all states *s* of the Markov chain $S(t)$ and for each segment j $(j = 1, \ldots, J)$, compute the expected number of transitions from state *s* to state s' denoted by $M_{s'}^{s}$ and the expected dwell time in state *s* denoted by D^{s}. $M_{s'}^{s}$ and D^{s} are functions of the entries in the intensity matrix *R*, which are in their turn functions of structural parameters Θ. For the sake of brevity, we refer the reader to Mark and Ephraim (2013) for expressions for $M_{s'}^{s}$ and D^{s}.

      ii. Given $M_{s'}^{s}$ and D^{s} computed in Step (i), maximize the expected log-likelihood of observed state transitions and durations in Equation 16 with respect to parameters Θ and the set of value-functions constraints.

End inner EM algorithm.

b) Update elements p_{ij} using Equation 17.

End outer EM algorithm.

Appendix D. Elasticity of Game Play

In this appendix we show elasticities of a 1% change in each utility component on the amount of product usage, at the observed data values. The results are shown in Table 2. We find that consumers react the most, in terms of relative change, to variations in the intrinsic utility, with values of about 2.5% to 3.5% across the three segments, for a 1% change in the baseline intrinsic utility. The arrival of an additional cue also causes a significant impact: Having one more opportunity to login leads to an increase in product usage of 1.7%, 4.1%, and 3.1%, for segments 1, 2, and 3, respectively. On the negative side, if the habit state disappears, there is a drop of 1.7%, 0.7%, and 0.8% for each segment, respectively.

Table 2. Elasticities of each component of the model (% change in hours spent in the active state)

	Segment 1	Segment 2	Segment 3
Flow utility of gaming, 1% increase	3.5	2.5	2.7
Additional utility from habit, 1% increase	0.1	0.6	0.2
Cost of starting a gaming session, 1% decrease	0.5	0.3	0.2
Probability of habit reduction, 1% increase	-1.7	-0.7	-0.8
Probability of getting into a tired state, 1% increase	0.1	0.2	0.0
Utility of staying idle when tired, 1% increase	0.1	0.2	0.0
Extrinsic utility, small reward, 1% increase	0.0	0.0	0.0
Extrinsic utility, large reward, 1% increase	0.0	0.0	0.0
Intrinsic utility of leveling up, 1% increase	0.1	0.2	0.1
Cue arrival rate in the idle state, h = 0	0.1	0.1	0.1
Cue arrival rate in the idle state, h = 1	1.7	0.8	1.0
Duration of flow in the active state, h = 0	1.7	4.1	3.1
Duration of flow in the active state, h = 1	0.1	0.4	0.2

Appendix E. Estimated Effects of Habit and Satiation on Consumer Choices

In this appendix, we explore the effects of habit and satiation on consumer gaming choices. To achieve this goal, we focus on the following statistics relevant to dynamic continuous-time discrete-choice models with state-specific arrival rates for decision opportunities:[1]
estimated probability of remaining in a state, estimated expected durations of a state, and ratio of state durations to calendar time. For a consumer at experience level $l = 32$ and different levels of habit and satiation, we show the above statistics in Table 3 and offer a brief discussion.

We start with the effects of satiation. We see that being satiated by a previous gaming session ($b = 1$) leads to a longer duration of the idle period. For example, a satiated Segment 1 consumer is expected to stay idle for 24.35 hours vs. 6.10 hours otherwise. Alternatively, we can compute the probability of remaining idle t hours after the end of a game session to assess the effect of satiation. Again, we see that satiation makes consumers stay idle longer: e.g., there is just 36% probability that a non-satiated Segment 1 consumer stays idle 5 hours after she stopped her most recent game session, while that probability is 80% for a satiated one. Other consumer segments demonstrate directionally similar effects of satiation.

