Supplemental material

Derivation of the Recursive Form of the Fisher Information Matrix

The fisher information matrix is defined by

$$\mathbf{G}_k = \mathrm{E}\Big[\big(\nabla_s \ln p(\boldsymbol{o}_{1:k}\,|\boldsymbol{s}_k\,)\big)\big(\nabla_s \ln p(\boldsymbol{o}_{1:k}\,|\boldsymbol{s}_k\,)\big)^{\mathrm{T}}\Big]. \tag{34}$$

The first order derivative of the log-density function is given by

$$\begin{aligned}&\frac{\partial \ln p(\boldsymbol{o}_{1:k}\,|\boldsymbol{s}_k\,)}{\partial s_i}\\ &= -\frac{1}{2}\sum_{m=1}^{k}\left(\frac{\partial \ln|\mathbf{C}\,(\boldsymbol{s}_k,m)|}{\partial s_i} + \frac{\partial}{\partial s_i}\Big[\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)^{\mathrm{T}}\mathbf{C}^{-1}(\boldsymbol{s}_k,m)\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)\Big]\right)\end{aligned} \tag{35}$$

Noting that $\mathbf{C}(\boldsymbol{s}_k,m)$ is diagonal matrix, we obtain [32]

$$\begin{aligned}&\frac{\partial \ln p(\boldsymbol{o}_{1:k}\,|\boldsymbol{s}_k\,)}{\partial s_i}\\ &= -\frac{1}{2}\sum_{m=1}^{k}\left(\mathrm{tr}\left[\mathbf{C}^{-1}(\boldsymbol{s}_k,m)\frac{\partial \mathbf{C}(\boldsymbol{s}_k,m)}{\partial s_i}\right] - 2\frac{\partial \boldsymbol{\eta}^{\mathrm{T}}(\boldsymbol{s}_k,m)}{\partial s_i}\mathbf{C}^{-1}(\boldsymbol{s}_k,m)\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)\right)\\ &\quad -\frac{1}{2}\sum_{m=1}^{k}\left(\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)^{\mathrm{T}}\frac{\partial \mathbf{C}^{-1}(\boldsymbol{s}_k,m)}{\partial s_i}\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)\right).\end{aligned} \tag{36}$$

Finally, noting that

$$\mathrm{E}\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big) = 0,$$

$$\mathrm{E}\Big[\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)^{\mathrm{T}}\Big] = \mathbf{C}(\boldsymbol{s}_k,m),$$

and

$$\mathrm{E}\Big[\big(\boldsymbol{o}_m - \boldsymbol{\eta}(\boldsymbol{s}_k,m)\big)\big(\boldsymbol{o}_l - \boldsymbol{\eta}(\boldsymbol{s}_k,l)\big)^{\mathrm{T}}\Big] = 0, m \neq l,$$

the Fisher information matrix can be written as

$$\begin{aligned}\left[\mathbf{G}_k\right]_{ij} =& \left[\mathbf{G}_{k-1}\right]_{ij} + \left[\frac{\partial \boldsymbol{\eta}(\boldsymbol{s}_k,k)}{\partial s_i}\right]^{\mathrm{T}}\mathbf{C}^{-1}(\boldsymbol{s}_k,k)\left[\frac{\partial \boldsymbol{\eta}(\boldsymbol{s}_k,k)}{\partial s_j}\right]\\ &+\frac{1}{2}\mathrm{tr}\left(\mathbf{C}^{-1}(\boldsymbol{s}_k,k)\frac{\partial \mathbf{C}(\boldsymbol{s}_k,k)}{\partial s_i}\mathbf{C}^{-1}(\boldsymbol{s}_k,k)\frac{\partial \mathbf{C}(\boldsymbol{s}_k,k)}{\partial s_j}\right).\end{aligned} \tag{37}$$

Then the Fisher information matrix can be calculated in a recursive form as

$$\mathbf{G}_k = \mathbf{G}_{k-1} + \Delta\mathbf{G}_k. \tag{38}$$

Furthermore, $\Delta\mathbf{G}_k$ is a matrix of 2×2 and its specific form is as following:

$$\left[\Delta\mathbf{G}_k\right]_{11} = \frac{\Delta x_{k,k}^2}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2 + h^2\right)^3\sigma_r^2} + \frac{\Delta y_{k,k}^2}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2\right)^2\sigma_\varphi^2} + \frac{8\Delta x_{k,k}^2}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2 + h^2\right)^2} \tag{39}$$

$$\left[\Delta\mathbf{G}_k\right]_{12} = \left[\mathbf{G}_k\right]_{21} = \frac{\Delta x_{k,k}\Delta y_{k,k}}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2 + h^2\right)^3\sigma_r^2} - \frac{\Delta x_{k,k}\Delta y_{k,k}}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2\right)^2\sigma_\varphi^2} + \frac{8\Delta x_{k,k}\Delta y_{k,k}}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2 + h^2\right)^2} \tag{40}$$

$$\left[\Delta\mathbf{G}_k\right]_{22} = \frac{\Delta y_{k,k}^2}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2 + h^2\right)^3\sigma_r^2} + \frac{\Delta x_{k,k}^2}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2\right)^2\sigma_\varphi^2} + \frac{8\Delta y_{k,k}^2}{\left(\Delta x_{k,k}^2 + \Delta y_{k,k}^2 + h^2\right)^2} \tag{41}$$