Appendix 1

The functional form of $D_k(j)$*:*

For k=1,

$$D_1(j) = \beta(t_0(j))\left\{\sum_{i=1}^{j-1}\int_{x_{i-1}}^{x_i} w(x)Q_x^*(t_0 - x)dx + \int_{x_{j-1}}^{t_0(j)} w(x)Q_x^*(t_0 - x)dx\right\},$$

for k=2,

$$D_2(j) = \beta\big(t_1(j)\big)\left[1 - \beta\big(t_0(j)\big)\left\{\sum_{i=1}^{j-1}\int_{x_{i-1}}^{x_i} w(x)\,Q_x^*(t_1 - x)dx + \int_{x_{j-1}}^{t_0(j)} w(x)Q_x^*(t_1 - x)dx\right\}\right.$$

$$\left. + \int_{t_0}^{t_1} w(x)Q_{t_0(j)}(t_1 - x)dx\right],$$

for k≥ 3,

$$D_k(j) = \beta\big(t_{k-1}(j)\big)\Big[\sum_{i=1}^{k-2}\Big\{\prod_{l=i}^{k-2}\Big(1 - \beta\big(t_l(j)\big)\Big)\Big\}\int_{t_{i-1}}^{t_i} w(x)\,Q_{t_{i-1}(j)}(t_{k-1} - x)dx \; +$$

$$\int_{t_{k-2}}^{t_{k-1}} w(x)Q_{t_{k-2}(j)}(t_{k-1} - x)dx \; + \; \prod_{l=0}^{k-2}\Big(1 - \beta\big(t_l(j)\big)\Big)\Big\{\sum_{i=1}^{j-1}\int_{x_{i-1}}^{x_i} w(x)\,Q_x^*(t_{k-1} - x)dx +$$

$$\int_{x_{j-1}}^{t_0(j)} w(x)Q_x^*(t_{k-1} - x)dx\Big\}\Big]$$

where $Q_x^*(\cdot)$ is survival distribution of sojourn time in S_p at age $x < t_0(j)$. In simulation, we set parameters of $Q_x^*(\cdot)$ to a known constant values when calculating $D_k(j)$. Since $\Delta_{\boldsymbol{k}} \leq \boldsymbol{\delta_j}$ for all k, and j $\{\mathrm{k} = 1, \cdots, \mathrm{K} - 1, \text{and } \mathrm{j} = 1, 2, \cdots \mathrm{g}\}$. the participants who belong to *jth* aged group at entry time is divided into at most two age groups at each examination time. Thus, the survival distribution of sojourn time in S_p at the *ith* examination in the jth age group is redefined as follows:

$$Q_{t_{i-1}(j)}(t) = \begin{cases} Q_s(t) & if\ t_{i-1}(j) \in A_s \\ Q_{s+1}(t) & if\ t_{i-1}(j) \in A_{s+1} \end{cases} \quad \text{and} \quad \beta\big(t_l(j)\big) = \begin{cases} \beta_s & if\ t_l(j) \in A_s \\ \beta_{s+1} & if\ t_l(j) \in A_{s+1} \end{cases},$$

where $Q_s(t)$denotes the survival distribution with mean μ_s.

The functional form of $I_k(j)$*:*

For k=1,

$$I_1(j) = \left(1-\beta\left(t_0(j)\right)\right)\left\{\sum_{i=1}^{j-1}\int_{x_{i-1}}^{x_i} w(x)\left(Q_x^*(t_0-x)-Q_x^*(t_1-x)\right)dx \right.$$

$$\left. + \int_{x_{j-1}}^{t_0(j)} w(x)\left(Q_x^*(t_0-x)-Q_x^*(t_1-x)\right)dx\right\} + \int_{t_0}^{t_1} w(x)\left(1-Q_{t_0(j)}(t_1-x)\right)dx,$$

for k>1,

$$I_k(j) = \left[\prod_{l=0}^{k-1}\left(1-\beta\left(t_l(j)\right)\right)\left\{\sum_{i=1}^{j-1}\int_{x_{i-1}}^{x_i} w(x)\left(Q_x^*(t_{k-1}-x)-Q_x^*(t_k-x)\right)dx \right.\right.$$

$$\left. + \int_{x_{j-1}}^{t_0(j)} w(x)\left(Q_x^*(t_{k-1}-x)-Q_x^*(t_k-x)\right)dx \right\}$$

$$+ \sum_{i=1}^{k-1}\prod_{l=i}^{k-1}\left(1-\beta\left(t_l(j)\right)\right)\int_{t_{i-1}}^{t_i} w(x)\left(Q_{t_{i-1}(j)}(t_{k-1}-x)-Q_{t_{i-1}(j)}(t_k-x)\right)dx$$

$$\left. + \int_{t_{k-1}}^{t_k} w(x)\left\{1-Q_{t_{k-1}(j)}(t_k-x)\right\}dx \right].$$

Here, $Q_x^*(\cdot)$, $Q_{t_{i-1}(j)}(\cdot)$ and $\beta\left(t_l(j)\right)$ defined the same as *the functional form of* $D_k(j)$.