PRICE-MATCHING GUARANTEES AS A DIRECT SIGNAL OF LOW PRICES

Samir Mamadehussene
Católica-Lisbon School of Business and Economics
samir@ucp.pt

WEB APPENDIX A - OMITTED PROOFS

Proof of Lemma 1

Consider a non-PMG store that charges $\bar{p}_{NO}$. If $\bar{p}_{NO} > z_U^{NO}$, then uninformed non-shoppers search further after visiting this store. Such consumers never return to purchase from the firm, because they will either find a PMG store (in which case they purchase and delay search to after purchase when they may have zero search costs) or they find a non-PMG store which charges a price lower than $\bar{p}_{NO}$ with probability one (notice that, using an argument similar to Varian 1980, the price distribution must be atomless). Informed non-shoppers only visit the store in case no store offers a PMG. In that case, informed non-shoppers behave the same way as uninformed non-shoppers. Hence, a non-PMG store that charges price $\bar{p}_{NO} > z_U^{NO}$ can only sell to shoppers (in case it has the lowest price in the market). But then a firm that charges price $\bar{p}_{NO}$ would prefer to offer a PMG, as it would be able to sell not only to shoppers (in case it has the lowest price in the market), but also to informed non-shoppers. This contradicts that $\bar{p}_{NO}$ is in the support of F_{NO}.

Proof of Lemma 2

Informed and uninformed non-shoppers face the same post-purchase search cost, s. However, whereas uninformed non-shoppers must search at random, informed non-shoppers can condition their search strategy on firms' PMG policies. It follows that the benefit of searching post-purchase must be weakly higher for informed non-shoppers than for uninformed non-shoppers. It then follows that if informed non-shoppers do not want to search post-purchase, uninformed non-shoppers will also refrain from post-purchase search. Hence, $z_I^{PMG} \leq z_U^{PMG}$

Proof of Lemma 3

By definition of EC_{PMG}, it follows that $EC_{PMG}\Big(min\{z_U^{NO},x\},2\Big) \leq min\{z_U^{NO},x\}+s$. Hence

$$z_U^{NO} = \alpha \int_{\underline{p}}^{\overline{p}} EC_{PMG}\Big(min\{z_U^{NO},x\},2\Big)\,\mathrm{d}F_{PMG}(x) + (1-\alpha)\int_{\underline{p}}^{\overline{p}} min\{z_U^{NO},x\}+s\,\mathrm{d}F_{NO}(x)$$
$$\leq \int_{\underline{p}}^{\overline{p}} min\{z_U^{NO},x\}+s\,\mathrm{d}F(x)$$

$$z_U^{NO} \leq \int_{\underline{p}}^{\overline{p}} min\{z_U^{NO},x\}+s\,\mathrm{d}F(x) \iff s \geq \int_{\underline{p}}^{z_U^{NO}} z_U^{NO}-x\,\mathrm{d}F(x) \tag{A.1}$$

Moreover, it follows from Kohn and Shavell (1974) that the threshold z_U^{PMG} is such that the expected benefit that uninformed non-shoppers derive from search after receiving price z_U^{PMG} must be equal to the search cost. Hence,

$$s = \int_{\underline{p}}^{z_U^{PMG}} z_U^{PMG}-x\,\mathrm{d}F(x) \tag{A.2}$$

Combining (A.1) and (A.2), it follows that

$$\int_{\underline{p}}^{z_U^{PMG}} z_U^{PMG}-x\,\mathrm{d}F(x) \geq \int_{\underline{p}}^{z_U^{NO}} z_U^{NO}-x\,\mathrm{d}F(x) \iff z_U^{PMG} \geq z_U^{NO}$$

Proof of Lemma 4

Suppose, by contradiction, that $\bar{p}_{PMG} > z_U^{PMG}$. It then follows from Lemmas 1 and 3 that $\bar{p}_{NO} \leq z_U^{NO} \leq z_U^{PMG} < \bar{p}_{PMG}$. Consider a firm that offers PMGs and charges price $\bar{p}_{PMG}$. All consumers who purchase from such store will search after purchase. It follows from Kohn and Shavell (1974) that they will follow a cutoff rule, such that they will stop searching either when they find a price lower than some threshold, which I denote by τ, or when they exhaust all stores. Let $\varphi(p/k)$ denote the expected price that consumers pay when they follow the cutoff search strategy, given that the firm's price is p and exactly $k+1$ firms (including the first firm they searched) offer PMGs.

I will show that the firm that offers a PMG and charges price $\bar{p}_{PMG}$ can increase its profit by reducing its price. At the time that the firm is choosing its price and PMG policy, the firm does not know how many firms will offer PMGs, it only knows the probability that each firm will offer a PMG. I will show that, for any realization k of the remaining $n-1$ stores offering PMGs, the firm can improve its profit by reducing its price, which implies that it will also want to reduce its price ex-ante.

I denote by $Emin(p/k)$ the expected minimum price in the market, given that exactly $k+1$ firms offer PMGs and one of those firms charges p. Let F_{PMG} and F_{NO} denote the price distributions of PMG and non-PMG firms, respectively.

Let $H_k(p) \equiv 1 - [1 - F_{NO}(p)]^{n-k-1}[1 - F_{PMG}(p)]^k$ so that

$$Emin(p) = \int_{\underline{p}}^{p} x \,\mathrm{d}H_k(x) + [1 - H_k(p)]p$$

Let $\Delta Emin(\bar{p}_{PMG}, \epsilon) \equiv Emin(\bar{p}_{PMG}/k) - Emin(\bar{p}_{PMG} - \epsilon/k)$. It follows that

$$\begin{aligned}
\Delta Emin(\bar{p}_{PMG}, \epsilon) &= \int_{\underline{p}}^{\bar{p}_{PMG}} x \,\mathrm{d}H_k(x) - \int_{\underline{p}}^{\bar{p}_{PMG-\epsilon}} x \,\mathrm{d}H_k(x) - [1 - H_k(\bar{p}_{PMG} - \epsilon)](\bar{p}_{PMG} - \epsilon) \\
&\leq \int_{\bar{p}_{PMG-\epsilon}}^{\bar{p}_{PMG}} \bar{p}_{PMG} \,\mathrm{d}H_k(x) - [1 - H_k(\bar{p}_{PMG} - \epsilon)](\bar{p}_{PMG} - \epsilon) \\
&= [1 - H_k(\bar{p}_{PMG} - \epsilon)]\bar{p}_{PMG} - [1 - H_k(\bar{p}_{PMG} - \epsilon)](\bar{p}_{PMG} - \epsilon) \\
&= [1 - H_k(\bar{p}_{PMG} - \epsilon)]\epsilon
\end{aligned} \tag{A.3}$$

Let $G_k(x) \equiv \frac{H_k(x) - H_k(\tau)}{1 - H_k(\tau)}$ and $W_k(x) \equiv \frac{H_k(x)}{H_k(\tau)}$, so that

$$\varphi(p/k) = H_k(\tau) \int_{\underline{p}}^{\tau} x \,\mathrm{d}W_k(x) + [1 - H_k(\tau)]\left\{ \int_{\tau}^{p} x \,\mathrm{d}G_k(x) + [1 - G_k(p)]p \right\}$$

Let $\Delta\varphi(\bar{p}_{PMG}, \epsilon) \equiv \varphi(\bar{p}_{PMG}/k) - \varphi(\bar{p}_{PMG} - \epsilon/k)$. It follows that

$$\begin{aligned}
\Delta\varphi(\bar{p}_{PMG}, \epsilon) &= [1 - H_k(\tau)]\left\{ \int_{\bar{p}_{PMG-\epsilon}}^{\bar{p}_{PMG}} x \,\mathrm{d}G_k(x) - [1 - G_k(\bar{p}_{PMG} - \epsilon)](\bar{p}_{PMG} - \epsilon) \right\} \\
&\leq [1 - H_k(\tau)]\left\{ \int_{\bar{p}_{PMG-\epsilon}}^{\bar{p}_{PMG}} \bar{p}_{PMG} \,\mathrm{d}G_k(x) - [1 - G_k(\bar{p}_{PMG} - \epsilon)](\bar{p}_{PMG} - \epsilon) \right\} \\
&= \int_{\bar{p}_{PMG-\epsilon}}^{\bar{p}_{PMG}} \bar{p}_{PMG} \,\mathrm{d}H_k(x) - [1 - H_k(\bar{p}_{PMG} - \epsilon)](\bar{p}_{PMG} - \epsilon) \\
&= [1 - H_k(\bar{p}_{PMG} - \epsilon)]\epsilon
\end{aligned} \tag{A.4}$$

Let $A \equiv (1-\lambda)\left(\frac{1-\phi}{n} + \frac{\phi}{k+1}\right)q$ and $B \equiv (1-\lambda)\left(\frac{1-\phi}{n} + \frac{\phi}{k+1}\right)(1-q)$. Let $\pi(p, PMG; k)$ denote the profit of a firm that offers a PMG and charges p when exactly k remaining stores in the market offer PMGs. WLOG, I normalize $c = 0$.

$$\pi(p, PMG; k) = \lambda[1 - F_{NO}(p)]^{n-k-1}[1 - F_{PMG}(p)]^k p + AEmin(p/k) + B\varphi(p/k)$$

Let $\epsilon < \frac{\lambda \bar{p}_{PMG}}{\lambda+A+B} = \frac{\lambda \bar{p}_{PMG}}{\lambda+(1-\lambda)\left(\frac{1-\phi}{n}+\frac{\phi}{k+1}\right)}$

I will show that a PMG firm that charges $\bar{p}_{PMG}$ can increase its profit by reducing its price. Let $\Delta\pi(\bar{p}_{PMG},\epsilon) \equiv \pi(\bar{p}_{PMG}-\epsilon, PMG) - \pi(\bar{p}_{PMG}, PMG)$. It follows that

$$\begin{aligned}
\Delta\pi(\bar{p}_{PMG},\epsilon) &= \lambda[1-H_k(\bar{p}_{PMG}-\epsilon)](\bar{p}_{PMG}-\epsilon) - A\Delta Emin(\bar{p}_{PMG},\epsilon) - B\Delta\varphi(\bar{p}_{PMG},\epsilon) \\
&\geq \lambda[1-H_k(\bar{p}_{PMG}-\epsilon)](\bar{p}_{PMG}-\epsilon) - [A+B][1-H_k(\bar{p}_{PMG}-\epsilon)]\epsilon \\
&= [1-F_{NO}(\bar{p}_{PMG}-\epsilon)]^{n-k-1}[1-F_{PMG}(\bar{p}_{PMG}-\epsilon)]^k\Big[\lambda\bar{p}_{PMG}-(\lambda+A+B)\epsilon\Big] \\
&> 0
\end{aligned} \tag{A.5}$$

where the weak inequality follows from Equations A.3 and A.4 and the strict inequality follows from the fact that $\epsilon < \frac{\lambda \bar{p}_{PMG}}{\lambda+A+B}$.

