Appendix 1

A simplified model of a TWIP robot, which consists of a rod connected to an axle incorporating two wheels, is shown in Figure A1. The angular position of the rod is measured from the positive vertical Y-axis and for stabilizing the system, the robot needs to move in the direction of the fall in the XZ plane. The sufficient force for the robot's movement is provided by two DC motors driving the wheels, and the angular position of the rod can be controlled by the degree of actuation of the driving motors. Table A1 provides the list of variables used to describe the system's dynamic motion equations.
The pendulum's rod center of gravity coordinate, from Figure A1, is

$$(x_{pG}, y_{pG}) = (x + l_p \sin\theta, r + l_p \cos\theta) \tag{A1}$$

where x is the horizontal coordinate of the robot. Newton's second law of motion can be written for the TWIP's horizontal motion as

$$F_M - Bv_x - F_{rb} = M_b \frac{dv_x}{dt} \tag{A2}$$

where F_{rb} is the force applied by the pendulum's rod to the base in the horizontal direction.

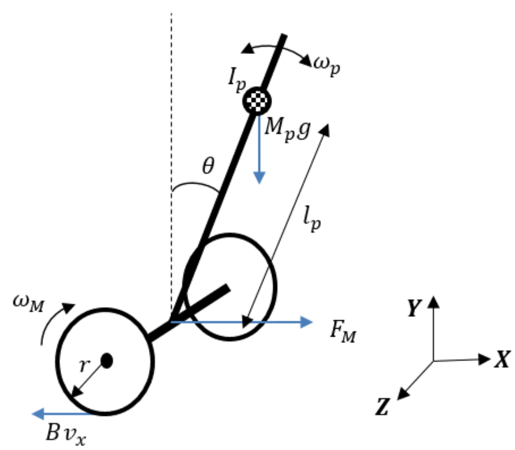

Figure A1. A simplified model of a TWIP

Table A1. Parameters label and description

Symbol	Description
B	Wheels' frictional coefficient (N/ms^{-1})
F_M	Applied force to the system by DC motors (N)
g	Gravitational constant (N/kg)
I_p	Inertia of pendulum (kg.m^2)
i	DC motor current (A)
k_t	Motor torque constant (N.m/A)
k_v	Motor electromagnetic force constant (V/rad.s^{-1})
L	Armature inductance of the DC motor (H)
M_b	Mass of base (kg)
M_p	Mass of pendulum (kg)
N	Motor transmission relation
l_p	Distance between pendulum centre of mass and pivot (m)
R	Armature resistance of the DC motor (Ω)
r	Radius of wheel (m)
V	Input voltage to the DC motors (V)
v_x	Linear speed of the base in the XZ plane (m/s)
θ	Angular position of the pendulum (rad)
ω_p	Angular speed of the pendulum (rad/s)
ω_M	Angular speed of the motor (rad/s)

The amount of this force is equal to

$$F_{rb} = M_p \frac{d^2 x_{pG}}{dt^2} = M_p \frac{d^2}{dt^2}(x + l_p \sin\theta) = M_p \left(\frac{dv_x}{dt} + l_p \cos\theta \frac{d\omega_p}{dt} - l_p \omega_p^2 \sin\theta\right) \tag{A3}$$

Since

$$\frac{d\theta}{dt} = \omega_p = f_1(\omega_p) \tag{A4}$$

The horizontal motion equation for the TWIP robot is derived as

$$(M_b + M_p)\frac{dv_x}{dt} = F_M - Bv_x - M_p l_p \cos\theta \frac{d\omega_p}{dt} + M_p l_p \omega_p^2 \sin\theta \tag{A5}$$

The motion equation for the pendulum's rod can be written based on Newton's second law of rotational motion as

$$F_Y l_p \sin\theta - F_{rb} l_p \sin\left(\frac{\pi}{2} - \theta\right) = I_p \frac{d\omega_p}{dt} \quad \text{(A6)}$$

where the amount of F_Y is

$$F_Y = M_p g + M_p \frac{d^2 y_{pG}}{dt^2} = M_p \left(g - l_p \sin\theta \frac{d\omega_p}{dt} - l_p \omega_p^2 \cos\theta\right) \quad \text{(A7)}$$

