Appendix A

The G-DINA Model

In CDMs, a **Q**-matrix that defines the relationship between items and attributes should be specified by content and domain experts. Let q_{jk} denote the entry in the **Q**-matrix for the indication of item *j* ($j = 1, \ldots, J$) loading on attribute *k* ($k = 1, \ldots, K$), and $q_{jk} = 1$ represents the *k*th attribute that is required to correctly answer the *j*th item, whereas $q_{jk} = 0$ otherwise. For convenient notation, let K_j^* denote the number of required attributes for item *j*, let $\boldsymbol{\alpha}_{lj}^* = (\alpha_{lj1}, \ldots \alpha_{ljK_j^*})$ be the reduced attribute vector for the latent class *l* and item *j*, and let $P(\boldsymbol{\alpha}_{lj(i)}^*)$ be the probability of a correct response to item *j* when examinee *i* has the attribute pattern of $\boldsymbol{\alpha}_{lj}^*$. The G-DINA model can be formulated as follows:

$$P\left(\boldsymbol{\alpha}_{lj(i)}^*\right) \equiv P\left(X_{ij} = 1 \mid \boldsymbol{\alpha}_{lj(i)}^*\right) = \delta_{j0} + \sum_{k=1}^{K_j^*} \delta_{jk}\alpha_{lk} + \sum_{k'=k+1}^{K_j^*} \sum_{k=1}^{K_j^*-1} \delta_{jkk'}\alpha_{lk}\alpha_{lk'} \ldots + \delta_{j12\ldots K_j^*} \prod_{k=1}^{K_j^*} \alpha_{lk}, \quad \text{(A1)}$$

where $X_{ij} = 1$ indicates a correct response to item *j* by examinee *i* and δ_{jo}, δ_{jk}, $\delta_{jkk'}$, and $\delta_{j12\ldots K_j^*}$ are the intercept term, main effect, first-order interaction effect, and K_j^*th-order interaction effect of item *j*, respectively.

Suppose five attributes are measured in a cognitive diagnostic test; each examinee can be assigned to one of 32 possible latent classes or attribute profiles. If item *j* measures the first three attributes, in the G-DINA model framework, the reduced attribute vector is denoted $\boldsymbol{\alpha}_{lj}^* = (\alpha_{lj1}, \alpha_{lj2}, \alpha_{lj3})$ and the number of latent classes can be reduced to eight, that is, $l = 1, 2, \ldots, 8$. When an examinee has mastered α_{lj1} and α_{lj3}, for example, the probability of a correct response to item *j* for him or her is $\delta_{j0} + \delta_{j1} + \delta_{j3} + \delta_{j13}$. Therefore, the eight latent classes identified by attribute profiles can be allowed to have different successful probabilities on the item response.

Various CDMs can be derived from the saturated G-DINA model (i.e., with $2^{K_j^*}$ parameters) by constraining certain parameters of this probability function to be equal to zero. For example, if only the intercept term and the highest-order effect are estimated, then the saturated G-DINA model reduces to the DINA model (Haertel, 1989; Junker & Sijtsma, 2001), and if only the main effects are retained together with the intercept term, the additive CDM arises. Readers who are interested in the variety of CDMs derived from the G-DINA model are referred to the original study by de la Torre (2011).

Appendix B

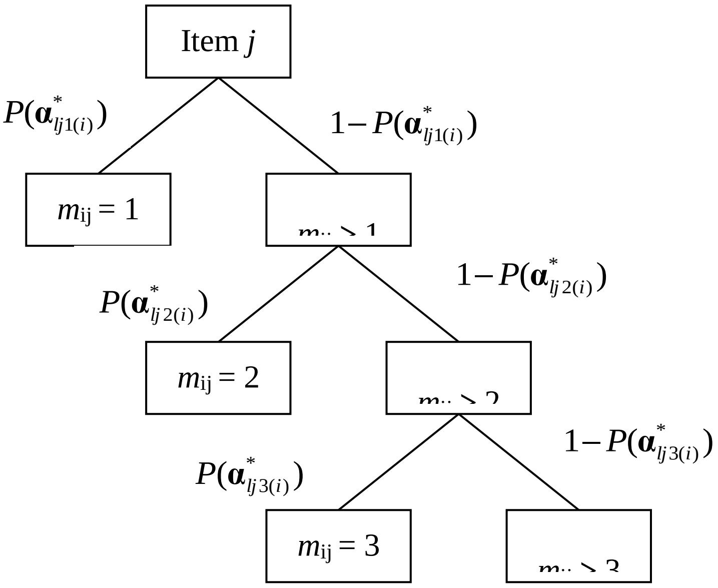

Figure B1. Tree diagram of a sequential CDM when the maximum number of attempts is three

Appendix C

Specification of **Q**-Matrix in this Study

Table C1. Q-Matrix for the Simulation Study

Item	Attribute 1	Attribute 2	Attribute 3	Attribute 4
1	1	0	0	0
2	1	0	0	0
3	1	1	0	0
4	1	1	0	0
5	1	1	0	0
6	1	1	0	0
7	1	1	0	0
8	1	1	0	0
9	0	1	0	0
10	0	1	0	1
11	1	1	1	1
12	1	1	1	1
13	1	1	1	1
14	1	1	1	1
15	1	1	1	1
16	0	0	1	0
17	1	1	1	1
18	0	1	0	1
19	1	0	0	1
20	1	0	0	0
21	1	0	0	0
22	1	0	0	0
23	1	0	1	0

Table C2. **Q**-Matrix for the Mathematical Test in the Empirical Data Analysis

Item	Attribute 1	Attribute 2	Attribute 3	Attribute 4
1	1	1	0	0
2	1	1	0	0
3	1	0	1	0
4	1	1	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	0	1