Proof of Lemma 5

I will show that for any $q > 0$ and $n = 2$, there exists (λ, ϕ) such that in equilibrium $\underline{p} < \hat{p} < \bar{p}$. This, in turn, implies the result stated in Lemma 5. It follows from Proposition 1 that an equilibrium exists and if $\phi > 0$ then $\hat{p} > \underline{p}$. So in order for an equilibrium with the property that $\underline{p} < \hat{p} < \bar{p}$ to exist, it is sufficient to show that no equilibrium where all firms offer PMGs (i.e $\hat{p} \geq \bar{p}$) exists.

Suppose, by contradiction, that there exists an equilibrium where all firms offer PMGs. Without loss of generality, I normalize $c = 0$. The profit of a firm that charges price $p \in (\underline{p}, \bar{p})$ is $\pi_{PMG}(p) = \lambda[1-F(p)]p + \frac{1-\lambda}{2}[qEmin(p) + (1-q)p]$. Because firms are indifferent between all prices in $(\underline{p}, \bar{p})$ it follows that

$$\frac{\partial \pi_{PMG}(x)}{\partial x} = 0 \iff -\lambda f(x)x + \left(\lambda + \frac{1-\lambda}{2}q\right)[1-F(x)] + \frac{(1-\lambda)(1-q)}{2} = 0$$

The solution to the above differential equation, together with the condition that $F(\underline{p}) = 0$ is $F(x) = \frac{1+\lambda}{\lambda(2-q)+q}\left[1-\left(\frac{x}{\underline{p}}\right)^{\frac{\lambda(q-2)-q}{2\lambda}}\right]$.

Because $n = 2$, it follows that $Emin(p) = \int_{\underline{p}}^{\bar{p}} p \, \mathrm{d}F(p)$. After some algebra, and using the fact that $F(\bar{p}) = 1$, it follows that $Emin(\bar{p}) = \frac{1-q}{q}\left[\left(\frac{1+\lambda}{(1-q)(1-\lambda)}\right)^{\frac{q(1-\lambda)}{2\lambda+q(1-\lambda)}} - 1\right]\bar{p}$. A sufficient condition for an equilibrium where all firms offer PMGs not to exist is that $\pi_{NO}(\bar{p}) > \pi_{PMG}(\bar{p}) \iff$ $\frac{(1-\lambda)(1-\phi)}{2}\bar{p} > \frac{1-\lambda}{2}[qEmin(\bar{p}) + (1-q)\bar{p}] \iff$

$$\phi < q^2[1-\eta(q)] \tag{A.6}$$

where $\eta(q) = (1-q)^{\frac{2\lambda}{2\lambda+q(1-\lambda)}}\left(\frac{1+\lambda}{1-\lambda}\right)^{\frac{q(1-\lambda)}{2\lambda+q(1-\lambda)}}$

It can be show that $\eta(q)$ is decreasing and $\eta(0) = 1$. Hence, for any $q > 0$, $\eta(q) < 1$. It follows that for any $q > 0$, $q^2[1-\eta(q)] > 0$, which in turn implies that there exists $\phi > 0$ that satisfies Equation A.6.

Proof of Lemma 6

When $q = 0$ it is straightforward that $\pi_{PMG}(p) - \pi_{NO}(p) > 0$ for any p. Hence, firms will choose to offer PMGs for any price they choose. When $\phi = 0$ it is easy to see that $\pi_{PMG}(p) - \pi_{NO}(p) \leq 0$ for any p. Hence, firms will never offer a PMG. Now suppose $\phi = 1$ and let $[\underline{p}, \overline{p}]$ denote the support of the equilibrium price distribution. It follows that $\underline{p} = Emin(\underline{p}/k)$ for any k, which implies that $\Delta(\underline{p}) > 0$, where Δ is defined in the proof of Proposition 1. It then follows that the probability that a firm offers a PMG, which I denote by α, is strictly positive. Suppose there was $p^* < \overline{p}$ such that $\Delta(p^*) = 0$ and $\Delta(p') > 0$ for all $p' < p^*$. Because $\phi = 1$, a firm that chooses not to offer a PMG only sells to uninformed consumers in case no firm offers a PMG, which happens with probability $(1-\alpha)^{n-1}$. It follows that $\pi(p^*, NO) = \lambda[1 - F(p^*)]^{n-1}(p^* - c) + (1-\alpha)^{n-1}\frac{1-\lambda}{n}(p^* - c)$

Now consider a firm that charges p^* and offers a PMG. The profit of the firm is

$$\begin{aligned}
\pi(p^*, PMG) &= \lambda[1 - F(p^*)]^{n-1}(p^* - c) + \sum_{k=0}^{n-1} g(k)\frac{1-\lambda}{k+1}(Emin(p^*, k) - c) \\
&\geq \lambda[1 - F(p^*)]^{n-1}(p^* - c) + g(0)(1-\lambda)(Emin(p^*, 0) - c) \\
&= \lambda[1 - F(p^*)]^{n-1}(p^* - c) + (1-\alpha)^{n-1}(1-\lambda)(p^* - c) \\
&> \lambda[1 - F(p^*)]^{n-1}(p^* - c) + (1-\alpha)^{n-1}\frac{(1-\lambda)}{n}(p^* - c) \\
&= \pi(p^*, NO)
\end{aligned}$$

which contradicts that $\Delta(p^*) = 0$. Notice that I have used the fact that $Emin(p^*, 0) = p^*$. This follows because $\Delta(p') > 0$ for all $p' < p^*$, i.e., all firms that charge prices lower than p^* offer PMGs. Hence, when no firm offers a PMG, the minimum price in the market cannot be lower than p^*.

Proof of Lemma 7

By definition of $\hat{p}$, it follows that $\pi_{NO}(\hat{p}) = \pi_{PMG}(\hat{p}) \iff \left[\frac{(1-\lambda)(1-\phi)}{n} + (1-\lambda)\phi\frac{(1-\alpha)^{n-1}}{n}\right]\hat{p} = \left[\frac{(1-\lambda)(1-\phi)}{n} + (1-\lambda)\phi\sum_{k=0}^{n-1}\frac{g(k)}{k+1}\right][qEmin(\hat{p}) + (1-q)\hat{p}] \iff$

$$\Psi(\alpha, \phi) = q\frac{Emin(\hat{p})}{\hat{p}} + 1 - q \tag{A.7}$$

where $\Psi(\alpha, \phi) \equiv \frac{(1-\lambda)(1-\phi)+(1-\lambda)\phi(1-\alpha)^{n-1}}{(1-\lambda)(1-\phi)+(1-\lambda)\phi\frac{1-(1-\alpha)^n}{\alpha}}$

It can be shown that Ψ is increasing in α and decreasing in ϕ.

Consider an equilibrium with price distribution F. An increase in q, holding $\underline{p}$ constant, leads to a reduction of the price distribution (i.e., the new price distribution $\tilde{F}$ is such that $\tilde{F}(x) \leq F(x)$). This then implies that $\frac{Emin(\hat{p})}{\hat{p}}$ decreases. It follows that an increase in q leads to a decrease of the RHS of Equation A.7. In order for Equation A.7 to hold, α must decrease. Hence, α is decreasing in q.

Similarly, an increase in ϕ, holding $\underline{p}$ constant, leads to an increase of the price distribution (i.e., the new price distribution $\tilde{F}$ is such that $\tilde{F}(x) \geq F(x)$). This then implies that $\frac{Emin(\hat{p})}{\hat{p}}$ increases. It follows that an increase in ϕ leads to an increase of the RHS and a decrease in the LHS Equation A.7. In order for Equation A.7 to hold, α must increase. Hence, α is increasing in ϕ.

Proof of Proposition 2

Let L denote the type of a firm that has cost c_L and H denote the type of a firm that has cost c_H. For $i \in \{L, H\}$, let $\pi_i(p, A)$ denote the profit of type i when it charges price p and PMG policy $A \in \{PMG, NO\}$. Let $\pi_i(p) = max\{\pi_i(p, PMG), \pi_i(p, NO)\}$ and let $\Delta_i(p) = \pi_i(p, PMG) - \pi_i(p, NO)$.

Notice that
$$\Delta_i(p) = \sum_{k=1}^{n-1} g(k)\left\{\tfrac{(1-\lambda)\phi}{k+1}\big(qEmin(p,k)+(1-q)p\big)-\tfrac{(1-\lambda)(1-\phi)q}{n}[p-Emin(p/k)]\right\}$$
$$+\, g(0)\left\{(1-\lambda)\phi\tfrac{n-1}{n}\big(qEmin(p,0)+(1-q)p\big)-\tfrac{(1-\lambda)q}{n}[p-Emin(p/0)]\right\}$$
$$-\, c_i\left[g(0)(1-\lambda)\phi\tfrac{n-1}{n}+\sum_{k=1}^{n-1} g(k)\tfrac{(1-\lambda)\phi}{k+1}\right]$$

It is straightforward to see that $\Delta_L(p) \geq \Delta_H(p)$. Moreover, $\Delta_i(\underline{p}) \geq 0$ and Δ_i is continuous and strictly concave. Let $\hat{p}_i$ be such that $\Delta_i(\hat{p}_i) = 0$. It follows that $\hat{p}_L \geq \hat{p}_H$.

Let $Q(x, A)$ and $P(x, A)$ denote the average quantity sold and price received by a firm that sets price x and chooses policy $A \in \{PMG, NO\}$.

Lemma 1 *Let x and y be in the support of F such that $x < y$. Let $A \in \{PMG, NO\}$.*

$$\pi_L(y, A) \geq \pi_L(x, A) \implies \pi_H(y, A) > \pi_H(x, A)$$
$$\pi_L(y, NO) \geq \pi_L(x, PMG) \implies \pi_H(y, NO) > \pi_H(x, PMG)$$

Proof. For the first part

$$\pi_L(y, A) \geq \pi_L(x, A)$$
$$\iff Q(y, A)[P(y, A) - c_L] \geq Q(x, A)[P(x, A) - c_L]$$
$$\implies Q(y, A)[P(y, A) - c_H] > Q(x, A)[P(x, A) - c_H]$$
$$\iff \pi_H(y, A) > \pi_H(x, A)$$

Where the implication follows from $Q(x, A) > Q(y, A)$

For the second part

$$\pi_L(y, NO) \geq \pi_L(x, PMG)$$
$$\iff Q(y, NO)[P(y, NO) - c_L] \geq Q(x, PMG)[P(x, PMG) - c_L]$$
$$\implies Q(y, NO)[P(y, NO) - c_H] > Q(x, PMG)[P(x, PMG) - c_H]$$
$$\iff \pi_H(y, NO) > \pi_H(x, PMG)$$

Where the implication follows from $Q(x, PMG) > Q(y, NO)$ □

Lemma 2 *Let $p' > \tilde{p}$. Let F_i denote the equilibrium price distribution of type i. If $\tilde{p}$ is in the support of both F_L and F_H then p' is not in the support of F_L.*

Proof. I will show that $\pi_L(p') = \pi_L(\tilde{p}) \implies \pi_H(p') > \pi_H(\tilde{p})$, which contradicts that $\tilde{p}$ is in the support of F_H. I split the proof in six cases, that depend on the location of $\tilde{p}$ and p'.

