Install R package:

- Install the latest R version(3.5.1)
- The Opt5PL package can be obtained and loaded in R using the following commands:
  R> install.packages("Opt5PL")
  R> library("Opt5PL")

Descriptions

1. RDOPT(LB,UB, P3,P4,P5,q,grid,r,epsilon,N_dose,log_scale)

Help document:
R> help(RDOPT)

Example:

1. We find the Robust D-optimal design for the case c_1 in Study 1 on Table A7 in Web Appendix A. The model parameter values for the 3PL, 4PL, and 5PL models need to be specified.

```
#Parameter values for the 5PL model
T5=c(30000,0.5,800,0.5,2)
#Parameter values for the 4PL model
T4=c(27264.92,0.67,3340.95,-225.55)
#Parameter values for the 3PL model
T3=c(26715.52,0.70,3204.92)

RDOPT(LB=1.95,UB=32000,P3=T3,P4=T4,P5=T5,q=c(1/3,1/3,1/3))

Computing the difference between the sensitivity function and the
upper bound
[1] 0.007210948
Computing the difference between the sensitivity function and the
upper bound
[1] 0.001938179
Computing the difference between the sensitivity function and the
upper bound
[1] 0.0005429934
D-optimal design
         [,1]   [,2]   [,3]     [,4]     [,5]
Dose   1.950 103.32 670.37 5529.370 31819.33
Weight 0.142   0.15   0.20    0.248     0.26

Note: Verification plot is given on the graphic window.
```

2. We find the D-optimal design for the 3PL model using the parameter values T3=(26715.52,0.70,3204.92) using the same range for the concentrations. For the D-optimal design for the 3PL model, the parameter values for the 3PL model are entered into p_1, p_2, p_3 in P5 and set p_4=0 *and* p_5=1.

```
#Set the parameter values for the 3PL model in the form of P5.
P5=c(26715.52,0.70,3204.92,0,1)
RDOPT(LB=1.95, UB=32000,P5=P5,q=c(1,0,0))
```

```
Computing the difference between the sensitivity function and the
 upper bound
[1] 1.464939e-06
D-optimal design
          [,1]     [,2]      [,3]
Dose   349.960 4664.940 31819.330
Weight   0.333    0.333     0.333

Note: Verification plot is given on the graphic window.
```

3. We find the D-optimal design for the 4PL model using the parameter values T4=(27264.92,0.67,3340.95,-225.55) using the same range for the concentrations. For the D-optimal design for the 4PL model, the parameter values for the 4PL model are entered into p_1, p_2, p_3, p_4 in P5 and set p_5=1.

```
#Set the parameter values for the 4PL model in the form of P5.
P5=c(27264.92,0.67,3340.95,-225.55,1)
RDOPT(LB=1.95, UB=32000,P5=P5,q=c(0,1,0))

Computing the difference between the sensitivity function and the
 upper bound
[1] 3.996803e-15
D-optimal design
       [,1]   [,2]    [,3]     [,4]
Dose   1.95 360.62 4855.32 31819.33
Weight 0.25   0.25    0.25     0.25

Note: Verification plot is given on the graphic window.
```

4. We find the D-optimal design for the 5PL model using the parameter values T5=(30000,0.5,800,0.5,2) using the same range for the concentrations. For the D-optimal design for the 5PL model, the parameter values for the 5PL model are entered into p_1, p_2, p_3, p_4, p_5 in P5.

```
#Set the parameter values for the 5PL model in the form of P5.
P5=c(30000,0.5,800,0.5,2)
RDOPT(LB=1.95, UB=32000,P5=P5,q=c(0,0,1))

Computing the difference between the sensitivity function and the
 upper bound
[1] 4.052776e-05
D-optimal design
       [,1]  [,2]   [,3]    [,4]     [,5]
Dose   1.95 75.78 878.16 7316.08 31819.33
Weight 0.20  0.20   0.20    0.20     0.20

Note: Verification plot is given on the graphic window.
```

