APPENDIX

Proof of Proposition 1

Firm's *i* objective function after the expression of the demand function has been replaced reads:

$$\pi_i = \frac{1}{2t}\begin{bmatrix} -2t(O_i + cD_i) + r(n(\sqrt{O_i} - \sqrt{O_j} + \sqrt{D_i} - \sqrt{D_j} + t)(\sqrt{D_i} - \delta\sqrt{O_j}) + \\ n\sqrt{O_i}(\sqrt{O_j} - \sqrt{O_i} - \sqrt{D_i} + \sqrt{D_j} + t)\theta) \end{bmatrix},\ i, j = 1,2,\ i \neq j.$$

The derivative with respect to O_i and D_i are:

$$\frac{1}{4t\sqrt{O_i}}(-2(2t + nr\theta)\sqrt{O_i} + nr((1-\theta)\sqrt{D_i} + \theta(\sqrt{D_j} + t) + (\theta - \delta)\sqrt{O_j})), \tag{11}$$

$$\frac{1}{4t\sqrt{D_i}}(2(2nr - 2ct)\sqrt{D_i} - nr(\sqrt{D_j} - t + \sqrt{O_j}(1+\delta) - (1-\theta)\sqrt{O_i})), \quad i, j=1, 2, i\neq j. \tag{12}$$

Equating the above expressions to zero, we get the first-order optimality condition for an interior equilibrium and solving for a symmetric solution (1) and (2) can be easily obtained. Replacing these last expressions in the demand and the firm's profits (3) and (4) are derived.

The second-order concavity conditions ensuring an interior maximum for the symmetric solution (O, D) read:

$$\sqrt{D} + t\theta + (\theta - \delta)\sqrt{O} > 0,$$

$$D - ((\theta - \delta)\sqrt{O} - t\theta)((\theta + \delta)\sqrt{O} - t) + (t(1-\theta) + (1+\theta^2)(\sqrt{O})\sqrt{D} < 0.$$

Proof of Proposition 2

Replacing $O_i = 0$ in (12) and equating to zero, D^{EDM} in (5) is obtained. D^{EDM} is positive if and only if $t > nr/(4c)$.

The optimal demand and profits in (6) and (7) can be easily obtained by substituting the expression of D^{EDM}. Therefore, if $t > nr/(2c)$, then D^{EDM}, q^{EDM} are positive and π^{EDM} is greater or equal to zero. Note that $nr/(2c)$ increases with n and r, and decreases with c.

The condition on the total demand $q_1+q_2 \leq n$, taking into account q^{EDM} reads: $(4c-nr)t \geq nr$. This last condition is only feasible if $4c-nr>0$, and if this is the case, it can be rewritten as: $t \geq nr/(4c-nr)$. Note that $nr/(4c-nr)$ increases with n and r, and decreases with c.

It can be easily checked that $nr/(4c-nr) \geq nr/(2c)$ if and only if $nr \leq 2c$. Therefore, the corner solution (O=0, D^{EDM}) requires condition ($nr \leq 2c$ and $t \geq nr/(2c)$) or condition ($2c<nr<4c$ and $t \geq nr/(4c-nr)$) in order to be feasible.

Proof of Proposition 3

Replacing $D_i = 0$ in (11) and equating to zero, O^{EOM} in (8) is obtained. The optimal demand and profits in (9) and (10) can be easily obtained substituting the expression of O^{EOM}.

If δ=θ, replacing in (9) and (10) one gets that the optimal demand is null and the optimal profits are negative. Therefore, this corner solution (O^{EOM}, D=0) is unfeasible when δ=θ.

From (9) and (10) the following conditions can be easily derived:

$$q^{EOM} > 0 \quad \text{if and only if} \quad \theta > \delta,$$

$$\pi^{EOM} \geq 0 \quad \text{if and only if} \quad 2t(\theta - 2\delta) + nr(\theta^2 - \delta^2) \geq 0.$$

Both conditions are satisfied simultaneously in the following two cases:

$$\theta > 2\delta$$

or

$$\delta < \theta < 2\delta \quad \text{and} \quad t \leq \frac{nr(\theta-\delta)(\theta+\delta)}{2(2\delta-\theta)} \tag{13}$$

The bound $\frac{nr(\theta-\delta)(\theta+\delta)}{2(2\delta-\theta)}$ increases with n, r and θ.

The condition on the total demand $q_1+q_2\leq n$, taking into account q^{EOM} reads: $(nr\theta(\theta-\delta)-4)t\leq nr(\theta+\delta)$. This last condition is fulfilled if one of the following two conditions is satisfied:

$$nr\theta(\theta-\delta)-4<0,$$

or

$$nr\theta(\theta-\delta)-4<0 \text{ and } t\leq\frac{nr(\delta+\theta)}{nr\theta(\theta-\delta)-4}. \tag{14}$$

$\frac{nr(\delta+\theta)}{nr\theta(\theta-\delta)-4}$ decreases with n, r and θ, and could decrease or increase with δ.

Furthermore,

$$\frac{nr(\theta-\delta)(\delta+\theta)}{2(2\delta-\theta)}>\frac{nr(\delta+\theta)}{nr\theta(\theta-\delta)-4} \text{ if and only if } nr<\frac{2}{(\theta-\delta)^2}. \tag{15}$$

Mixing conditions (13) and (14) taking into account (15), five possibilities as described below characterize the feasibility of equilibrium (O^{EOM}, D=0). The two firms exclusively undertake offensive marketing at the equilibrium if one of the following conditions is satisfied:

- $\theta\geq 2\delta$ and $nr\leq\frac{4}{\theta(\theta-\delta)}$.
- $\theta\geq 2\delta$, $nr>\frac{4}{\theta(\theta-\delta)}$ and $t\leq\frac{nr(\delta+\theta)}{nr\theta(\theta-\delta)-4}$.
- $\delta<\theta<2\delta$, $nr\leq\frac{4}{\theta(\theta-\delta)}$ and $t\leq\frac{nr(\theta-\delta)(\delta+\theta)}{2(2\delta-\theta)}$.
- $\delta<\theta<2\delta$, $\frac{4}{\theta(\theta-\delta)}<nr\leq\frac{2}{(\theta-\delta)^2}$ and $t\leq\frac{nr(\theta-\delta)(\delta+\theta)}{2(2\delta-\theta)}$,
- $\delta<\theta<2\delta$, $nr>\frac{2}{(\theta-\delta)^2}$ and $t\leq\frac{nr(\delta+\theta)}{nr\theta(\theta-\delta)-4}$. (16)