Appendix I. Simplified Spectator-Grandstand Interaction Model Matrices and Vectors.

This appendix details the matrices and vector of the equation system which reflects the formulation of the proposed spectator-grandstand interaction model. Herein $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{F}$ is the force vector, and $\mathbf{z}$ is the vertical displacement vector.

$$\mathbf{M} = \begin{bmatrix} M_1 + \phi_{num,1}(\mathbf{x}_s) \cdot m_s \cdot \phi_{num,1}(\mathbf{x}_s) & \cdots & \phi_{num,1}(\mathbf{x}_s) \cdot m_s \cdot \phi_{num,n}(\mathbf{x}_s) & 0 \\ \cdots & \cdots & \cdots & 0 \\ \phi_{num,n}(\mathbf{x}_s) \cdot m_s \cdot \phi_{num,1}(\mathbf{x}_s) & \cdots & M_n + \phi_{num,n}(\mathbf{x}_s) \cdot m_s \cdot \phi_{num,n}(\mathbf{x}_s) & 0 \\ 0 & 0 & 0 & m_a \end{bmatrix} \tag{13}$$

$$\mathbf{C} = \begin{bmatrix} C_1 + \phi_{num,1}(\mathbf{x}_s) \cdot c_s \cdot \phi_{num,1}(\mathbf{x}_s) & \cdots & \phi_{num,1}(\mathbf{x}_s) \cdot c_s \cdot \phi_{num,n}(\mathbf{x}_s) & -\phi_{num,1}(\mathbf{x}_s) \cdot c_s \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{num,n}(\mathbf{x}_s) \cdot c_s \cdot \phi_{num,1}(\mathbf{x}_s) & \cdots & C_n + \phi_{num,n}(\mathbf{x}_s) \cdot c_s \cdot \phi_{num,n}(\mathbf{x}_s) & -\phi_{num,n}(\mathbf{x}_s) \cdot c_s \\ -\phi_{num,1}(\mathbf{x}_s) \cdot c_s & \cdots & -\phi_{num,n}(\mathbf{x}_s) \cdot c_s & c_s \end{bmatrix} \tag{14}$$

$$\mathbf{K} = \begin{bmatrix} K_1 + \phi_{num,1}(\mathbf{x}_s) \cdot k_s \cdot \phi_{num,1}(\mathbf{x}_s) & \cdots & \phi_{num,1}(\mathbf{x}_s) \cdot k_s \cdot \phi_{num,n}(\mathbf{x}_s) & -\phi_{num,1}(\mathbf{x}_s) \cdot k_s \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{num,n}(\mathbf{x}_s) \cdot k_s \cdot \phi_{num,1}(\mathbf{x}_s) & \cdots & K_n + \phi_{num,n}(\mathbf{x}_s) \cdot k_s \cdot \phi_{num,n}(\mathbf{x}_s) & -\phi_{num,n}(\mathbf{x}_s) \cdot k_s \\ -\phi_{num,1}(\mathbf{x}_s) \cdot k_s & \cdots & -\phi_{num,n}(\mathbf{x}_s) \cdot k_s & k_s \end{bmatrix} \tag{15}$$

$$\boldsymbol{F} = \begin{bmatrix} \phi_{num,1}(\mathbf{x}_s) \cdot F_{s,ver} \\ \cdots \\ \phi_{num,n}(\mathbf{x}_s) \cdot F_{s,ver} \\ 0 \end{bmatrix} \tag{16}$$

$$\boldsymbol{z} = \begin{bmatrix} z_1 \\ \cdots \\ z_n \\ z_a \end{bmatrix} \tag{17}$$

Appendix II. Basics of Finite Element Model Updating

The FE model-updating technique, based on the modal domain (Friswell and Mottershead 1995), allows adjusting the numerical modal parameters obtained from a FE model of a structure to its real experimental modal parameters. After the updating process, the numerical model better characterizes the actual behavior of the structure. Herein, the FE model updating was applied under the maximum likelihood approach (Marwala, 2010). According to this approach, the updating process may be formulated via a multi-objective optimization problem. The main aim of this optimization problem is to minimize the value of the different terms of the multi-objective function. These terms are defined as the relative differences between the experimental and numerical modal parameters. Concretely, the natural frequencies and associated vibration modes are considered herein. As design variables of the optimization problem, the physical parameters, θ, with greater influence on the dynamic behavior of the structure are considered. A search domain for each design variable is established in order to reduce the simulation time of the optimization algorithm. As optimization method, the NSGA-II algorithm (Srinivas and Deb 1994), has been used herein due to both its robustness and independence on the starting point pre-selected to initiate the search process (Nocendal and Wright 1999). The multi-objective function may be defined as:

$$\min(f_1(\theta) \quad f_2(\theta)) = min\left(\frac{1}{2}\sqrt{\sum_{j=1}^{m_f} r_j^f(\theta)^2} \quad \frac{1}{2}\sqrt{\sum_{j=1}^{m_m} r_j^m(\theta)^2}\right) \tag{18}$$
$$\theta_l \leq \theta \leq \theta_u$$

$$r_j^f(\theta) = \frac{f_{num,j}(\theta) - f_{exp,j}}{f_{exp,j}} \quad j = 1,2, \dots, m_f \tag{19}$$

$$r_j^m(\theta) = \sqrt{\frac{\left(1 - MAC_j(\theta)\right)^2}{MAC_j(\theta)}} \quad j = 1,2, \dots, m_m \tag{20}$$

where $f_1(\theta)$ and $f_2(\theta)$ are the first and the second sub-objective function of the multi-objective function approach, θ_l and θ_u are the lower and upper limit of the search domain, $r_j^f(\theta)$ and $r_j^m(\theta)$ are the residuals associated with the natural frequencies and vibration modes respectively, m_f and m_m are the number of natural frequencies and vibration modes considered in the updating process, $f_{num,j}(\theta)$ and $f_{exp,j}$ are the numerical and experimental natural frequencies associated with the vibration mode j and $MAC_j(\theta)$ is the modal assurance criterion between the numerical and experimental vibration mode j (Allemang and Brown, 1982).

The optimization process consists of the following steps: (i) several solutions are randomly generated and the multi-objective function is evaluated; (ii) a new generation is created based on a ratio of selected solutions; (iii) the multi-objective function is evaluated in terms of this new generation; and (iv) the steps (ii) and (iii) are repeated iteratively until a previously defined stop criteria is met. Herein the roulette-wheel selection has been used as stochastic selection function (Mohamed et al., 2008). Each new generation is obtained through the crossover mechanism in which a new solution is obtained from two previous ones. Additionally, the mutation mechanism is used to explore new areas of the search domain (modifying randomly a design parameter of the considered new solution). As stop criteria, a maximum multi-objective function tolerance of 10^{-5} has been considered. The optimization algorithms runs until the average relative change in the multi-objective function value is less than the established tolerance. As result of the optimization process, a Pareto's front is obtained. Each point of the Pareto's front reflects a possible optimal solution. Finally, as optimal solution, the point of the Pareto's front which better balances the change of the two considered residuals is considered (Jin et al., 2014).

Finally, the good results of the updating process are checked through the comparison of the experimental and numerical natural frequencies and modal shapes by computing the relative differences between the numerical and experimental natural frequencies, $\Delta f_j(\theta)$, and the modal assurance criterion, $MAC_j(\theta)$.

The relative differences, $\Delta f_j(\theta)$, between natural frequencies may be defined as:

$$\Delta f_j(\theta) = \frac{f_{num,j}(\theta) - f_{exp,j}}{f_{exp,j}} \cdot 100 \quad j = 1,2,\ldots,m_f \tag{21}$$

The modal assurance criterion$MAC_j(\theta)$, , may be defined as:

$$MAC_j(\theta) = \frac{\left(\phi_{num,j}(\theta)^T \cdot \phi_{exp,j}(\theta)\right)^2}{\left(\phi_{num,j}(\theta)^T \cdot \phi_{num,j}(\theta)\right) \cdot \left(\phi_{exp,j}(\theta)^T \cdot \phi_{exp,j}(\theta)\right)} \quad j = 1,2,\ldots,m_m \tag{22}$$

where $\phi_{num.j}(\theta)$ and $\phi_{exp.j}$ are the numerical and experimental vibration modes to be compared, and ^{T}denotes the transpose. A good correlation between numerical and experimental vibration modes is achieved when their relative differences, $\Delta f_j(\theta)$, are below 5.00 % and their$MAC_j(\theta)$ ratios are above 0.90 (Zivanovic et al., 2007). The reliability of the modal parameters identified experimentally was carefully checked to avoid converge problems of the iterative process. In the same way, the grid of measurements was sufficiently dense to avoid spatial aliasing problems associated with the determination of the $MAC_j(\theta)$ ratio (Jiménez-Alonso et al., 2016).