5. We find the Robust D-optimal design for the case c_1 in Study 2 on Table A9 in Web Appendix A. The model parameter values for the 3PL, 4PL, and 5PL models need to be specified.

```
#Parameter values for the 5PL model
T5=c(100,0.81,40.14,0,1.63)
#Parameter values for the 4PL model
T4=c(95.61 , 0.95, 76.13, -2.32,1)
#Parameter values for the 3PL model
T3=c(91.93,  1.03, 73.68,0,1)

RDOPT(LB=7,UB=300,P3=T3,P4=T4,P5=T5,q=c(1/3,1/3,1/3))
Computing the difference between the sensitivity function and the
upper bound
0.002997727
0.0008149701
$`D-optimal design`
        [,1]   [,2]   [,3]   [,4]    [,5]    [,6]
Dose   7.000 15.890 17.050 44.970 121.010 297.650
Weight 0.191  0.054  0.116  0.168   0.215   0.256

Note: Verification plot is given on the graphic window.
```

6. We find the D-optimal design for the 3PL model using the parameter values T3=(91.93,1.03,73.68) using the same range for the concentrations. For the D-optimal design for the 3PL model, the parameter values for the 3PL model are entered into p_1, p_2, p_3 in P5 and set p_4=0 *and* p_5=1.

```
#Set the parameter values for the 3PL model in the form of P5.
P5=c(91.93,  1.03, 73.68,0,1)
RDOPT(LB=7, UB=300,P5=P5,q=c(1,0,0))

Computing the difference between the sensitivity function and the
 upper bound
7.101123e-06
$`D-optimal design`
         [,1]   [,2]    [,3]
Dose   15.420 87.870 297.650
Weight  0.333  0.333   0.333

Note: Verification plot is given on the graphic window.
```

7. We find the D-optimal design for the 4PL model using the parameter values T4=c(95.61 , 0.95, 76.13, -2.32,1)using the same range for the concentrations. For the D-optimal design for the 4PL model, the parameter values for the 4PL model are entered into p_1, p_2, p_3, p_4 in P5 and set p_5=1.

```
#Set the parameter values for the 4PL model in the form of P5.
P5=c(95.61 , 0.95, 76.13, -2.32,1)
RDOPT(LB=7, UB=300,P5=P5,q=c(0,1,0))

Computing the difference between the sensitivity function and the
 upper bound
9.103829e-15
$`D-optimal design`
       [,1]   [,2]   [,3]   [,4]   [,5]
```

```
Dose   7.00 26.470 26.730 109.50 297.65
Weight 0.25  0.049  0.201   0.25   0.25

Note: Verification plot is given on the graphic window.
```

8. We find the D-optimal design for the 5PL model using the parameter values T5=(100,.81,40.14,0,1.63) using the same range for the concentrations. For the D-optimal design for the 5PL model, the parameter values for the 5PL model are entered into p_1, p_2, p_3, p_4, p_5 in P5.

```
#Set the parameter values for the 5PL model in the form of P5.
P5=c(100,0.81,40.14,0,1.63)
RDOPT(LB=7, UB=300,P5=P5,q=c(0,0,1))

Computing the difference between the sensitivity function and the
 upper bound
0.0001508013
$`D-optimal design`
       [,1]  [,2]  [,3]   [,4]   [,5]
Dose    7.0 15.58 47.75 143.44 297.65
Weight  0.2  0.20  0.20   0.20   0.20

Note: Verification plot is given on the graphic window.
```

2. Deff(weight,dose,model,P,LB,UB,grid,N_dose)

Help document:
R> help(Deff)

Example.