Case 1: $\tilde{p} < \hat{p}_H$ and $p' < \hat{p}_H$

Because in this case both firms find it optimal to offer PMG, it follows from Lemma 1 that $\pi_L(p') = \pi_L(\tilde{p}) \implies \pi_H(p') > \pi_H(\tilde{p})$

Case 2: $\tilde{p} < \hat{p}_H$ and $\hat{p}_H \leq p' \leq \hat{p}_L$

In this case it is optimal for firm L to offer PMG at prices $\tilde{p}$ and p'. Hence, it must be that $\pi_L(\tilde{p}, PMG) = \pi_L(p', PMG)$. It then follows from Lemma 1 that $\pi_H(p') \geq \pi_H(p', PMG) > \pi_H(\tilde{p}, PMG) = \pi_H(\tilde{p})$

Case 3: $\tilde{p} < \hat{p}_H$ and $p' > \hat{p}_L$

In this case, firm L finds it optimal to offer PMG at price $\tilde{p}$ but not at price p'. Hence, $\pi_L(\tilde{p}) = \pi_L(p') \iff \pi_L(\tilde{p}, PMG) = \pi_L(p', NO)$. It then follows from Lemma 1 that $\pi_H(\tilde{p}, PMG) < \pi_H(p', NO)$. Moreover, because $\tilde{p} < \hat{p}_H$, it follows that $\pi_H(\tilde{p}) = \pi_H(\tilde{p}, PMG)$. We can conclude that $\pi_H(p') > \pi_H(\tilde{p})$

Case 4: $\hat{p}_H \leq \tilde{p} \leq \hat{p}_L$ and $p' < \hat{p}_L$

In this case it is optimal for firm L to offer PMG at prices $\tilde{p}$ and p'. Hence, it must be that $\pi_L(\tilde{p}, PMG) = \pi_L(p', PMG)$.

It is straightforward to see that $\frac{\partial \Delta_L}{\partial p} = \frac{\partial \Delta_H}{\partial p}$. Because $\Delta_H(\underline{p}) \geq 0$, $\Delta_H(\hat{p}_H) = 0$ and Δ_H is strictly concave, it follows that Δ_H is decreasing in p after $\hat{p}_H$. It then follows that Δ_L is decreasing in p after $\hat{p}_H$. Hence

$\Delta_L(p') \leq \Delta_L(\tilde{p}) \iff \pi_L(p', PMG) - \pi_L(p', NO) \leq \pi_L(\tilde{p}, PMG) - \pi_L(\tilde{p}, NO)$. Because $\pi_L(\tilde{p}, PMG) = \pi_L(p', PMG)$, it follows that $\pi_L(p', NO) \geq \pi_L(\tilde{p}, NO)$. Lemma 1 then implies that $\pi_H(p', NO) > \pi_H(\tilde{p}, NO)$.

Because it is optimal for firm H not to offer PMG at prices $\tilde{p}$ and p', we have that $\pi_H(p') = \pi_H(p', NO) > \pi_H(\tilde{p}, NO) = \pi_H(\tilde{p})$

Case 5: $\hat{p}_H \leq \tilde{p} \leq \hat{p}_L$ and $p' \geq \hat{p}_L$

In this case, firm L finds it optimal to offer PMG at price $\tilde{p}$ but not at price p'. Hence $\pi_L(p', NO) = \pi_L(\tilde{p}, PMG) \geq \pi_L(\tilde{p}, NO)$. It then follows from Lemma 1 that $\pi_H(p', NO) > \pi_H(\tilde{p}, NO)$. Moreover, it is optimal for firm H not to offer PMG at prices $\tilde{p}$ and p'. Hence, $\pi_H(p') = \pi_H(p', NO) > \pi_H(\tilde{p}, NO) = \pi_H(\tilde{p})$

Case 6: $\tilde{p} > \hat{p}_L$ and $p' > \hat{p}_L$

In this case, firm l finds it optimal not to offer PMG at prices $\tilde{p}$ and p'. Hence, $\pi_L(\tilde{p}, NO) = \pi_L(p', NO)$. It then follows from Lemma 1 that $\pi_H(p', NO) > \pi_H(\tilde{p}, NO)$. Because $\tilde{p} > \hat{p}_L \geq \hat{p}_H$, it follows that firm H finds it optimal not to offer PMG at price $\tilde{p}$. Hence $\pi_H(p') = \pi_H(p', NO) > \pi_H(\tilde{p}, NO) = \pi_H(\tilde{p})$ □

It follows from Lemma 2 that there exists $\underline{p} < \tilde{p} < \overline{p}$ such that type L plays prices in $[\underline{p}, \tilde{p}]$ and type H plays prices in $[\tilde{p}, \overline{p}]$.

To see that there exists a price threshold, $\hat{p}$, as defined in the Proposition, notice that, because it was already shown that $\hat{p}_L \geq \hat{p}_H$, we can restrict the analysis to three cases: i) $\hat{p}_L \geq \hat{p}_H \geq \tilde{p}$. In this case, firms of type L always offer a PMG and firms of type H offer a PMG when they charge a price lower than $\hat{p}_H$. In this case, we can set $\hat{p} = \hat{p}_H$; ii) $\hat{p}_L > \tilde{p} > \hat{p}_H$. In this case, firms of type

L always offer a PMG and firms of type H never do. We can then set $\hat{p} = \tilde{p}$; iii) $\tilde{p} \geq \hat{p}_L \geq \hat{p}_H$. In this case, firms of type H never offer a PMG and firms of type L offer a PMG when they charge a price lower than $\hat{p}_L$. We can then set $\hat{p} = \hat{p}_L$

Proof of Lemma 8

By a similar argument as in the proof of Proposition 1, it can be shown that an equilibrium exists. Moreover, by the same reasoning as in the proof of Proposition 1, it is straightforward that if $\phi > 0$ then $\hat{p} > \underline{p}$. So in order for an equilibrium with the property that $\underline{p} < \hat{p} < v$ to exist, it is sufficient to show that no equilibrium where all firms offer PMGs (i.e $\hat{p} \geq v$) exists.

Let F denote the price distribution (unconditional on firms' marginal costs). Let $\sigma(p; q)$ denote the expected value of the minimum price in the market, given that one firm charges p, in an equilibrium where all firms offer PMGs. It follows from Proposition 2 that there exists $\tilde{p}$ such that for $p \in (\tilde{p}, v)$

$\pi_H(p) = \lambda[1 - F(p)]^{n-1}(p - c) + \frac{1-\lambda}{n}[q\sigma(p; q) + (1 - q)p - c]$

is constant. It follows that $\frac{\partial \pi_H(p)}{\partial p} = 0 \iff$

$$-(n-1)\lambda[1-F(p)]^{n-2}f(p)(p-c)+\lambda[1-F(p)]^{n-1}+\frac{1-\lambda}{n}-\frac{1-\lambda}{n}q\big[1-[1-F(p)]^{n-1}\big] = 0 \quad \text{(A.8)}$$

Let $q_1 > q_2$ and let F_1 and F_2 denote the respective equilibrium price distributions for the case where every firm offers a PMG. It is easy to see from Equation A.8 that if $F_1(p) = F_2(p)$ then $f_1(p) < f_2(p)$. Moreover, we know that $F_1(v) = F_2(v) = 1$. It then follows that F_2 first-order stochastically dominates F_1. Hence, $\sigma(p; q)$ is decreasing in q.

I will now characterize conditions for the non-existence of an equilibrium where all firms offer PMGs. It follows from Proposition 2 that firms of type H charge v. In an equilibrium where all firms offer PMGs, the profit of firms of type H is $\frac{1-\lambda}{n}[q\sigma(v; q) + (1 - q)v - c]$. If a firm of type H decided not to offer a PMG and list price v, its profit would be $\frac{(1-\lambda)(1-\phi)}{n}(v - c)$. Hence, a sufficient condition for an equilibrium where all firms offer PMGs not to exist is $\frac{(1-\lambda)(1-\phi)}{n}(v-c) > \frac{1-\lambda}{n}[q\sigma(v; q) + (1 - q)v - c]$. Equivalently,

$$q[v - \sigma(v; q)] > \phi(v - c) \quad \text{(A.9)}$$

Because $\sigma(v; q)$ is decreasing in q, it follows that the LHS of Equation A.9 is increasing in q. We can then define $\tilde{q}$ to be the value of q such that $q[v - \sigma(v; q)] = \phi(v - c)$.

It follows from Equation A.8 that as q approaches 1, $F(p)$ approaches 1 for $p > c$. It follows that as q approaches 1, $\sigma(v; q)$ converges to a value lower than c. It follows that $\tilde{q} < 1$.

I now show that property iii) also holds. First notice that the equilibrium price distribution for $p \in (\tilde{p}, v)$ is defined in Equation A.8 and does not depend on Δ. Moreover, it is required that firms of type L are indifferent between all prices in $(\underline{p}, \tilde{p})$. It follows that

$$-(n-1)\lambda[1-F(p)]^{n-2}f(p)(p-c)+\left[\lambda+\frac{1-\lambda}{n}q\right][1-F(p)]^{n-1}+\frac{1-\lambda}{n}(1-q)-\Delta(n-1)\lambda[1-F(p)]^{n-2}f(p) = 0 \quad \text{(A.10)}$$

Let $\Delta_1 > \Delta_2$ and let F_1 and F_2 denote the respective equilibrium price distributions for the case where every firm offers a PMG. It is easy to see from Equation A.10 that if $F_1(p) = F_2(p)$ then $f_1(p) < f_2(p)$. Moreover, we know that $F_1(\tilde{p}) = F_2(\tilde{p}) = 1 - \gamma$. It then follows that F_2 first-order stochastically dominates F_1. It follows that $\sigma(p; q)$ is decreasing in Δ. It is then immediate from Equation A.9 that $\tilde{q}$ is decreasing in Δ.

Web Appendix B - Introducing PMGs in the setup of Varian (1980)

In this appendix, I show that, when firms are able to offer PMGs in the setup of Varian (1980), they cannot act as a signal of low prices. In the setup of Varian (1980), a fraction λ of consumers are informed about prices, so they go to the store with the lowest price and purchase the product there. The remaining fraction $1-\lambda$ of consumers are uninformed about firms' prices, and they can only visit one store. They purchase the product at the store they visit provided that the price is not higher than their reservation price, v.

Firms are able to offer PMGs, and uninformed consumers observe firms' PMG strategy. They can then choose whether to visit a PMG store or a store that does not offer such policy.

I will show that there is no rational expectations equilibrium under which PMG stores offer low prices. In order to do that, I assume that consumers believe PMG stores to offer low prices, so uninformed consumers choose to visit a PMG store. I will show that, under this assumption, PMG stores charge higher prices, which is against consumers' belief and, hence, cannot be a rational expectations equilibrium.

Let n denote the number of firms and let k denote the number of firms that offer PMGs. I take k as exogenous. In order for PMGs to signal low prices, both PMG and non-PMG stores must exist. To simplify the analysis, I consider the case where at least 2 stores do not offer PMGs, i.e., $n-k \geq 2$. I consider a symmetric equilibrium, under which all PMG firms play the same strategy and all non-PMG firms play the same strategy. I denote the equilibrium price distributions of PMG and non-PMG firms as F_{PMG} and F_{NO}, respectively.

