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# Online Supplement for "shape invariant mixture model for clustering non-linear longitudinal growth trajectories"

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**Table E1.** Comparison of candidates SITAR mixture models for pregnant women dataset

| Number of clusters | Constrained variance |         |         | Unconstrained variance |                |                |
|--------------------|----------------------|---------|---------|------------------------|----------------|----------------|
|                    | $\bar{D}$            | $p_D$   | DIC     | $\bar{D}$              | $p_D$          | DIC            |
| 1                  | 1225.13              | 12.632  | 1237.76 | -                      | -              | -              |
| 2                  | 1230.39              | 90.1415 | 1320.53 | <b>1087.9</b>          | <b>51.2841</b> | <b>1139.19</b> |
| 3                  | 1244.98              | 36.7511 | 1281.73 | 1194.27                | 44.6451        | 1238.91        |

**Table E2.** Clustering performance of Mixture SITAR using different number of knots and knots locations

| Number of Knots | Equally spaced of<br>the sample quantiles |       | Equally time spaced |       |
|-----------------|-------------------------------------------|-------|---------------------|-------|
|                 | cRate                                     | aRand | cRate               | aRand |
| 2               | 0.83                                      | 0.32  | 0.83                | 0.32  |
| 3               | 0.86                                      | 0.45  | 0.88                | 0.49  |
| 4               | 0.88                                      | 0.50  | 0.88                | 0.50  |
| 5               | 0.88                                      | 0.50  | 0.86                | 0.46  |
| 6               | 0.88                                      | 0.52  | 0.88                | 0.52  |
| 7               | 0.88                                      | 0.52  | 0.88                | 0.52  |
| 8               | 0.88                                      | 0.52  | 0.88                | 0.52  |

cRate: correct classification rate

aRand: adjusted rand index

**Table E3.** Parameter settings in Simulation I

|                | Scenario 1 |         | Scenario 2 |         | Scenario 3 |         |
|----------------|------------|---------|------------|---------|------------|---------|
|                | $k = 1$    | $k = 2$ | $k = 1$    | $k = 2$ | $k = 1$    | $k = 2$ |
| $\mu_{k1}$     | -1         | 3       | 0          | 4       | 1          | 2       |
| $\mu_{k2}$     | -1         | -3      | 0          | -5      | 0          | -5      |
| $\mu_{k3}$     | -0.05      | -0.5    | -0.05      | -0.2    | 0          | -0.2    |
| $\Sigma_{k11}$ | 0.0114     | 0.06    | 0.0114     | 0.06    | 0.0114     | 0.06    |
| $\Sigma_{k12}$ | 0.0119     | -0.081  | 0.0119     | -0.081  | 0.0119     | -0.081  |
| $\Sigma_{k13}$ | 0.0054     | -0.027  | 0.0054     | -0.027  | 0.0054     | -0.027  |
| $\Sigma_{k22}$ | 0.0137     | 0.1301  | 0.0137     | 0.1301  | 0.0137     | 0.1301  |
| $\Sigma_{k23}$ | 0.0067     | 0.0245  | 0.0067     | 0.0245  | 0.0067     | 0.0245  |
| $\Sigma_{k33}$ | 0.0035     | 0.0326  | 0.0035     | 0.0326  | 0.0035     | 0.0326  |
| $\gamma_{k1}$  | 9.8        | 4.2     | 9.8        | 7       | 9.8        | 7       |
| $\gamma_{k2}$  | 10.5       | 5.6     | 10.5       | 7       | 10.5       | 7       |
| $\gamma_{k3}$  | 7.7        | 7       | 7.7        | 5.6     | 7.7        | 5.6     |
| $\gamma_{k4}$  | 17.5       | 5.6     | 17.5       | 2.1     | 17.5       | 2.1     |
| $\gamma_{k5}$  | 6.3        | 4.2     | 6.3        | 1.4     | 6.3        | 1.4     |
| $\sigma_k^2$   | 1.5        | 1.5     | 1.5        | 1.5     | 1.5        | 1.5     |

$\Sigma_{kij}$  indicates the  $i^{th}$  row and  $j^{th}$  column of  $\Sigma_k$