1. Under the 5PL model with the parameter values Θ=(30000,0.5,800,0.5,2), we find the D-efficiency of the broad range design in Study 1 in the paper. This obtains e_D^{5PL} of ξ_{S1} for the case c_1 in Table 6.

```
#The broad range design
dose=c(1.95,7.8,31.25,125,500,2000,8000,32000)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(30000,0.5,800,0.5,2)
#Compute D-efficiency
Deff(weight,dose,model=5,P=T5,LB=1.95,UB=32000,grid=.01)

D-optimal design
       [,1]  [,2]   [,3]    [,4]     [,5]
Dose   1.95 75.78 878.16 7316.08 31819.33
Weight 0.20  0.20   0.20    0.20     0.20
D-efficiency
[1] 0.88
%More Samples Needed
[1] 13.69
```

2. Under the 4PL model with the parameter values Θ=(27264.92,0.67,3340.95,-225.55), we find the D-efficiency of the broad range design in Study 1 in the paper. This obtains e_D^{4PL} of ξ_{S1} for the case c_1 in Table 6. For the D-efficiency for the 4PL model, the parameter values for the 4PL model are entered into p_1, p_2, p_3, p_4 in P and set p_5=1.

```
#The broad range design
dose=c(1.95,7.8,31.25,125,500,2000,8000,32000)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(27264.92,0.67,3340.95,-225.55,1)
#Compute D-efficiency
Deff(weight,dose,model=4,P=T5,LB=1.95,UB=32000,grid=.01)

D-optimal design
        [,1]   [,2]    [,3]     [,4]
Dose    1.95 360.62 4855.32 31819.33
Weight 0.25   0.25    0.25     0.25
D-efficiency
[1] 0.83
%More Samples Needed
[1] 20.77
```

3. Under the 3PL model with the parameter values Θ=(26715.52,0.70,3204.92), we find the D-efficiency of the broad range design in Study 1 in the paper. This obtains e_D^{3PL} of ξ_{S1} for the case c_1 in Table 6. For the D-efficiency for the 3PL model, the parameter values for the 3PL model are entered into p_1, p_2, p_3 in P and set p_4=0 and p_5=1.

```
#The broad range design
dose=c(1.95,7.8,31.25,125,500,2000,8000,32000)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(26715.52,0.70,3204.92,0,1)
#Compute D-efficiency
Deff(weight,dose,model=3,P=T5,LB=1.95,UB=32000,grid=.01)

D-optimal design
           [,1]     [,2]      [,3]
Dose    349.960 4664.940 31819.330
Weight    0.333    0.333     0.333
D-efficiency
[1] 0.6
%More Samples Needed
[1] 67.6
```

4. Under the 5PL model with the parameter values Θ=(100,.81,40.14,0,1.63), we find the D-efficiency of the design in Study 2 in the paper. This obtains e_D^{5PL} of ξ_{S2} for the case c_1 in Table 6.

```
#The actual design
dose=c(7.09,13.24,24.71,46.12,86.10,160.7,300)
dlength=length(dose)
```

```
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(100,0.81,40.14,0,1.63)
#Compute D-efficiency
Deff(weight,dose,model=5,P=T5,LB=7,UB=300,grid=.01)

D-optimal design
       [,1]  [,2]  [,3]   [,4]   [,5]
Dose    7.0 15.58 47.75 143.44 297.65
Weight  0.2  0.20  0.20   0.20   0.20
D-efficiency
[1] 0.92
%More Samples Needed
[1] 8.78
```

5. Under the 4PL model with the parameter values Θ=(95.61 , 0.95, 76.13, -2.32), we find the D-efficiency of the broad range design in Study 2 in the paper. This obtains e_D^{4PL} of ξ_{S2} for the case c_1 in Table 6. For the D-efficiency for the 4PL model, the parameter values for the 4PL model are entered into p_1, p_2, p_3, p_4 in P and set p_5=1.

```
#The actual design
dose=c(7.09,13.24,24.71,46.12,86.10,160.7,300)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(95.61 , 0.95, 76.13, -2.32,1)
#Compute D-efficiency
Deff(weight,dose,model=4,P=T5,LB=7,UB=300,grid=.01)

D-optimal design
       [,1]   [,2]   [,3]   [,4]   [,5]
Dose   7.00 26.470 26.730 109.50 297.65
Weight 0.25  0.049  0.201   0.25   0.25
D-efficiency
[1] 0.86
%More Samples Needed
[1] 16.28
```