Because uninformed consumers purchase at PMG stores, non-PMG stores can only sell to informed consumers. Informed consumers purchase from the firm with the lowest price.

First notice that F_{NO} can have no atoms at prices higher than marginal cost. By contradiction, suppose F_{NO} had an atom at some $\tilde{p} > c$. Non-PMG firms would prefer to play a price slightly lower than $\tilde{p}$. This leads to a discontinuous jump in the probability of selling to informed consumers.

Moreover, F_{NO} must be degenerate. If F_{NO} was non-degenerate, a non-PMG firm that charges the upper bound on the support of F_{NO} will never be the firm with the lowest price, and will make zero profit, so it would prefer to charge a lower price.

It then follows that F_{NO} is degenerate at price equal to marginal cost, c.

PMG stores make positive profits in equilibrium, because they can choose to charge any price between c and v in which case they sell to uninformed consumers that enter their store, a fraction $\frac{1-\lambda}{k}$. Hence, it follows that PMG stores charge prices strictly higher than c. But then PMG stores never sell to informed consumers and would prefer to charge v and extract all surplus from uninformed consumers. The following Proposition summarizes the results.

Proposition B.1 *There is no rational expectations equilibrium under which PMGs signal low prices. When consumers believe that PMG stores charge lower prices, non-PMG stores charge c and PMG stores charge v, which is inconsistent with consumers' belief.*

Web Appendix C - A Survey on Consumers' Awareness of PMG Policies

Because the assumption that consumers are heterogeneous regarding their information of firms' PMG policies is new in the PMG literature, I test it by conducting a survey on consumers' information regarding firms' PMG policies. 112 US-based Amazon Mechanical Turk workers participated in the survey. The survey starts by asking respondents whether they are familiar with Price-Matching Guarantees. 5 respondents were not familiar with this policy and were eliminated from the sample. Participants were presented with 4 of the major firms in the Retail chain category (Sears, Target, Costco, and Macy's) and 4 of the major firms in the Toys/Hobbies category (ToysRUs, GameStop, American Girl and LEGO) according to Top 500 list by *Internet Retailer* in 2014.

First, respondents were asked whether they were familiar with each firm. Participants were then asked about the PMG policy of each firm (there were three options: "the firm offers a PMG", "the firm does not offer a PMG" and "I do not know whether or not the firm offers a PMG").

For each firm, I only consider respondents familiar with it. All stores considered were known by the majority of respondents (with the exception of American Girl, for which 60% of respondents are familiar with, all stores are familiar to more than 80% of respondents).

A share of 13% of answers were incorrect (i.e. respondents who stated that firms that offer PMGs do not offer them and vice-versa). Such incorrect responses were removed from the sample. The results presented in Table C.1 support the assumption that only some consumers are aware of firms' PMG policies. For stores that offer PMGs, an average of 40% of respondents are aware that the store offers such policy.

Table C.1: Consumer awareness of firms' PMG policies

	Informed	Uninformed
PMG Stores		
ToysRUs	41%	59%
Sears	23%	77%
Target	54%	46%
Non-PMG Stores		
Gamestop	24%	76%
American Girl	22%	78%
Lego	21%	79%
Costco	28%	72%
Macy's	25%	75%

Web Appendix D - When s is small

In the base model, I show that, in equilibrium, uninformed non-shoppers who purchase from a PMG store do not search after purchase when they have post-purchase search cost s. However, informed non-shoppers have a larger incentive to search post-purchase, as they can go directly to another PMG store, where prices are expected to be lower. I have assumed that s is large enough so that informed non-shoppers also refrain from searching after purchase when their post-purchase search cost is s. In this appendix, I show that the main result holds even when this is not the case, i.e., I show that when informed non-shoppers who visit a PMG store that charges $\bar{p}_{PMG}$ search post-purchase even if they have high post-purchase search costs, the result that there exists a threshold such that PMG firms charge prices below the threshold and non-PMG stores charge prices above the threshold still holds.

Let z_I^{PMG} denote the price that makes informed non-shoppers indifferent between searching another PMG store at search cost s and stop searching, i.e., informed non-shoppers search post-purchase even when they have search cost s when the price they pay is higher than z_I^{PMG}. In that case, because they expect PMG stores to have lower prices, they immediately search a PMG store. They keep searching PMG stores until either they find a price lower than z_I^{PMG} or they exhaust all PMG stores. Let $V_k(p)$ denote the expected price paid by a consumer who follows this search rule, when k remaining stores offer PMGs.

The profit of a PMG firm that charges price $p \leq z_I^{PMG}$ is the same as defined in Equation 3. The profit of a PMG firm that charges price $p > z_I^{PMG}$ is

$$\pi_{PMG}(p) = \sum_{k=0}^{n-1} g(k) \Bigg[\lambda L_k(p)(p-c) + \frac{(1-\lambda)(1-\phi)}{n}\big(qEmin(p/k) + (1-q)p - c\big) + \\ + \frac{(1-\lambda)\phi}{k+1}\big(qEmin(p/k) + (1-q)V_k(p) - c\big) \Bigg] \tag{D.1}$$

The profit of a non-PMG firm is the same as defined in Equation 2.

For $i \in \{A, B\}$ let $\zeta_i(p,k) = \begin{cases} p & \text{if } i = A \\ V_k(p) & \text{if } i = B \end{cases}$.

For $i \in \{A, B\}$ let

$$\Delta_i(p) = \sum_{k=1}^{n-1} g(k) \left\{ \frac{(1-\lambda)\phi}{k+1}\big(qEmin(p/k) + (1-q)\zeta_i(p,k) - c\big) - \frac{(1-\lambda)(1-\phi)q}{n}[p - Emin(p/k)] \right\} \\ + g(0) \left\{ (1-\lambda)\phi\frac{n-1}{n}\big(qEmin(p/0) + (1-q)p - c\big) - \frac{(1-\lambda)q}{n}[p - Emin(p/0)] \right\}$$

Define $\Delta(p) = \pi_{PMG}(p) - \pi_{NO}(p)$. It follows that

$$\Delta(p) = \begin{cases} \Delta_A(p) & \text{if } p \leq z_I^{PMG} \\ \Delta_B(p) & \text{if } p > z_I^{PMG} \end{cases}$$

Because both $Emin(p/k)$ and $V_k(p)$ are concave, it follows that $\Delta_A(p)$ and $\Delta_B(p)$ are concave. In addition, because $V_k(\underline{p}) = \underline{p}$ for all k, it follows that $\Delta_A(\underline{p}) = \Delta_B(\underline{p})$. Also, it is straightforward to see that if $\phi > 0$ then $\Delta_A(\underline{p}) = \Delta_B(\underline{p}) > 0$. It then follows that if $\phi > 0$, for $i \in \{A, B\}$ there exists $\hat{p}_i > \underline{p}$ such that $\Delta_i(p) > 0$ for $p < \hat{p}_i$ and $\Delta_i(p) \leq 0$ for $p \geq \hat{p}_i$. Moreover, because $V_k(p) \leq p$, it follows that $\Delta_A(p) \geq \Delta_B(p)$ for all p. Hence, $\hat{p}_B \leq \hat{p}_A$.

Now suppose that $z_I^{PMG} < \bar{p}_{PMG}$, i.e., PMG firms charge prices above z_I^{PMG} with positive probability. Then there exists $p > z_I^{PMG}$ such that $\Delta_B(p) > 0$. It then follows that $p < \hat{p}_B \leq \hat{p}_A$. Hence, $\Delta_A(p) > 0$ for all $p \leq z_I^{PMG}$, i.e., firms that charge prices lower than z_I^{PMG} offer PMGs. It then follows that firms that do not offer PMGs charge prices above $\hat{p}_B$ whereas firms that offer PMGs charge prices below $\hat{p}_B$.

Web Appendix E - Details of Example 1

Let $\alpha = 0.5$ and let $\underline{p} = \frac{(1-\lambda)(1-\alpha\phi)}{2\lambda+(1-\lambda)(1+\phi-\alpha\phi)}v \approx 0.31$, $p_1 = \frac{(1-\lambda)(1-\alpha\phi)}{2\lambda(1-\alpha)+(1-\lambda)(1+\phi-\alpha\phi)}v \approx 0.46$, $p_2 = \frac{(1-\lambda)(1-\alpha\phi)}{2\lambda(1-\alpha)+(1-\lambda)(1-\alpha\phi)}v \approx 0.49$ and $\bar{p} = v = 1$.

Let the equilibrium price distribution, denoted by F, be as follows.

$$F(p) = \begin{cases} 0 & \text{if } p < \underline{p} \\ \frac{2\lambda+(1-\lambda)(1+\phi-\alpha\phi)}{2\lambda} - \frac{(1-\lambda)(1-\alpha\phi)}{2\lambda}\frac{v}{p} & \text{if } \underline{p} \leq p < p_1 \\ \alpha & \text{if } p_1 \leq p < p_2 \\ \frac{2\lambda+(1-\lambda)(1-\alpha\phi)}{2\lambda} - \frac{(1-\lambda)(1-\alpha\phi)}{2\lambda}\frac{v}{p} & \text{if } p_2 \leq p < v \\ 1 & \text{if } p \geq v \end{cases}$$

Consumer search strategy

Shoppers can become informed about prices for free, so they purchase from the firm with the lowest price. Uninformed non-shoppers will visit a store at random. Because $s > v - \int_{\underline{p}}^{v} p \, \mathrm{d}F(p)$, it follows that non-shoppers purchase even if they sample price v. Hence, non-shoppers purchase at the random store they visit. Informed non-shoppers believe that PMG stores charge lower prices, so they choose to visit a PMG store (when it exists). Because $n = 2$, there are three cases to consider: i) no firm offers a PMG, in which case informed non-shoppers behave like uninformed non-shoppers; ii) only one firm offers a PMG, in which case informed non-shoppers purchase from the PMG store and never search after purchase because they expect that the price of the PMG store is always lower than the price of a non-PMG store, so there is no benefit to search after purchase; iii) both stores offer PMGs, in which case informed non-shoppers purchase at a random PMG store. In this case, there is a benefit of searching post-purchase, as it is possible that the store they did not visit has a lower price. The benefit from searching after purchasing at price x (gross of search costs), denoted by $B(x)$, is $B(x) = \int_{\underline{p}}^{x} x - p \, \mathrm{d}F_{PMG}(p)$[1]. After some algebra, it follows that $B(p_1) = s_L$. Hence, consumers only search after purchase when they have search cost s_L if the price they paid is higher than p_1. Because PMG stores always charge prices lower than p_1, it then follows that consumers never search post-purchase.

Hence, given the price strategy described above, the consumer search strategy described in Example 1 is optimal. I will now show that, given the consumer search strategy, the price strategy is optimal.