Therefore, the vertical motion equation for the TWIP robot is derived as

$$\left(I_p + M_p l_p^2\right) \frac{d\omega_p}{dt} = M_p g l_p \sin\theta - M_p l_p \cos\theta \frac{dv_x}{dt} \quad \text{(A8)}$$

The torque value provided by an ideal DC motor is equal to $Nk_t i$, and since two similar DC motors are driving the robot, the amount of applied force to the system is

$$F_M = 2N \frac{k_t i}{r} \quad \text{(A9)}$$

For the sake of simplicity, the inertia of wheels ignored in the equation (A9). By combining equations (A5), (A8) and (A9), we get

$$\frac{d\omega_p}{dt} = \frac{M_p l_p \left(2N \frac{k_t i}{r} \cos\theta - B v_x \cos\theta + M_p l_p \omega_p^2 \cos\theta \sin\theta - g\left(M_b + M_p\right) \sin\theta\right)}{M_p^2 l_p^2 \cos^2\theta - \left(M_b + M_p\right)\left(I_p + M_p l_p^2\right)} = f_2\left(\theta, \omega_p, v_x, i\right) \quad \text{(A10)}$$

and

$$\frac{dv_x}{dt} = \frac{-\left(I_p + M_p l_p^2\right)\left(2N \frac{k_t i}{r} - B v_x + M_p l_p \omega_p^2 \sin\theta\right) + M_p^2 l_p^2 g \cos\theta \sin\theta}{M_p^2 l_p^2 \cos^2\theta - \left(M_b + M_p\right)\left(I_p + M_p l_p^2\right)} = f_3\left(\theta, \omega_p, v_x, i\right) \quad \text{(A11)}$$

Finally, the relation between the motor current and the input voltage is

$$\frac{di}{dt} = \frac{1}{L}\left(V - Ri - \frac{k_v \omega_M}{N}\right) = \frac{1}{L}\left(V - Ri - k_v \frac{r}{N} v_x\right) = f_4(v_x, i, V) \quad \text{(A12)}$$

The model of the TWIP given by equations (A4), (A10), (A11), and (A12) is nonlinear. For the purpose of designing a linear controller, the achieved model has to be linearized about its steady-state point, which is $\boldsymbol{X_0} = \left[\theta\ \omega_p\ v_x\ i\right]^T = [0\ 0\ 0\ 0]^T$. For the state vector $\boldsymbol{X} = \left[\theta\ \omega_p\ v_x\ i\right]^T$, the linearized model is achieved as

$$\frac{d}{dt}\begin{pmatrix}\theta \\ \omega_p \\ v_x \\ i\end{pmatrix} = \left.\begin{pmatrix} \frac{\partial f_1}{\partial \theta} & \frac{\partial f_1}{\partial \omega_p} & \frac{\partial f_1}{\partial v_x} & \frac{\partial f_1}{\partial i} \\ \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial \omega_p} & \frac{\partial f_2}{\partial v_x} & \frac{\partial f_2}{\partial i} \\ \frac{\partial f_3}{\partial \theta} & \frac{\partial f_3}{\partial \omega_p} & \frac{\partial f_3}{\partial v_x} & \frac{\partial f_3}{\partial i} \\ \frac{\partial f_4}{\partial \theta} & \frac{\partial f_4}{\partial \omega_p} & \frac{\partial f_4}{\partial v_x} & \frac{\partial f_4}{\partial i} \end{pmatrix}\right|_{\boldsymbol{X_0}} \begin{pmatrix}\theta \\ \omega_p \\ v_x \\ i\end{pmatrix} + \left.\begin{pmatrix} \frac{\partial f_1}{\partial V} \\ \frac{\partial f_2}{\partial V} \\ \frac{\partial f_3}{\partial V} \\ \frac{\partial f_4}{\partial V} \end{pmatrix}\right|_{\boldsymbol{X_0}} V \quad \text{(A13)}$$

Despite the simple structure of this TWIP robot and ignoring many factors such as the joint and motor damping effects, still, laborious derivations require for linearizing the model. Nevertheless, calculating inertia and center of mass for more complex structures are demanding tasks.