**Table E4.** Percentage of correctly selecting the number of clusters using DIC in different scenarios of Simulation I

| Sample size | max.obs | Proportion | Scenario 1<br>(Low separation) | Scenario 2<br>(Median separation) | Scenario 3<br>(High separation) |
|-------------|---------|------------|--------------------------------|-----------------------------------|---------------------------------|
| 100         | 4       | (0.5, 0.5) | 0                              | 0                                 | 93.3                            |
|             |         | (0.7, 0.3) | 0                              | 0                                 | 93.3                            |
|             | 8       | (0.5, 0.5) | 0                              | 16.7                              | 100                             |
|             |         | (0.7, 0.3) | 3.3                            | 13.3                              | 100                             |
|             | 12      | (0.5, 0.5) | 46.7                           | 60                                | 100                             |
|             |         | (0.7, 0.3) | 16.7                           | 53.3                              | 100                             |
| 200         | 4       | (0.5, 0.5) | 0                              | 0                                 | 100                             |
|             |         | (0.7, 0.3) | 0                              | 0                                 | 100                             |
|             | 8       | (0.5, 0.5) | 0                              | 3.3                               | 100                             |
|             |         | (0.7, 0.3) | 0                              | 10                                | 96.7                            |
|             | 12      | (0.5, 0.5) | 20                             | 76.7                              | 100                             |
|             |         | (0.7, 0.3) | 0                              | 86.7                              | 96.7                            |

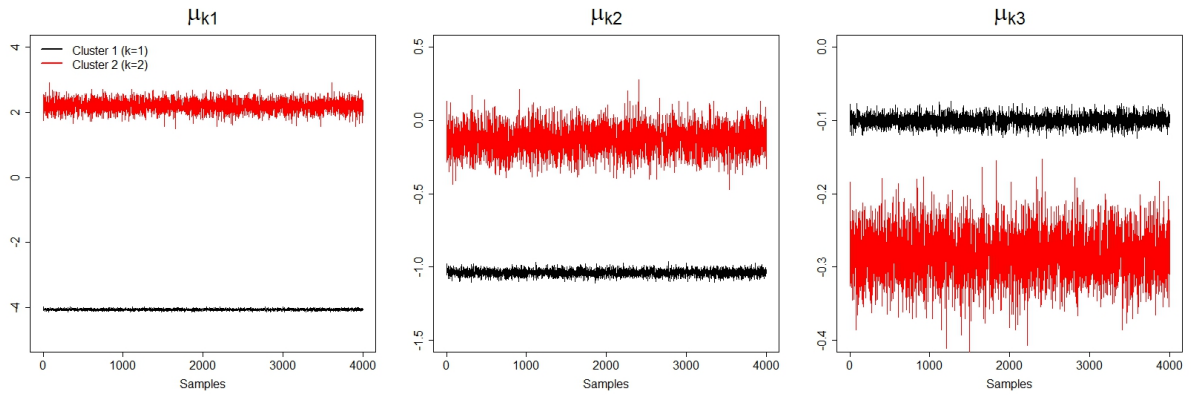
**Table E5.** Percentage of correctly selecting the number of clusters using R & M criteria in different scenarios of Simulation I