Firm strategy

Let $\pi_{PMG}(p)$ ($\pi_{NO}(p)$) denote the expected profit of a firm that offers (does not offer) a PMG and charges price p. Let $E(p/p < p_1) = \int_{\underline{p}}^{p_1} p \, \mathrm{d}F_{PMG}(p)$. Given the consumer search strategy described

[1]For $p \in [\underline{p}, p_1]$, $F_{PMG}(p) = \frac{1}{\alpha}F(p)$

above, it follows that

$$\pi_{NO}(p) = \begin{cases} \lambda[1 - F(p)]p + \frac{(1-\lambda)(1-\alpha\phi)}{2}p & \text{if } p \leq v \\ 0 & \text{if } p > v \end{cases}$$

$$\pi_{PMG}(p) = \begin{cases} \lambda[1 - F(p)]p + \frac{(1-\lambda)(1+\phi-\alpha\phi)}{2}p & \text{if } p \leq p_1 \\ \lambda[1 - F(p)]p + \frac{(1-\lambda)(1+\phi-\alpha\phi)}{2}[q\alpha E(p/p < p_1) + (1 - q\alpha)p] & \text{if } p_1 < p \leq v \\ 0 & \text{if } p > v \end{cases}$$

Let $\pi^* \equiv \frac{(1-\alpha)(1-\alpha\phi)}{2}v = 0.2375$. After some algebra, it follows that $\pi_{NO}(x) = \pi^*$ for $x \in [p_2, v]$, $\pi_{NO}(x) \leq \pi^*$ for $x \notin [p_2, v]$, $\pi_{PMG}(x) = \pi^*$ for $x \in [\underline{p}, p_1]$, and $\pi_{PMG}(x) \leq \pi^*$ for $x \notin [\underline{p}, p_1]$. It follows that the pricing strategy is optimal.

Web Appendix F - Robustness

F.1 Sequential Choice of PMG and Price

The literature is not consensual regarding the timing that firms use to choose prices and PMG policies. Whereas Png and Hirshleifer (1987), Jain and Srivastava (2000), and Janssen and Parakhonyak (2013) use the same timing as the model in this paper, Moorthy and Winter (2006), Moorthy and Zhang (2006), and Jiang, Kumar, and Ratchford (2016) assume that firms first decide on whether or not to offer a PMG, and only choose prices after observing which firms offer PMGs.

The argument to model the decision of PMG policies and prices as sequential is that prices are more flexible than PMGs, and usually firms stick to their PMG policy for a long period of time.

Similar to previous literature that finds PMGs as a signal of low prices (namely Moorthy and Winter 2006 and Moorthy and Zhang 2006), a feature of this model is that, because PMGs are used to transmit information to consumers, they lead to an increase in consumer welfare. In this paper, because I have considered a model of unit demands, total welfare is not affected by the presence or absence of PMGs, it is always $v - c$. Hence, an increase in consumer welfare leads to a reduction in industry profit.

Because PMGs are pro-competitive, when the choice of PMG and price is modeled as a two-stage game, firms may refrain from offering PMGs in the first stage, in order to soften price competition in the second stage. The analysis of the sequential game features two questions: i) under what conditions will firms offer PMGs? ii) what prices will PMG and non-PMG stores charge, in case some firms offer PMGs?

The main goal of this paper is not to find conditions under which firms will adopt PMGs, but rather to understand the pricing incentives of PMG and non-PMG firms in markets where PMGs exist. To abstract from the first question, the base model assumes a simultaneous choice of prices and PMG policies. In the simultaneous game, there is always a positive probability that a firm offers a PMG (as long as $\phi > 0$).

Even though the analysis of the simultaneous game provides cleaner results regarding the pricing incentives for PMG and non-PMG stores, such incentives are similar in the sequential game. Indeed, when firms choose prices in the simultaneous game they do not know the PMG policies of the other firms, but they hold rational beliefs regarding the probability that each firm offers a PMG. Such belief generates a probability distribution over the number of stores that offer PMGs (which I denote by k). Firms choose prices taking into account the probability distribution over k.

If, instead, PMG policies were chosen before prices, firms would observe the actual realization of k before choosing prices. Instead of choosing prices based on a (correct) belief on the distribution of k, firms would choose prices based on the actual realization of k. The underlying intuition of the model and the main results do not change.

I analyze a two-stage model, where firms choose PMG policies first and choose prices only after observing each others' PMG policies. I make the simplifying assumption that the fraction $1 - \lambda$ of non-shoppers buy at the first store they visit, as long as the price of such store provides them with nonnegative consumer surplus. Effectively, this is an assumption that the search cost of visiting another retailer is larger than the consumer surplus they expect to get by doing so. Jain and Srivastava (2000), Moorthy and Winter (2006), and Moorthy and Zhang (2006) also make similar assumptions. To facilitate the exposition, and without loss of generality, I assume zero marginal costs of production.

The main result is summarized below. k^* denotes the equilibrium number of firms that offer PMGs. Moreover, let $q_1 \equiv \frac{1}{1+\frac{1-\phi}{\phi n}\left(\frac{(1-\lambda)(1-\phi)}{\lambda n+(1-\lambda)(1-\phi)}\right)^{\frac{(1-\lambda)[1+\phi(n-1)]}{\lambda n}}}$ and $q_2 \equiv \frac{1}{1+\frac{n\lambda}{n-(1-\lambda)(1-\phi)}\left(\frac{(1-\lambda)(1-\phi)}{1+\lambda(n-1)}\right)^{1+\frac{1-\lambda}{n}}}$.

Proposition F.1 *When $\lambda < \frac{(n-1)(1-\phi)}{n+(n-1)(1-\phi)}$ and $q > \max\{q_1, q_2\}$, both PMG and non-PMG stores coexist, i.e., $1 \leq k^* \leq n-1$. When $k^* = 1$, the expected price of the PMG store is lower than the expected price of a non-PMG store, provided that ϕ is small and q is large. When $k^* \geq 2$, PMG stores choose prices from $[\underline{p}, v]$ and non-PMG stores choose price v.*

Below, I provide the analysis that leads to the above result. I start by introducing some notation: let $Emin(x, F)$ denote the expected value of the minimum between x and $(k-1)$ draws from the distribution F.

The following Lemma will be useful throughout the analysis

Lemma F.1 *Let $F(x) = 1 - \left(\frac{\underline{p}}{x}\right)^T$. Then $Emin(v, F) = \frac{1}{1-T(k-1)}\left(\frac{\underline{p}}{v}\right)^{T(k-1)} v - \frac{(k-1)T}{1-T(k-1)}\underline{p}$*

Proof. Let $G(x)$ denote the cumulative distribution of the minimum between v and $(k-1)$ draws from F. It follows that, for $x \leq v$, $G(x) = 1 - [1-F(x)]^{k-1}$. Hence

$Emin(v, F) = \int_{\underline{p}}^{v} xg(x)\,\mathrm{d}x + [1-G(v)]v$

The result follows after some algebra □

Second stage: choice of prices

I start by analyzing the equilibrium in the pricing stage, after the number of firms that offer PMGs, denoted by k, is known.

k=0

When no firm offers PMGs, the equilibrium in the pricing stage is similar to Varian (1980). All firms make profit $\frac{1-\lambda}{n}v$

k=1

For the case where only one firm offers a PMG, I show that there exists an equilibrium such that the PMG firm and one non-PMG firm play mixed strategies on prices, whereas the remaining non-PMG firms play price v. In such equilibrium, the profit of a non-PMG firms is $\frac{(1-\lambda)(1-\phi)}{n}v$. Let F_{PMG} denote the equilibrium price distribution of the PMG firm and let F_{NO} denote the equilibrium price distribution of the non-PMG firm that plays mixed strategies. Using an argument similar to Varian (1980), it can be shown that both distributions have the same lower bound, which I denote by $\underline{p}$.

In order for the non-PMG firm to be indifferent between all prices in $[\underline{p}, v]$, it must be that

$\lambda[1 - F_{PMG}(x)]x + \frac{(1-\lambda)(1-\phi)}{n}x = \frac{(1-\lambda)(1-\phi)}{n}v \iff F_{PMG}(x) = \frac{\lambda n + (1-\lambda)(1-\phi)}{\lambda n} - \frac{(1-\lambda)(1-\phi)}{\lambda n}\frac{v}{x}$

Because $F_{PMG}(\underline{p}) = 0$, it follows that $\underline{p} = \frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)}v$. It then follows that

$\pi_{PMG} = \pi_{PMG}(\underline{p}) = \left[\lambda + (1-\lambda)\phi + \frac{(1-\lambda)(1-\phi)}{n}\right]\frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)}v$

The profit of the PMG firm when it chooses price x is

$\pi_{PMG}(x) = \lambda[1 - F_{NO}(x)]x + \left[(1-\lambda)\phi + \frac{(1-\lambda)(1-\phi)}{n}\right]\left(qEmin(x) + (1-q)x\right)$

In order for the PMG firm to be indifferent between all prices in $[\underline{p}, v]$, it must be that $\frac{\partial \pi_{PMG}(x)}{\partial x} = 0$. Hence

$-\lambda f_{NO}(x)x + \left[\lambda + \left((1-\lambda)\phi + \frac{(1-\lambda)(1-\phi)}{n}\right)\right][1 - F_{NO}(x)] + \left[(1-\lambda)\phi + \frac{(1-\lambda)(1-\phi)}{n}\right](1-q) = 0$

Solving the above differential equation, together with the condition that $F_{NO}(\underline{p}) = 0$, we have that

$F_{NO}(x) = \frac{\lambda n + (1-\lambda)[1+\phi(n-1)]}{\lambda n + (1-\lambda)[1+\phi(n-1)]q}\left[1 - \left(\frac{\underline{p}}{x}\right)^{1 + \frac{(1-\lambda)[1+\phi(n-1)]q}{\lambda n}}\right]$

Using the expressions for F_{PMG} and F_{NO}, we can compute the expected price of the PMG and non-PMG stores:

$E(p_{PMG}) = \frac{(1-\lambda)(1-\phi)}{\lambda n} ln\left(1 + \frac{\lambda n}{(1-\lambda)(1-\phi)}\right)v$

$E(p_{NO}) = \left[A - \frac{\lambda n + (1-\lambda)[1+\phi(n-1)](1-q)}{(1-\lambda)[1+\phi(n-1)]q}\left(\frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)}\right)^{\frac{\lambda n + (1-\lambda)[1+\phi(n-1)]q}{\lambda n}}\right]v$

where $A = \frac{\lambda n + (1-\lambda)[1+\phi(n-1)]}{[1+\phi(n-1)]q}\frac{1-\phi}{\lambda n + (1-\lambda)(1-\phi)} - \frac{(1-\lambda)[1+\phi(n-1)](1-q)}{\lambda n + (1-\lambda)[1+\phi(n-1)]q}$

It can be shown that, for $\phi = 0$ and $q = 1$, $E(p_{NO}) > E(p_{PMG})$. It then follows from continuity that for q large enough and ϕ small enough, $E(p_{NO}) > E(p_{PMG})$.

$1 < k < n$

I will find conditions for there to be an equilibrium where all non-PMG firms play v and all PMG firms play prices according to a distribution F_{PMG}. Under this equilibrium, $\pi_{NO} = \frac{(1-\lambda)(1-\phi)}{n}v$. Let F_{PMG} be such that

$\pi_{PMG} = \lambda[1 - F_{PMG}(x)]^{k-1}x + (1-\lambda)\left[\frac{\phi}{k} + \frac{1-\phi}{n}\right]\left(qEmin(x, F_{PMG}) + (1-q)x\right)$

Given F_{PMG} described above, PMG firms are indifferent between all prices in $[\underline{p}, v]$. In order to show that this is an equilibrium, it is left to verify that non-PMG firms prefer to play price v than any other price.