We now turn to the effects of habit. Consumer builds up her habit by interacting with the game and can reduce her habit level during an idle period by consistently abstaining from gaming when cues nudging her to game arrive. Therefore, consumer starts her idle period in a high habit state $h = 1$ after having interacted with the product. The high habit state during an idle period is relatively short-lived. For example, for a satiated Segment 1 consumer, the expected

[1] Analyzing choice probabilities alone is not sufficient for these types of models since consumer change of state is a function of both choice probabilities and arrival rates for decision opportunities. E.g., switching from an idle state to an active gaming state is a function of consumer choice probabilities in the idle state and a habit state-specific cue arrival rate, which takes on different values depending on habit level.

duration is under an hour. Eventually, an idle consumer in a high habit state either starts a new gaming session or remains idle and may transition into the low habit state $h = 0$. If a non-satiated Segment 1 consumer indeed enters the low habit state, the expected duration of her idle period is 10.15 hours. It is considerably longer than the expected duration computed without conditioning on necessarily entering the low habit state, which is just 6.10 hours. The main driver behind the difference in those durations is the consumer's propensity to get back to gaming, which differs between the two habit states: the probability that the consumer is back to gaming in the next 30 minutes is 15.92% for state $h = 1$, and just 3.76% for state $h = 0$. The effect is directionally consistent across satiation levels and all consumer segments.

A user who does not exit the high habit state $h = 1$ during her idle period but starts a new gaming session in that habit state ends up spending larger share of her calendar time gaming. The reason is that the heightened level of habit pushes a gamer to start a new gaming session after a rather short break. For example, Segment 1 gamer devotes 62% of her calendar time to gaming if resumes gaming in the high habit state, though the expected duration of a new gaming session in that case tends to be shorter, 0.99 hours vs. 1.55 hours. On the other hand, an identical gamer who happens to have transitioned into the low habit state $h = 0$ during her idle period spends just 13% of her calendar time actively gaming. Again, the effect of the habit on the share of calendar time devoted to active gaming is directionally consistent across satiation levels and consumer segments.

		Habit *(h)* and satiation (*b*) levels: *0 = low, 1 = high*	Segment 1	Segment 2	Segment 3
1	Estimated expected duration of the idle state ($g = 0$), hours	$b = 0$	6.10	5.93	4.77
		$b = 1$	24.35	34.36	6.51
2	Estimated probability of remaining idle ($g = 0$) 5 hours after ending a game session	$b = 0$	0.36	0.35	0.29
		$b = 1$	0.80	0.84	0.39
4	Estimated expected duration of the high habit state during an idle period ($g = 0$, $h = 1$), hours[2]	$b = 0$	0.60	0.58	0.52
		$b = 1$	0.92	0.92	0.64
5	Estimated expected duration of an idle session ($g = 0$), conditional on consumer having transitioned into a low habit state $h = 0$ over the course of the idle period	$b = 0$	10.15	10.38	9.44
		$b = 1$	25.91	36.32	10.20
6	Probability of being in an active game session in 30 minutes, conditional on current habit *h* and satiation states *b*	$h = 1$, $b = 0$	15.92%	19.83%	22.31%
		$h = 0$, $b = 0$	3.76%	3.87%.	4.17%
		$h = 1$, $b = 1$	1.78%	1.77%	14.57%
		$h = 0$, $b = 1$	1.46%	1.09%	3.90%
7	Estimated expected duration of an active game session ($g = 1$), in hours	$h = 0$	1.55	2.63	2.40
		$h = 1$	0.99	2.23	1.19
8	Estimated probability to be gaming ($g = 1$), 2 hours after the start of a gaming session	$h = 0$	0.27	0.48	0.45
		$h = 1$	0.13	0.41	0.18
9	Estimated share of gaming time (out of calendar time[3]) if a gaming session was started in the habit state *h* after having spent the idle period in satiation state *b*	$h = 1$, $b = 0$	62.26%	76.63%	69.59%
		$h = 1$, $b = 1$	51.83%	70.79%	65.03%
		$h = 0$, $b = 0$	13.25%	20.22%	20.27%
		$h = 0$, $b = 1$	5.64%	6.75%	19.05%

[2] Satiated consumer tends to stay idle longer, while a non-satiated consumer is drawn back into active gaming sooner, including the cases when she starts a new game session in the high habit state. Therefore, it should not be concluded from the numbers in this line that satiation causes a longer duration of the high habit state. These numbers are used as inputs for line 8.

[3] Calendar time = idle period duration + game session duration.