6. Under the 3PL model with the parameter values Θ=(91.93, 1.03, 73.68), we find the D-efficiency of the broad range design in Study 2 in the paper. This obtains e_D^{3PL} of ξ_{S2} for the case c_1 in Table 6. For the D-efficiency for the 3PL model, the parameter values for the 3PL model are entered into p_1, p_2, p_3 in P and set p_4=0 and p_5=1.

```
#The actual design
dose=c(7.09,13.24,24.71,46.12,86.10,160.7,300)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(91.93,  1.03, 73.68,0,1)
#Compute D-efficiency
Deff(weight,dose,model=3,P=T5,LB=7,UB=300,grid=.01)

D-optimal design
```

2. Under the 4PL model with the parameter values Θ=(27264.92,0.67,3340.95,-225.55), we find the D-efficiency of the broad range design in Study 1 in the paper. This obtains e_D^{4PL} of ξ_{S1} for the case c_1 in Table 6. For the D-efficiency for the 4PL model, the parameter values for the 4PL model are entered into p_1, p_2, p_3, p_4 in P and set p_5=1.

```
#The broad range design
dose=c(1.95,7.8,31.25,125,500,2000,8000,32000)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(27264.92,0.67,3340.95,-225.55,1)
#Compute D-efficiency
Deff(weight,dose,model=4,P=T5,LB=1.95,UB=32000,grid=.01)

D-optimal design
        [,1]    [,2]     [,3]      [,4]
Dose   1.95 360.62 4855.32 31819.33
Weight 0.25   0.25    0.25      0.25
D-efficiency
[1] 0.83
%More Samples Needed
[1] 20.77
```

3. Under the 3PL model with the parameter values Θ=(26715.52,0.70,3204.92), we find the D-efficiency of the broad range design in Study 1 in the paper. This obtains e_D^{3PL} of ξ_{S1} for the case c_1 in Table 6. For the D-efficiency for the 3PL model, the parameter values for the 3PL model are entered into p_1, p_2, p_3 in P and set p_4=0 and p_5=1.

```
#The broad range design
dose=c(1.95,7.8,31.25,125,500,2000,8000,32000)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(26715.52,0.70,3204.92,0,1)
#Compute D-efficiency
Deff(weight,dose,model=3,P=T5,LB=1.95,UB=32000,grid=.01)

D-optimal design
          [,1]      [,2]       [,3]
Dose   349.960 4664.940 31819.330
Weight   0.333    0.333     0.333
D-efficiency
[1] 0.6
%More Samples Needed
[1] 67.6
```

4. Under the 5PL model with the parameter values Θ=(100,.81,40.14,0,1.63), we find the D-efficiency of the design in Study 2 in the paper. This obtains e_D^{5PL} of ξ_{S2} for the case c_1 in Table 6.

```
#The actual design
dose=c(7.09,13.24,24.71,46.12,86.10,160.7,300)
dlength=length(dose)
```

```
Dose   7.00 26.470 26.730 109.50 297.65
Weight 0.25  0.049  0.201   0.25   0.25

Note: Verification plot is given on the graphic window.
```

8. We find the D-optimal design for the 5PL model using the parameter values T5=(100,.81,40.14,0,1.63) using the same range for the concentrations. For the D-optimal design for the 5PL model, the parameter values for the 5PL model are entered into p_1, p_2, p_3, p_4, p_5 in P5.

```
#Set the parameter values for the 5PL model in the form of P5.
P5=c(100,0.81,40.14,0,1.63)
RDOPT(LB=7, UB=300,P5=P5,q=c(0,0,1))

Computing the difference between the sensitivity function and the
 upper bound
0.0001508013
$`D-optimal design`
       [,1]  [,2]  [,3]   [,4]   [,5]
Dose    7.0 15.58 47.75 143.44 297.65
Weight  0.2  0.20  0.20   0.20   0.20

Note: Verification plot is given on the graphic window.
```

2. Deff(weight,dose,model,P,LB,UB,grid,N_dose)

Help document:
R> help(Deff)

Example.