Notice that, for $x \in [\underline{p}, v]$

$\pi_{NO}(x) = \lambda[1 - F_{PMG}(x)]^k x + \frac{(1-\lambda)(1-\phi)}{n}x$

$\pi_{PMG}(x) = \lambda[1 - F_{PMG}(x)]^{k-1}x + (1-\lambda)\left[\frac{\phi}{k} + \frac{1-\phi}{n}\right]\left(qEmin(x, F_{PMG}) + (1-q)x\right)$

It follows that

$\pi_{NO}(x) - \pi_{PMG}(x) = -\lambda[1 - F_{PMG}(x)]^{k-1}F_{PMG}(x)x + W(x)$

where $W(x) = \frac{(1-\lambda)(1-\phi)}{n}q[x - Emin(x, F_{PMG})] - \frac{(1-\lambda)\phi}{k}[qEmin(x) + (1-q)x]$

Because $\pi_{PMG}(x)$ is constant, the optimal price for a non-PMG firms is the one that maximizes $\pi_{NO}(x) - \pi_{PMG}(x)$. Notice that W is convex, so the maximum of W is achieved either at $\underline{p}$ or at v. Moreover, the maximum of the first term, $-\lambda[1-F_{PMG}(x)]^{k-1}F_{PMG}(x)x$, is achieved both at $\underline{p}$ and at v. Hence, if a non-PMG firm prefers to charge a price other than v, it will prefer to charge $\underline{p}$. It follows that, in order for the above strategies to be an equilibrium, it is only left to verify that $\pi_{NO}(v) \geq \pi_{NO}(\underline{p})$

Let $\tilde{F}$ be such that

$\pi_{PMG} = \lambda[1-\tilde{F}(x)]^{k-1}x + (1-\lambda)\left[\frac{\phi}{k} + \frac{1-\phi}{n}\right]Emin(x,\tilde{F})$

It is clear that $\tilde{F}(x) \leq F_{PMG}(x)$ for all x and $\tilde{F}(\underline{p}) = F(\underline{p})$. Moreover, differentiating the above expression yields

$$(k-1)[1-\tilde{F}(x)]^{k-2}\tilde{f}(x)x = \left[1 + \frac{1-\lambda}{\lambda}\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)\right][1-\tilde{F}(x)]^{k-1} \tag{F.1}$$

The solution to the above differential equation, together with the condition that $\tilde{F}(\underline{p}) = 0$, is

$\tilde{F}(x) = 1 - \left(\frac{\underline{p}}{x}\right)^{\frac{1}{k-1} + \frac{1-\lambda}{\lambda(k-1)}\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)}$

We can then use Lemma F.1 to get that $Emin(v,\tilde{F}) = \frac{1+U}{U}\underline{p} - \frac{1}{U}\underline{p}\left(\frac{\underline{p}}{v}\right)^U$

where $U = \frac{1-\lambda}{\lambda}\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)$

Because $\tilde{F}(x) \leq F_{PMG}(x)$ for all x, it follows that $Emin(v,\tilde{F}) \geq Emin(v, F_{PMG})$. Hence,

$$\begin{aligned}
\pi_{PMG} &= \pi_{PMG}(v) \\
&= (1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)\left[qEmin(v,F_{PMG}) + (1-q)v\right] \\
&\leq (1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)\left[qEmin(v,\tilde{F}) + (1-q)v\right] \\
&= (1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)\left[q\frac{1+U}{U}\underline{p} - \frac{q}{U}\underline{p}\left(\frac{\underline{p}}{v}\right)^U + (1-q)v\right]
\end{aligned}$$

Because $\pi_{PMG} = \pi_{PMG}(\underline{p}) = \left[\lambda + \frac{(1-\lambda)\phi}{k} + \frac{(1-\lambda)(1-\phi)}{n}\right]\underline{p}$, it follows that

$$\left[\lambda + \frac{(1-\lambda)\phi}{k} + \frac{(1-\lambda)(1-\phi)}{n}\right]\underline{p} \leq (1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)\left[q\frac{1+U}{U}\underline{p} - \frac{q}{U}\underline{p}\left(\frac{\underline{p}}{v}\right)^U + (1-q)v\right] \tag{F.2}$$

Let $\gamma \equiv \frac{\underline{p}}{v}$. The above equation becomes

$$\gamma(1+U)(1-q) + q\gamma^{1+U} \leq U(1-q) \tag{F.3}$$

Notice that the LHS of Equation F.3 is increasing in γ. Hence, let $\bar{\gamma}$ be such that

$\bar{\gamma}(1+U)(1-q) + q\bar{\gamma}^{1+U} = U(1-q)$

Then Equation F.2 is equivalent to $\gamma \leq \bar{\gamma}$

In order for the non-PMG firm to prefer to charge v over $\underline{p}$, it is required that

$\frac{(1-\lambda)(1-\phi)}{n} v \geq \left[\lambda + \frac{(1-\lambda)(1-\phi)}{n}\right]\underline{p} \iff \gamma \leq \frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)}$

Hence, a non-PMG firm will prefer to charge v over $\underline{p}$ as long as $\frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)} \geq \bar{\gamma}$. Let $T \equiv \frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)}$. Because the LHS of Equation F.3 is increasing in γ, it follows that $T \geq \bar{\gamma}$ if

$T(1+U)(1-q) + qT^{1+U} > U(1-q)$

Replacing U and T we get $q > \frac{1}{1+k\frac{1-\phi}{\phi n}\left(\frac{(1-\lambda)(1-\phi)}{\lambda n+(1-\lambda)(1-\phi)}\right)^{\frac{1-\lambda}{\lambda}\left(\frac{\phi}{k}+\frac{1-\phi}{n}\right)}}$

Because the RHS of the above expression is decreasing with k, a sufficient condition is obtained by replacing $k = 1$. Hence, we require that

$$q > \frac{1}{1 + \frac{1-\phi}{\phi n}\left(\frac{(1-\lambda)(1-\phi)}{\lambda n+(1-\lambda)(1-\phi)}\right)^{\frac{(1-\lambda)[1+\phi(n-1)]}{\lambda n}}} \tag{F.4}$$

We find that, under (F.4), there exists an equilibrium as described in Proposition F.1.

k=n

Let F_{PMG} be such that

$\pi_{PMG} = \lambda[1 - F_{PMG}(x)]^{n-1}x + \frac{1-\lambda}{n}\left(qEmin(x, F_{PMG}) + (1-q)x\right)$

Given F_{PMG} described above, PMG firms are indifferent between all prices in $[\underline{p}, v]$.

Let $\tilde{F}$ be such that

$\pi_{PMG} = \lambda[1 - \tilde{F}(x)]^{n-1}x + \frac{1-\lambda}{n}Emin(x, \tilde{F})$

It is clear that $\tilde{F}(x) \leq F_{PMG}(x)$ for all x and $\tilde{F}(\underline{p}) = F(\underline{p})$. Moreover, differentiating the above expression yields

$$(n-1)[1-\tilde{F}(x)]^{n-2}\tilde{f}(x)x = \left[1 + \frac{1-\lambda}{\lambda n}\right][1-\tilde{F}(x)]^{k-1} \tag{F.5}$$

The solution to the above differential equation, together with the condition that $\tilde{F}(\underline{p}) = 0$, is

$\tilde{F}(x) = 1 - \left(\frac{\underline{p}}{x}\right)^{\frac{1}{n-1}+\frac{1-\lambda}{n\lambda(n-1)}}$

We can then use Lemma F.1 to get that $Emin(v, \tilde{F}) = \left(1 + \frac{n\lambda}{1-\lambda}\right)\underline{p} - \frac{n\lambda}{1-\lambda}\underline{p}\left(\frac{\underline{p}}{v}\right)^{\frac{1-\lambda}{n\lambda}}$

Because $\tilde{F}(x) \leq F_{PMG}(x)$ for all x, it follows that $Emin(v, \tilde{F}) \geq Emin(v, F_{PMG})$. Hence,

$$\begin{aligned}
\pi_{PMG} &= \pi_{PMG}(v) \\
&= \frac{1-\lambda}{n}\Big[qEmin(v, F_{PMG}) + (1-q)v\Big] \\
&\leq \frac{1-\lambda}{n}\Big[qEmin(v, \tilde{F}) + (1-q)v\Big] \\
&= \frac{1-\lambda}{n}\Bigg[q\Bigg(1+\frac{n\lambda}{1-\lambda}\Bigg)\underline{p} - q\frac{n\lambda}{1-\lambda}\underline{p}\Bigg(\frac{\underline{p}}{v}\Bigg)^{\frac{1-\lambda}{n\lambda}} + (1-q)v\Bigg]
\end{aligned}$$

Because $\pi_{PMG} = \pi_{PMG}(\underline{p}) = \Bigg[\lambda + \frac{1-\lambda}{n}\Bigg]\underline{p}$, it follows that

$$\Bigg[\lambda + \frac{1-\lambda}{n}\Bigg]\underline{p} \leq \frac{1-\lambda}{n}\Bigg[q\Bigg(1+\frac{n\lambda}{1-\lambda}\Bigg)\underline{p} - q\frac{n\lambda}{1-\lambda}\underline{p}\Bigg(\frac{\underline{p}}{v}\Bigg)^{\frac{1-\lambda}{n\lambda}} + (1-q)v\Bigg] \tag{F.6}$$

Let $\omega \equiv \frac{\underline{p}}{v}$. The above equation becomes

$$\omega\Bigg(\lambda + \frac{1-\lambda}{n}\Bigg)(1-q) + \lambda q\omega^{1+\frac{1-\lambda}{n}} \leq \frac{1-\lambda}{n}(1-q) \tag{F.7}$$

Notice that the LHS of Equation F.7 is increasing in ω. Hence, let $\bar{\omega}$ be such that

$$\bar{\omega}\Bigg(\lambda + \frac{1-\lambda}{n}\Bigg)(1-q) + \lambda q\bar{\omega}^{1+\frac{1-\lambda}{n}} = \frac{1-\lambda}{n}(1-q) \tag{F.8}$$

Then Equation F.6 is equivalent to $\omega \leq \bar{\omega}$

We then have that $\pi_{PMG} \leq \Bigg[\lambda + \frac{1-\lambda}{n}\Bigg]\bar{\omega}v$

First stage: choice of PMG policies

I now analyze the first stage of the game, where firms choose whether or not to offer a PMG. I consider pure strategies, so that the number of PMG stores is deterministic. A number k of PMG stores is an equilibrium if no firm can gain by unilaterally changing its policy. Let $\pi_{PMG}(k)$ and $\pi_{NO}(k)$ denote, respectively, the profit of PMG and non-PMG firms when exactly k stores offer a PMG. There exists an equilibrium where no firm offers PMGs ($k^* = 0$) if $\pi_{NO}(k = 0) \geq \pi_{PMG}(k = 1)$. There exists an equilibrium where all firms offer PMGs if $\pi_{PMG}(k = n) \geq \pi_{NO}(k = n-1)$. An interior equilibrium, where $1 \leq k^* \leq n-1$, is such that $\pi_{PMG}(k^*) \geq \pi_{NO}(k^* - 1)$ and $\pi_{NO}(k^*) \geq \pi_{PMG}(k^* + 1)$.