|             |         |            | Scenario 1 (Low separation)    |                 |                  |                 |                  |                 |
|-------------|---------|------------|--------------------------------|-----------------|------------------|-----------------|------------------|-----------------|
|             |         |            | $\delta = 1$                   |                 | $\delta = 2$     |                 | $\delta = 4$     |                 |
| Sample size | max.obs | Proportion | $\varphi = 0.05$               | $\varphi = 0.1$ | $\varphi = 0.05$ | $\varphi = 0.1$ | $\varphi = 0.05$ | $\varphi = 0.1$ |
| 100         | 4       | (0.5, 0.5) | 86.7                           | 96.7            | 16.7             | 20.0            | 6.7              | 10.0            |
|             |         | (0.7, 0.3) | 0.0                            | 0.0             | 16.7             | 10.0            | 20.0             | 26.7            |
|             | 8       | (0.5, 0.5) | 100.0                          | 100.0           | 56.7             | 70.0            | 60.0             | 63.3            |
|             |         | (0.7, 0.3) | 70.0                           | 70.0            | 60.0             | 70.0            | 66.7             | 86.7            |
|             | 12      | (0.5, 0.5) | 96.7                           | 100.0           | 93.3             | 100.0           | 46.7             | 66.7            |
|             |         | (0.7, 0.3) | 100.0                          | 86.7            | 96.7             | 83.3            | 43.3             | 66.7            |
| 200         | 4       | (0.5, 0.5) | 100                            | 100             | 100              | 100             | 100              | 60              |
|             |         | (0.7, 0.3) | 30.0                           | 0.0             | 40.0             | 0.0             | 46.7             | 63.3            |
|             | 8       | (0.5, 0.5) | 73.3                           | 80.0            | 46.7             | 63.3            | 40.0             | 56.7            |
|             |         | (0.7, 0.3) | 60.0                           | 80.0            | 80.0             | 90.0            | 56.7             | 76.7            |
|             | 12      | (0.5, 0.5) | 100.0                          | 100.0           | 90.0             | 100.0           | 90.0             | 93.3            |
|             |         | (0.7, 0.3) | 100.0                          | 100.0           | 100.0            | 96.7            | 93.3             | 100.0           |
|             |         |            | Scenario 2 (Median separation) |                 |                  |                 |                  |                 |
|             |         |            | $\delta = 1$                   |                 | $\delta = 2$     |                 | $\delta = 4$     |                 |
| Sample size | max.obs | Proportion | $\varphi = 0.05$               | $\varphi = 0.1$ | $\varphi = 0.05$ | $\varphi = 0.1$ | $\varphi = 0.05$ | $\varphi = 0.1$ |
| 100         | 4       | (0.5, 0.5) | 33.3                           | 23.3            | 30.0             | 20.0            | 33.3             | 20.0            |
|             |         | (0.7, 0.3) | 30.0                           | 23.3            | 16.7             | 10.0            | 30.0             | 23.3            |
|             | 8       | (0.5, 0.5) | 66.7                           | 60.0            | 43.3             | 56.7            | 43.3             | 73.3            |
|             |         | (0.7, 0.3) | 90.0                           | 63.3            | 56.7             | 70.0            | 33.3             | 63.3            |
|             | 12      | (0.5, 0.5) | 20.0                           | 56.7            | 3.3              | 43.3            | 16.7             | 46.7            |
|             |         | (0.7, 0.3) | 40.0                           | 86.7            | 36.7             | 50.0            | 23.3             | 46.7            |
| 200         | 4       | (0.5, 0.5) | 100.00                         | 96.67           | 100.00           | 100.00          | 83.33            | 90.00           |
|             |         | (0.7, 0.3) | 86.67                          | 0.00            | 93.33            | 3.33            | 96.67            | 13.33           |
|             | 8       | (0.5, 0.5) | 26.67                          | 73.33           | 26.67            | 60.00           | 10.00            | 33.33           |
|             |         | (0.7, 0.3) | 76.67                          | 83.33           | 63.33            | 80.00           | 60.00            | 80.00           |
|             | 12      | (0.5, 0.5) | 0.00                           | 36.67           | 0.00             | 26.67           | 0.00             | 30.00           |
|             |         | (0.7, 0.3) | 26.67                          | 83.33           | 30.00            | 80.00           | 26.67            | 80.00           |
|             |         |            | Scenario 3 (High separation)   |                 |                  |                 |                  |                 |
|             |         |            | $\delta = 1$                   |                 | $\delta = 2$     |                 | $\delta = 4$     |                 |
| Sample size | max.obs | Proportion | $\varphi = 0.05$               | $\varphi = 0.1$ | $\varphi = 0.05$ | $\varphi = 0.1$ | $\varphi = 0.05$ | $\varphi = 0.1$ |
| 100         | 4       | (0.5, 0.5) | 96.7                           | 100.0           | 100.0            | 20.0            | 23.3             | 30.0            |
|             |         | (0.7, 0.3) | 100.0                          | 100.0           | 46.7             | 46.7            | 23.3             | 23.3            |
|             | 8       | (0.5, 0.5) | 63.3                           | 80.0            | 36.7             | 16.7            | 6.7              | 6.7             |
|             |         | (0.7, 0.3) | 76.7                           | 93.3            | 13.3             | 33.3            | 3.3              | 10.0            |
|             | 12      | (0.5, 0.5) | 23.3                           | 3.3             | 0.0              | 0.0             | 0.0              | 0.0             |
|             |         | (0.7, 0.3) | 0.0                            | 16.7            | 6.7              | 16.7            | 6.7              | 23.3            |
| 200         | 4       | (0.5, 0.5) | 96.7                           | 100.0           | 93.3             | 100.0           | 6.7              | 10.0            |
|             |         | (0.7, 0.3) | 100.0                          | 100.0           | 96.7             | 63.3            | 30.0             | 43.3            |
|             | 8       | (0.5, 0.5) | 33.3                           | 80.0            | 26.7             | 60.0            | 13.3             | 43.3            |
|             |         | (0.7, 0.3) | 56.7                           | 90.0            | 40.0             | 80.0            | 36.7             | 83.3            |
|             | 12      | (0.5, 0.5) | 0.0                            | 23.3            | 3.3              | 20.0            | 0.0              | 23.3            |
|             |         | (0.7, 0.3) | 10.0                           | 50.0            | 20.0             | 56.7            | 10.0             | 56.7            |

