

Supplementary material

Supplementary material 1 | Distribution of population, employment, LFDI stocks and LFDI flows in the French urban system

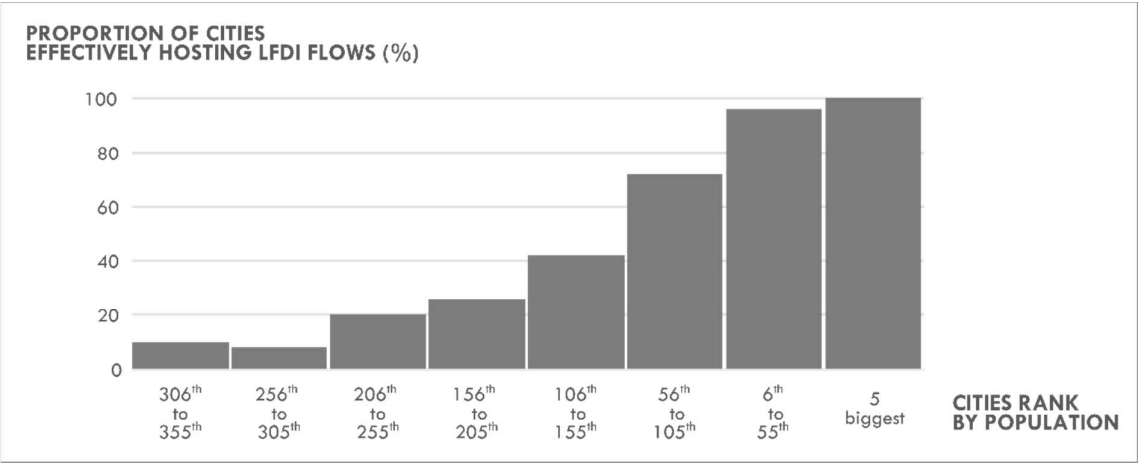
Category of cities by number of inhabitants	Number of cities	% of total urban population	% of total urban employment	% of total urban LFDI stocks*	% of total urban LFDI flows*
[Paris]	1	24.2	26.3	34.4	45.7
[1 million; Paris[	6	16.8	16.5	18.2	20.6
[500,000; 1 million[	10	12.7	12.0	10.5	10.8
[200,000; 500,000[	29	18.0	17.0	15.0	9.2
[100,000; 200,000[	40	10.1	10.0	8.5	5.3
[50,000; 100,000[	61	8.8	8.3	5.8	5.2
[25,000; 50,000[	77	5.2	5.2	4.1	2.4
[min; 25,000[	131	4.2	4.7	3.4	0.9
	Total number of cities	Total urban inhabitants	Total urban employment	Total urban LFDI stocks*	Total urban LFDI flows*
	355	51,118,905	21,878,056	1,981,279	138,093

* LFDI stocks and flows measured as an amount of jobs

Supplementary material 2 | Statistical characteristics of urban economic attributes as observed in the 355 French cities

	Urban population		Urban employment		Urban LFDI stocks		Urban LFDI flows		
	Raw data	Log	Raw data	Log	Raw data	Log	Raw data	Raw data excluding 0-values	Log
Elements	355	355	355	355	355	355	355	142	142
Total	51,118,905		21,878,056		1,981,279		138,093	138,093	
Mean	143,996.9	4.7	61,628.3	4.3	5,581.1	3.0	389.0	972.5	2.2
Median	36,271	4.6	15,437.3	4.2	947	3.0	0.0	141.0	2.2
Standard deviation	689,203.9	0.5	317,278.5	0.5	37,280.2	0.7	3424.9	5373.8	0.7
Coefficient of variation	4.8	0.1	5.1	0.1	6.7	0.2	8.8	5.5	0.3
Maximum proportion of the total	24.2% (Paris, rank 1)		26.3% (Paris, rank 1)		34.4% (Paris, rank 1)		45.7% (Paris, rank 1)		
Proportion of 0-values	0.0%		0.0%		0.0%		60.0%	0.0%	
Scaling parameters			$\beta = 0.94$ $CI_{95\%} = [0.92 ; 0.96]$ $R^2 = 0.69$		$\beta = 1.13$ $CI_{95\%} = [1.05 ; 1.21]$ $R^2 = 0.69$		$\beta = 0.90$ $CI_{95\%} = [0.74 ; 1.07]$ $R^2 = 0.45$		

Supplementary material 3 | Proportion of cities hosting LFDI flows according to their rank in the system



Supplementary material 4 | Maximum Likelihood exponents (with BIC values) for all three attributes and a Gaussian noise specification