Lemma F.2 *There always exists an equilibrium k^**

Proof. First notice from the analysis of the second stage that

$\pi_{NO}(k=0) > \pi_{NO}(k=1) = \pi_{NO}(k=2) = ... = \pi_{NO}(k=n-1)$.

Let $\tilde{\pi} \equiv \pi_{NO}(k=1)$

As discussed above, if $\pi_{NO}(k=0) \geq \pi_{PMG}(k=1)$ then $k^* = 0$ is an equilibrium. So assume that $\pi_{NO}(k=0) < \pi_{PMG}(k=1)$.

Moreover, if $\pi_{PMG}(k=n) \geq \pi_{NO}(k=n-1)$ then $k^* = n$ is an equilibrium. So assume that $\pi_{PMG}(k=n) < \pi_{NO}(k=n-1)$. Equivalently, $\pi_{PMG}(k=n) < \tilde{\pi}$.

Now suppose that $\pi_{NO}(k=1) \geq \pi_{PMG}(k=2)$. Because we are already assuming that $\pi_{NO}(k=0) < \pi_{PMG}(k=1)$, this then would imply that $k^* = 1$ is an equilibrium. So let us assume that $\pi_{NO}(k=1) < \pi_{PMG}(k=2)$. Equivalently, $\pi_{PMG}(k=2) > \tilde{\pi}$.

We then have that $\pi_{PMG}(k=2) > \tilde{\pi} > \pi_{PMG}(k=n)$. This then implies that there exists some $\tilde{k} \in \{2, 3, ..., n-1\}$ such that $\pi_{PMG}(\tilde{k}) \geq \tilde{\pi}$ and $\pi_{PMG}(\tilde{k}+1) < \tilde{\pi}$. It immediately follows that such $\tilde{k}$ is an equilibrium.

□

The result from Lemma F.2 implies that, in order for an interior equilibrium to exist, it suffices to verify that $k^* \neq 0$ and $k^* \neq n$.

We have seen in the analysis of the second stage that, when no firm offers PMGs, the profit of each firm is $\frac{1-\lambda}{n}v$. When only one firm offers a PMG, the profit of the PMG firm is $\left[\lambda + (1-\lambda)\phi + \frac{(1-\lambda)(1-\phi)}{n}\right]\frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)}v$. It follows that $k^* \neq 0$ if

$\pi_{PMG}(k=1) > \pi_{NO}(k=0) \iff \left[\lambda + (1-\lambda)\phi + \frac{(1-\lambda)(1-\phi)}{n}\right]\frac{(1-\lambda)(1-\phi)}{\lambda n + (1-\lambda)(1-\phi)}v > \frac{1-\lambda}{n}v \iff \lambda < \frac{(n-1)(1-\phi)}{n+(n-1)(1-\phi)}$

In the analysis of the second stage it was found that, when all firms offer a PMG, the profit of each firm is bounded above by $\left[\lambda + \frac{1-\lambda}{n}\right]\bar{\omega}v$, where $\bar{\omega}$ is defined in Equation F.8. When all but one store offer PMGs, the profit of the non-PMG store is $\frac{(1-\lambda)(1-\phi)}{n}v$. It follows that $k^* \neq n$ if

$\pi_{NO}(k=n-1) > \pi_{PMG}(k=n) \impliedby \frac{(1-\lambda)(1-\phi)}{n}v > \left[\lambda + \frac{1-\lambda}{n}\right]\bar{\omega}v \iff \bar{\omega} < \frac{(1-\lambda)(1-\phi)}{1+\lambda(n-1)}$

Because the LHS of Equation F.7 is increasing, we have that $\bar{\omega} < \frac{(1-\lambda)(1-\phi)}{1+\lambda(n-1)}$ if

$$\frac{(1-\lambda)(1-\phi)}{1+\lambda(n-1)}(\lambda+\frac{1-\lambda}{n})(1-q)+\lambda q\left(\frac{(1-\lambda)(1-\phi)}{1+\lambda(n-1)}\right)^{1+\frac{1-\lambda}{n}} > \frac{1-\lambda}{n}(1-q) \iff q > \frac{1}{1+\frac{n\lambda}{n-(1-\lambda)(1-\phi)}\left(\frac{(1-\lambda)(1-\phi)}{1+\lambda(n-1)}\right)^{1+\frac{1-\lambda}{n}}}$$

The result in Proposition F.1 has the unappealing feature that all non-PMG stores charge the same price. This is in contrast to the simultaneous model, where all firms play mixed strategies: PMG stores choose prices from $[\underline{p}, \hat{p}]$ and non-PMG stores choose prices from $[\hat{p}, \bar{p}]$.

Let $\bar{p}_{PMG}$ denote the upper bound on the price distribution of PMG stores. In the simultaneous game, a non-PMG store that charges a price higher than $\bar{p}_{PMG}$ may sell to shoppers because there is a probability that no firm in the market will offer a PMG. Choosing a price higher than $\bar{p}_{PMG}$ leads to a trade-off: it reduces the probability that the firm sells to shoppers, but it increases the surplus that the firm extracts from the remaining consumers.

In contrast, in the sequential game, a non-PMG store that charges a price above $\bar{p}_{PMG}$ never sells to shoppers (as these consumers will purchase from a PMG store that charges a lower price) nor to informed non-shoppers. It only sells to uninformed non-shoppers. Hence, if a non-PMG store is charging a price equal or higher than $\bar{p}_{PMG}$, it is optimal to charge the consumer reservation value.

The result that all non-PMG stores charge the same price follows from the simplicity of the model presented here. It is easy to extend the model in realistic directions, so that all firms play mixed strategies on prices.

To illustrate this, I consider one such extension. Suppose there is another consumer segment, which consists of consumers who are partially informed about prices. The existence of consumers who are informed about a subset of prices is usually assumed in models of advertising, such as Butters (1977), Grossman and Shapiro (1984), and Robert and Stahl (1993). The intuition is that advertisements do not reach the entire population, and hence some consumers will be exposed only to a subset of advertised prices.

When this consumer segment is present, non-PMG firms that charge a price higher than $\bar{p}_{PMG}$ may still sell to partially informed consumers. Choosing a price higher than $\bar{p}_{PMG}$ leads to a trade-off: it reduces the probability that the firm sells to partially informed consumers, but it increases the surplus that the firm extracts from uninformed non-shoppers.

The analysis of the pricing stage in the presence of partially informed consumers is characterized below.

F.2 - Sequential choice of PMG policy and price in presence of partially informed consumers

In this section, I assume the existence of consumers who are informed about a subset of prices. To simplify the analysis, I assume that such consumers are informed about two prices (including the first search that is free, i.e., if a partially informed consumer wants to get a third price quotation, he must incur the search cost of $s > 0$).

Table F.1 depicts the four consumer segments, and their respective sizes.

Consumer Segment	Size
Shoppers	$\lambda(1-\beta)$
Partially informed	$\lambda\beta$
Informed non-shoppers	$(1-\lambda)\phi$
Uninformed non-shoppers	$(1-\lambda)(1-\phi)$

Table F.1

The main result is stated below.

Proposition F.2 *Suppose $2 \leq k \leq n-2$. If the share of partially informed consumers is not too large and q is large enough, then there exists $\underline{p} < \hat{p} < v$ such that in equilibrium PMG stores choose prices from $[\underline{p}, \hat{p}]$ and non-PMG stores choose prices from $[\hat{p}, v]$.*

The proof of Proposition F.2 proceeds in two steps. In the first step, I construct F_{PMG} and F_{NO} that make PMG stores indifferent between all prices in $[\underline{p}, \hat{p}]$ and make non-PMG stores indifferent between all prices in $[\hat{p}, v]$.

In a second step, I show that PMG stores do not want to deviate to prices outside of $[\underline{p}, \hat{p}]$ and non-PMG stores do not want to deviate to prices outside of $[\hat{p}, v]$.

First step

Using an argument similar to Varian (1980), it follows that F_{NO} and F_{PMG} have no mass points. I start by characterizing F_{NO}.

A non-PMG firm that charges a price in $[\hat{p}, v]$ never sells to shoppers, because they will purchase from PMG stores that charge a price lower than $\hat{p}$. A non-PMG store may, however, sell to partially informed consumers. Because it is assumed that such consumers are informed about two random price quotations, a share $\frac{2}{n}$ of partially informed consumers receive a price quotation for a given firm. A non-PMG firm can only sell to such consumers in the event that the other price quotation that they have is from another non-PMG store. It follows that the profit of a non-PMG store that charges a price $x \in [\hat{p}, v]$ is

$$\pi_{NO}(x) = \lambda\beta\frac{2}{n}\frac{n-k-1}{n-1}[1-F_{NO}(x)]x + \frac{(1-\lambda)(1-\phi)}{n}x$$

Because F_{NO} has no mass points, it follows that a non-PMG firms that charges price v never sells to partially informed consumers. Hence, the profit of a non-PMG store that charges price v is

$\frac{(1-\lambda)(1-\phi)}{n}v$. In order for non-PMG stores to be indifferent between any price in $[\hat{p}, v]$, it must be that

$$\pi_{NO}(x) = \frac{(1-\lambda)(1-\phi)}{n}v \iff$$

$$F_{NO}(x) = \frac{2\beta\lambda(n-k-1)+(n-1)(1-\lambda)(1-\phi)}{2\beta\lambda(n-k-1)} - \frac{(n-1)(1-\lambda)(1-\phi)}{2\beta\lambda(n-k-1)}\frac{v}{x} \tag{F.9}$$

Because $F_{NO}(\hat{p}) = 0$, it follows from (F.9) that $\hat{p} = \frac{(n-1)(1-\lambda)(1-\phi)}{2\beta\lambda(n-k-1)+(n-1)(1-\lambda)(1-\phi)}v$

I now characterize F_{PMG}. A PMG firm that charges a price $x \in [\underline{p}, \hat{p}]$ sells to shoppers in case all other PMG firms charge a price higher than x. It also sells to partially informed consumers when one of the following conditions is met: i) the partially informed consumers that get a price quotation from the store get the other price quotation from a non-PMG store; ii) the partially informed consumers that get a price quotation from the store get the other price quotation from a PMG store that charges a price higher than x. It follows that the profit of a PMG store that charges price $x \in [\underline{p}, \hat{p}]$ is

$$\pi_{PMG}(x) = \lambda(1-\beta)[1-F_{PMG}(x)]^{k-1}x + \lambda\beta\frac{2}{n}\left[\frac{n-k}{n-1} + \frac{k-1}{n-1}[1-F_{PMG}(x)]\right]x + (1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)\left[qEmin(x, F_{PMG}) + (1-q)x\right]$$

Because PMG firms are indifferent between any price $x \in [\underline{p}, \hat{p}]$, it follows that $\frac{\partial \pi_{PMG}(x)}{\partial x} = 0$. This condition characterizes F_{PMG}.