**Table E6.** Parameter setting in Simulation II

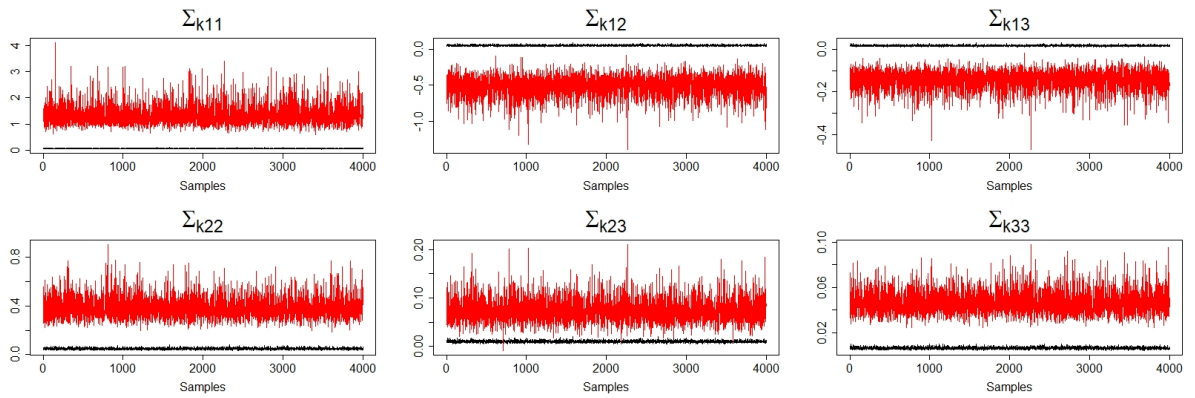
|                | Scenario 1 (Two clusters) |         | Scenario 2 (Three clusters) |         |         | Scenario 3 (Four clusters) |         |         |         |
|----------------|---------------------------|---------|-----------------------------|---------|---------|----------------------------|---------|---------|---------|
|                | $k = 1$                   | $k = 2$ | $k = 1$                     | $k = 2$ | $k = 3$ | $k = 1$                    | $k = 2$ | $k = 3$ | $k = 4$ |
| $\mu_{k1}$     | -2                        | 3       | -2                          | 3       | 1       | -2                         | 3       | 1       | 6       |
| $\mu_{k2}$     | -1                        | 0       | -1                          | 0       | 0.8     | -1                         | 0       | 0.8     | 0.2     |
| $\mu_{k3}$     | -0.2                      | 0.5     | -0.2                        | 0.5     | -0.5    | -0.2                       | 0.5     | -0.5    | -0.1    |
| $\Sigma_{k11}$ | 0.0525                    | 0.81    | 0.0525                      | 0.81    | 0.65    | 0.0525                     | 0.81    | 0.65    | 0.17    |
| $\Sigma_{k12}$ | 0.0425                    | -0.521  | 0.0425                      | -0.521  | -0.001  | 0.0425                     | -0.521  | -0.001  | 0.05    |
| $\Sigma_{k13}$ | 0.016                     | -0.099  | 0.016                       | -0.099  | -0.085  | 0.016                      | -0.099  | -0.085  | -0.005  |
| $\Sigma_{k22}$ | 0.0525                    | 0.4101  | 0.0525                      | 0.4101  | 0.0901  | 0.0525                     | 0.4101  | 0.0901  | 0.0225  |
| $\Sigma_{k23}$ | 0.016                     | 0.0465  | 0.016                       | 0.0465  | 0.0035  | 0.016                      | 0.0465  | 0.0035  | -0.006  |
| $\Sigma_{k33}$ | 0.0054                    | 0.0326  | 0.0054                      | 0.0326  | 0.0126  | 0.0054                     | 0.0326  | 0.0126  | 0.0029  |
| $\gamma_{k1}$  | 9                         | 18      | 9                           | 18      | 6       | 9                          | 18      | 6       | 10      |
| $\gamma_{k2}$  | 8                         | 8       | 8                           | 8       | 8       | 8                          | 8       | 8       | 4       |
| $\gamma_{k3}$  | -2                        | -5      | -2                          | -5      | -2      | -2                         | -5      | -2      | 4       |
| $\gamma_{k4}$  | 12                        | 8       | 12                          | 8       | 8       | 12                         | 8       | 8       | 10      |
| $\gamma_{k5}$  | 8                         | 10      | 8                           | 10      | -8      | 8                          | 10      | -8      | -1      |
| $\gamma_{k6}$  | 15                        | 8       | 15                          | 8       | 0       | 15                         | 8       | 0       | 2       |
| $\gamma_{k7}$  | -8                        | 10      | -8                          | 10      | 8       | -8                         | 10      | 8       | 2       |
| $\sigma_k^2$   | 1.5                       | 1.5     | 1.5                         | 1.5     | 1.5     | 1.5                        | 1.5     | 1.5     | 1.5     |