Appendix 2

1. (theta==NL) & (Omega==NL) => (AppliedVoltage=NL) (1)
2. (theta==NL) & (Omega==NM) => (AppliedVoltage=NL) (1)
3. (theta==NL) & (Omega==NS) => (AppliedVoltage=NL) (1)
4. (theta==NL) & (Omega==ZE) => (AppliedVoltage=NL) (1)
5. (theta==NL) & (Omega==PS) => (AppliedVoltage=NL) (1)
6. (theta==NL) & (Omega==PM) => (AppliedVoltage=NL) (1)
7. (theta==NL) & (Omega==PL) => (AppliedVoltage=NL) (1)
8. (theta==PL) & (Omega==NL) => (AppliedVoltage=PL) (1)
9. (theta==PL) & (Omega==NM) => (AppliedVoltage=PL) (1)
10. (theta==PL) & (Omega==NS) => (AppliedVoltage=PL) (1)
11. (theta==PL) & (Omega==ZE) => (AppliedVoltage=PL) (1)
12. (theta==PL) & (Omega==PS) => (AppliedVoltage=PL) (1)
13. (theta==PL) & (Omega==PM) => (AppliedVoltage=PL) (1)
14. (theta==PL) & (Omega==PL) => (AppliedVoltage=PL) (1)
15. (theta==ZE) & (Omega==ZE) => (AppliedVoltage=ZE) (1)
16. (theta==ZE) & (Omega==NS) => (AppliedVoltage=NS) (1)
17. (theta==ZE) & (Omega==NM) => (AppliedVoltage=NS) (1)
18. (theta==ZE) & (Omega==NS) => (AppliedVoltage=NS) (1)
19. (theta==ZE) & (Omega==PS) => (AppliedVoltage=PS) (1)
20. (theta==ZE) & (Omega==PM) => (AppliedVoltage=PS) (1)
21. (theta==ZE) & (Omega==PL) => (AppliedVoltage=PS) (1)
22. (theta==PM) & (Omega==PL) => (AppliedVoltage=PL) (1)
23. (theta==PM) & (Omega==PM) => (AppliedVoltage=PL) (1)
24. (theta==PM) & (Omega==PS) => (AppliedVoltage=PL) (1)
25. (theta==PM) & (Omega==ZE) => (AppliedVoltage=PL) (1)
26. (theta==PM) & (Omega==NS) => (AppliedVoltage=PL) (1)
27. (theta==PM) & (Omega==NM) => (AppliedVoltage=PL) (1)
28. (theta==PM) & (Omega==NL) => (AppliedVoltage=PS) (1)
29. (theta==NM) & (Omega==NL) => (AppliedVoltage=NL) (1)
30. (theta==NM) & (Omega==NM) => (AppliedVoltage=NL) (1)
31. (theta==NM) & (Omega==NS) => (AppliedVoltage=NL) (1)
32. (theta==NM) & (Omega==ZE) => (AppliedVoltage=NL) (1)
33. (theta==NM) & (Omega==PS) => (AppliedVoltage=NL) (1)
34. (theta==NM) & (Omega==PM) => (AppliedVoltage=NL) (1)
35. (theta==NM) & (Omega==PL) => (AppliedVoltage=NS) (1)
36. (theta==NS) & (Omega==NL) => (AppliedVoltage=NS) (1)
37. (theta==NS) & (Omega==NM) => (AppliedVoltage=NS) (1)
38. (theta==NS) & (Omega==NS) => (AppliedVoltage=NS) (1)
39. (theta==NS) & (Omega==ZE) => (AppliedVoltage=NS) (1)
40. (theta==NS) & (Omega==PS) => (AppliedVoltage=NS) (1)
41. (theta==NS) & (Omega==PM) => (AppliedVoltage=NS) (1)
42. (theta==NS) & (Omega==PL) => (AppliedVoltage=NS) (1)
43. (theta==PS) & (Omega==PL) => (AppliedVoltage=PS) (1)
44. (theta==PS) & (Omega==PM) => (AppliedVoltage=PS) (1)
45. (theta==PS) & (Omega==PS) => (AppliedVoltage=PS) (1)
46. (theta==PS) & (Omega==ZE) => (AppliedVoltage=PS) (1)
47. (theta==PS) & (Omega==NS) => (AppliedVoltage=PS) (1)
48. (theta==PS) & (Omega==NM) => (AppliedVoltage=PS) (1)
49. (theta==PS) & (Omega==NL) => (AppliedVoltage=PS) (1)