Second step

I start by showing that PMG stores do not want to deviate to prices outside of $[\underline{p}, \hat{p}]$. It is clear that they do not want to deviate to prices lower than $\underline{p}$ (that would reduce the price without any increase in demand) nor to prices higher than v (in which case they do not sell any product). For $x \in [\hat{p}, v]$

$$\pi_{PMG}(x) = \lambda\beta\frac{2}{n}\frac{n-k}{n-1}[1-F_{NO}(x)]x + (1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)\left[qEmin(x, F_{PMG}) + (1-q)x\right]$$

It then follows that

$$\pi_{PMG}(x) - \pi_{PMG}(\hat{p}) = \lambda\beta\frac{2}{n}\frac{n-k}{n-1}\left[[1-F_{NO}(x)]x - \hat{p}\right] + (1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)(1-q)(x-\hat{p})$$

Moreover,

$$\pi_{NO}(x) - \pi_{NO}(\hat{p}) = \lambda\beta\frac{2}{n}\frac{n-k-1}{n-1}\left[[1-F_{NO}(x)]x - \hat{p}\right] + \frac{(1-\lambda)(1-\phi)}{n}(x-\hat{p})$$

Because $\pi_{NO}(x) - \pi_{NO}(\hat{p}) = 0$ for all $x \in [\hat{p}, v]$, a sufficient condition for PMG stores to prefer to charge $\hat{p}$ over any $x \in [\hat{p}, v]$ is that $\pi_{NO}(x) - \pi_{NO}(\hat{p}) \geq \pi_{PMG}(x) - \pi_{PMG}(\hat{p})$.

Because $[1-F_{NO}(x)]x - \hat{p} < 0$ - as it can be seen from (F.9) -, a sufficient condition for $\pi_{NO}(x) - \pi_{NO}(\hat{p}) \geq \pi_{PMG}(x) - \pi_{PMG}(\hat{p})$ is that $(1-\lambda)\left(\frac{\phi}{k} + \frac{1-\phi}{n}\right)(1-q)(x-\hat{p}) \leq \frac{(1-\lambda)(1-\phi)}{n}(x-\hat{p}) \iff q \geq 1 - \frac{1}{1+\frac{n}{k}\frac{\phi}{1-\phi}}$.

Because the LHS of the above expression is decreasing in k, and because we are assuming that

$k \geq 2$, a sufficient condition that holds for all k can be obtained replacing $k = 2$. Hence, we require that

$$q > 1 - \frac{1}{1 + \frac{n}{2}\frac{\phi}{1-\phi}} \tag{F.10}$$

We thus conclude that, under Equation F.10, PMG stores do not want to deviate.

In order to see that non-PMG stores also do not want to deviate on prices, notice that as β approaches zero, the equilibrium converges to the one described in the case $1 < k < n$ of Web Appendix F.1. In particular it is easy to see that $\lim_{\beta \to 0} \hat{p} = v$. In that section, it was shown that for q large enough, $\pi_{NO}(v) > \pi_{NO}(x)$ for all $x < v$. It then follows from continuity that as long as β is small enough, there exists $\bar{q}$ such that if $q > \bar{q}$ then $\pi_{NO}(\hat{p}) > \pi_{NO}(x)$ for all $x < \hat{p}$.

F.3. - Endogenous Decision to Learn Firms' PMG Policies

The literature that explains PMGs as a signal device assumes that all consumers are informed about firms' PMG policies (Jain and Srivastava 2000, Moorthy and Winter 2006, Moorthy and Zhang 2006). This seems too strong an assumption. Even though firms may advertise their PMG policies, consumers who which to learn them still need to incur search costs. Consumer heterogeneity may lead only some of them to choose to become informed. For example, consumers who purchase the product frequently may find it beneficial to become informed, but it may not be worthwhile for occasional buyers to learn which firms offer PMGs.

In the model presented in this paper, I allow for the coexistence of consumers who are informed and uninformed regarding firms' PMG policies. For simplicity, I assume that the fraction of informed consumers is exogenous. However, the decision to become informed about firms' PMG policies can be modeled in a similar way that Varian (1980) models the decision to become informed about prices.

Suppose consumers are heterogeneous in their cost of becoming informed. Let P_i denote the expected price paid by a consumer who is informed about which firms offer PMGs, and let P_u denote the average price paid by consumers who are uninformed about firms' PMG policies. A consumer who becomes informed about firms' PMG policies saves, on average, $P_u - P_i$. Consumers whose cost of becoming informed is lower than $P_u - P_i$ will choose to learn which firms offer PMGs and will go directly to one of those stores to purchase the product. Consumers whose cost of becoming informed is higher than $P_u - P_i$ will prefer to remain uninformed regarding firms' PMG policies and will visit a firm at random.

F.4. - Price Change During the Post-purchase Search Period

I analyze a dynamic model under an assumption that makes it more tractable: when the post-purchase search period is infinite. In this setting, a consumer who searches post-purchase observes an arbitrarily large number of prices. In a mixed-strategy equilibrium where firms choose prices from a distribution with support $[\underline{p}, \bar{p}]$, a consumer who searches post-purchase will find a price arbitrarily close to $\underline{p}$ with probability one. Hence, there are no longer incentives for PMG firms not to lower their prices in subsequent periods because, regardless of the firm's prices in subsequent periods, consumers who search post-purchase always find a price quotation arbitrarily close to $\underline{p}$. The main result is stated below.

Proposition F.3 *Suppose the post-purchase search period is infinite. In equilibrium, firms play mixed strategies over prices on a set $[\underline{p}, \bar{p}]$. There exists a price threshold, $\hat{p}$, such that firms offer a PMG only when they choose a price below $\hat{p}$. If $\phi \in (0,1)$ and $q > \frac{\phi n\lambda + (1-\lambda)\phi}{n\lambda + (1-\lambda)\phi}$ then $\underline{p} < \hat{p} < \bar{p}$.*

By a similar argument as in the proof of Proposition 1, it can be shown that an equilibrium exists. Moreover, by the same reasoning as in the proof of Proposition 1, it is straightforward that a price threshold $\hat{p}$ exists and if $\phi > 0$ then $\hat{p} > \underline{p}$. So in order for an equilibrium with the property that $\underline{p} < \hat{p} < \bar{p}$ to exist, it is sufficient to show that no equilibrium where all firms offer PMGs (i.e $\hat{p} \geq \bar{p}$) exists.

Suppose there is an equilibrium under which all firms offer a PMG. By a similar argument as in Varian (1980), it follows that in a symmetric equilibrium, the price distribution is atomless. It then follows that $\pi_{PMG}(\underline{p}) = \left[\lambda + \frac{1-\lambda}{n}\right](\underline{p} - c)$ and $\pi_{PMG}(\bar{p}) = \frac{1-\lambda}{n}[q\underline{p} + (1-q)\bar{p} - c]$.

Because $\pi_{PMG}(\underline{p}) = \pi_{PMG}(\bar{p})$ it follows that $\bar{p} - c = \frac{\lambda n + (1-\lambda)(1-q)}{(1-\lambda)(1-q)}(\underline{p} - c)$.

In order for an equilibrium where all firms offer PMGs not to exist, it is sufficient that a firm prefers charging $\bar{p}$ and not offering a PMG over charging $\bar{p}$ and offering a PMG. A firm that charges $\bar{p}$ and does not offer a PMG makes profit $\frac{(1-\lambda)(1-\phi)}{n}(\bar{p} - c)$. Hence, a sufficient condition for the equilibrium to feature $\hat{p} < \bar{p}$ is

$$\pi_{NO}(\bar{p}) > \pi_{PMG}(\bar{p}) \iff \frac{(1-\lambda)(1-\phi)}{n}(\bar{p} - c) > \frac{1-\lambda}{n}[q\underline{p} + (1-q)\bar{p} - c] \iff q > \frac{\phi n\lambda + (1-\lambda)\phi}{n\lambda + (1-\lambda)\phi}$$

Web Appendix G - PMGs documented in the text

Figure G.1

Our promise to guests

Price Match Policy

Price Match Guarantee

We'll match the price if you buy a qualifying item at Target then find the identical item for less at Target.com, select online competitors, or in Target's or competitor's local print ad. Price matches may be requested at time of purchase or within 14 days after purchase. The full list of online competitors is available online or at Guest Services.

If you find a current lower price within 14 days after purchase, just bring in the proof and we will adjust your payment to the lower price, upon request. Target store price matches and adjustments are completed at the store on any lane. For Target.com purchases, call Target.com Guest Services at 1-800-591-3869.

Figure G.2

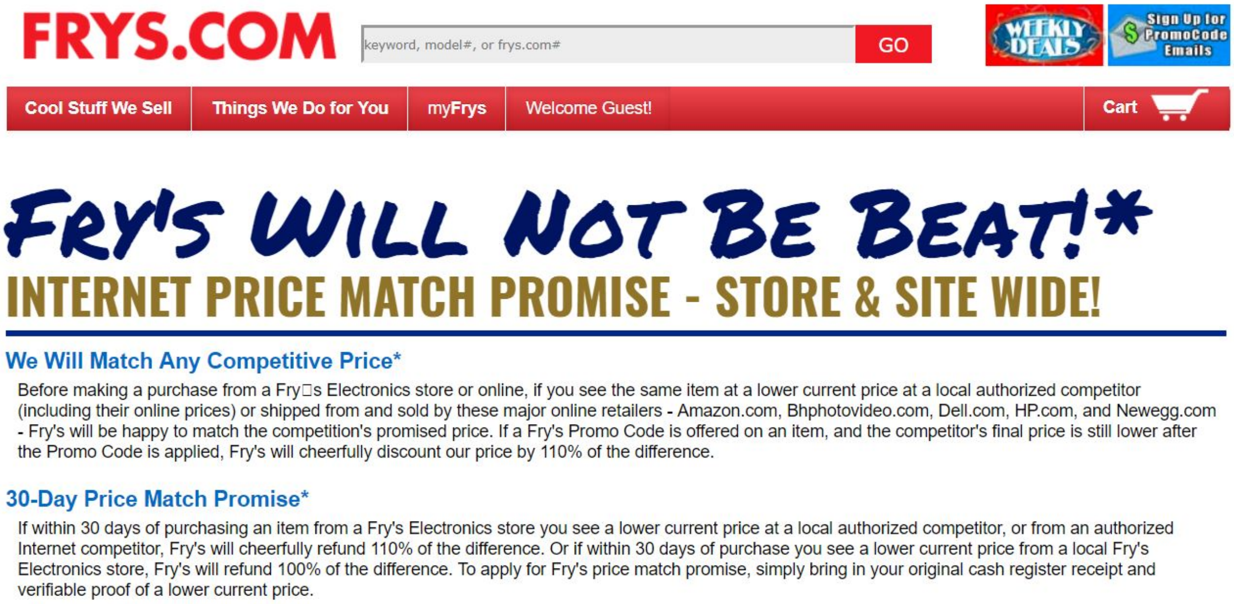

Figure G.3

Figure G.4


Buy any set of 4 tires and get...
FREE
Alignment check
FREE
Tire disposal, valve stems, wheel weights, and tire rotations for life
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Figure G.5

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