$\Sigma_{kij}$  indicates the  $i^{th}$  row and  $j^{th}$  column of  $\Sigma_k$

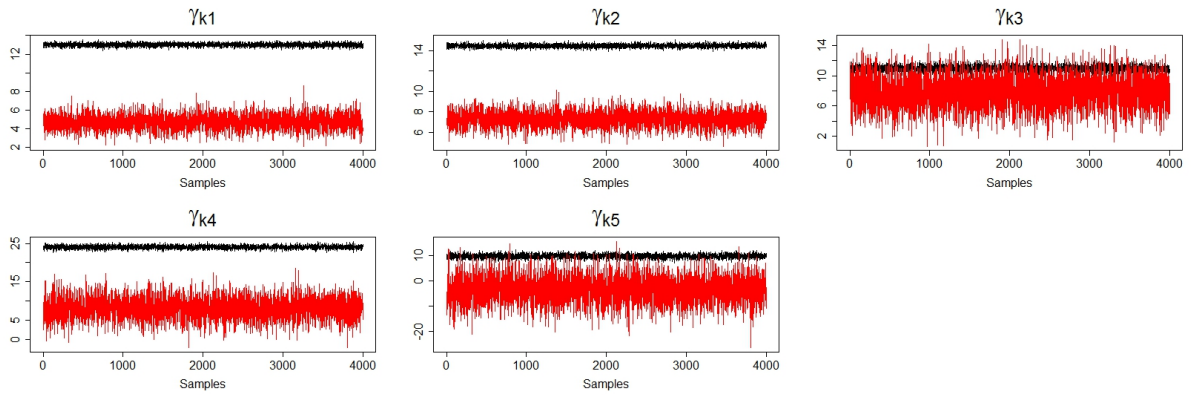


**Figure E1.** Trace plots of  $\mu_k$  for Hormone profile dataset





**Figure E2.** Trace plots of elements of  $\Sigma_k$  for Hormone profile dataset



**Figure E3.** Trace plots of  $\gamma_k$  for Hormone profile dataset