1. Under the 5PL model with the parameter values Θ=(30000,0.5,800,0.5,2), we find the D-efficiency of the broad range design in Study 1 in the paper. This obtains e_D^{5PL} of ξ_{S1} for the case c_1 in Table 6.

```
#The broad range design
dose=c(1.95,7.8,31.25,125,500,2000,8000,32000)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values
T5=c(30000,0.5,800,0.5,2)
#Compute D-efficiency
Deff(weight,dose,model=5,P=T5,LB=1.95,UB=32000,grid=.01)

D-optimal design
       [,1]  [,2]   [,3]    [,4]     [,5]
Dose   1.95 75.78 878.16 7316.08 31819.33
Weight 0.20  0.20   0.20    0.20     0.20
D-efficiency
[1] 0.88
%More Samples Needed
[1] 13.69
```

5.DsOPT(LB,UB,P,grid,r,epsilon,N_dose,log_scale)

Help document:
R> help(DsOPT)

Example:

1. We find the Ds-optimal design ξ_{D_s} for estimating θ_5 for the case c_1 under Study 1 in Table A6 in Web Appendix A.

```
#Parameter values for the 5PL model
T5=c(30000,0.5,800,0.5,2)
#Find the Ds-optimal design
DsOPT(LB=1.95, UB=32000, P=T5,grid=.01,epsilon=.001)

Computing the difference between the sensitivity function and the
upper bound
11339.37
0.002566188
1e-20
$`Ds-optimal design`
        [,1]   [,2]    [,3]     [,4]     [,5]
Dose   1.950 55.580 914.000 9394.030 31819.33
Weight 0.184  0.292   0.206    0.208     0.11

Note: Verification plot is given on the graphic window.
```

2. We find the Ds-optimal design ξ_{D_s} for estimating θ_5 for the case c_1 under Study 2 in Table A7 in Web Appendix A.

```
#Parameter values for the 5PL model
T5=c(100,0.81,40.14,0,1.63)
#Find the Ds-optimal design
DsOPT(LB=7, UB=300, P=T5,grid=.01,epsilon=.001)

Computing the difference between the sensitivity function and the
upper bound
81484.33
47979.08
0.003089908
1e-20
$`Ds-optimal design`
        [,1]   [,2]  [,3]    [,4]    [,5]
Dose   7.000 13.820 48.71 161.730 297.650
Weight 0.151  0.269  0.23   0.231   0.119

Note: Verification plot is given on the graphic window.
```

6.Dseff(weight,dose,P,LB,UB,r,epsilon,grid,N_dose)

Help document:

R> help(Dseff)

Example:

1. Under the 5PL model, we find the Ds-efficiency of the broad range design in Study 1 in the paper. This obtains e_{D_s} of ξ_{S1} for the case $c_1$in Table 5.

```
#The broad range design
dose=c(1.95,7.8,31.25,125,500,2000,8000,32000)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values for the 5PL model
T5=c(30000,0.5,800,0.5,2)
#Ds efficiency for original design
Dseff(weight,dose,P=T5,LB=1.95,UB=32000,grid=.01)
Ds-optimal design
        [,1]   [,2]    [,3]     [,4]     [,5]
Dose   1.950 55.580 914.000 9394.030 31819.33
Weight 0.184  0.292   0.206    0.208     0.11
Ds-efficiency
[1] 0.59
%More Samples Needed
[1] 70.02
```

2. Under the 5PL model, we find the Ds-efficiency of the broad range design in Study 2 in the paper. This obtains e_{D_s} of ξ_{S2} for the case $c_1$in Table 5.

```
#The actual design
dose=c(7.09,13.24,24.71,46.12,86.10,160.7,300)
dlength=length(dose)
weight=rep(1/dlength,dlength-1)
#Parameter values for the 5PL model
T5=c(100,0.81,40.14,0,1.63)
#Ds efficiency for original design
Dseff(weight,dose,P=T5,LB=7,UB=300,grid=.01)

Ds-optimal design
        [,1]   [,2]  [,3]    [,4]    [,5]
Dose   7.000 13.820 48.71 161.730 297.650
Weight 0.151  0.269  0.23   0.231   0.119
Ds-efficiency
[1] 0.66
%More Samples Needed
[1] 52.03
```