Definition of the urban system (N top cities)	Total employment		LFDI stocks		LFDI flows	
	Free β (BIC)	Fixed value $\beta = 1$ (BIC)	Free β (BIC)	Fixed value $\beta = 1$ (BIC)	Free β (BIC)	Fixed value $\beta = 1$ (BIC)
355	0.97 (6,913.4)	1 (6,917.8)	1.10 (5,977.9)	1 (5,989.7)	1.17 (2,145.8)	1 (2,155.0)
330	0.98 (6,432.6)	1 (6,435.7)	1.10 (5,601.2)	1 (5,613.0)	1.18 (2,121.3)	1 (2,131.3)
305	0.99 (5,970.1)	1 (5,968.3)	1.11 (5,241.5)	1 (5,257.0)	1.18 (2,085.5)	1 (2,095.8)
280	0.99 (5,520.1)	1 (5,517.3)	1.12 (4,852.8)	1 (4,869.8)	1.17 (2,073.9)	1 (2,084.3)
255	0.99 (5,072.9)	1 (5,069.0)	1.12 (4,473.1)	1 (4,491.1)	1.17 (2,037.9)	1 (2,048.8)
230	0.99 (4,618.0)	1 (4,613.1)	1.12 (4,057.4)	1 (4,071.4)	1.17 (1,973.8)	1 (1,985.6)
205	1.00 (4,185.3)	1 (4,179.9)	1.13 (3,676.3)	1 (3,692.4)	1.17 (1,911.3)	1 (1,923.5)
180	1.00 (3,733.2)	1 (3,728.2)	1.14 (3,278.0)	1 (3,299.3)	1.20 (1,828.1)	1 (1,841.4)
155	1.02 (3,264.8)	1 (3,262.9)	1.14 (2,869.9)	1 (2,895.3)	1.21 (1,712.2)	1 (1,726.2)
130	1.02 (2,780.8)	1 (2,780.3)	1.15 (2,398.9)	1 (2,414.6)	1.21 (1,634.3)	1 (1,648.4)
105	1.02 (2,278.1)	1 (2,276.5)	1.14 (1,979.7)	1 (1,988.0)	1.21 (1,426.4)	1 (1,440.9)
80	1.03 (1,769.7)	1 (1,768.7)	1.13 (1,542.9)	1 (1,545.7)	1.29 (1,136.6)	1 (1,142.8)
55	1.04 (1,253.0)	1 (1,260.4)	1.16 (1,098.6)	1 (1,107.5)	1.31 (847.2)	1 (857.5)
30	1.04 (699.6)	1 (707.7)	1.16 (622.9)	1 (626.8)	1.33 (493.9)	1 (505.3)
10	1.04 (240.4)	1 (241.6)	1.13 (224.9)	1 (228.5)	1.22 (178.7)	1 (184.2)

Supplementary material 5 | Maximum Likelihood exponents (with BIC values) for all three attributes and a Lognormal noise specification

Definition of the urban system (N top cities)	Total employment		LFDI stocks		LFDI flows	
	Free β (BIC)	Fixed value $\beta = 1$ (BIC)	Free β (BIC)	Fixed value $\beta = 1$ (BIC)	Free β (BIC)	Fixed value $\beta = 1$ (BIC)
355	0.96 (6,881.9)	1 (6,907.7)	1.07 (5,823.0)	1 (5,826.5)	0.90 (1,921.1)	1 (1,917.9)
330	0.97 (6,420.6)	1 (6,432.0)	1.08 (5,466.7)	1 (5,472.4)	0.94 (1,891.8)	1 (1,887.4)
305	0.98 (5,966.2)	1 (5,967.3)	1.09 (5,115.9)	1 (5,123.3)	0.94 (1,862.8)	1 (1,858.4)
280	0.98 (5,527.4)	1 (5,526.0)	1.10 (4,742.5)	1 (4,753.7)	0.93 (1,855.5)	1 (1,851.2)
255	0.99 (5,085.7)	1 (5,082.5)	1.10 (4,381.3)	1 (4,391.9)	0.94 (1,826.4)	1 (1,822.0)
230	0.99 (4,628.2)	1 (4,623.6)	1.11 (3,987.5)	1 (3,997.2)	1.00 (1,763.0)	1 (1,758.1)
205	1.00 (4,194.6)	1 (4,189.3)	1.11 (3,622.1)	1 (3,631.9)	1.04 (1,704.7)	1 (1,700.1)
180	1.00 (3,743.2)	1 (3,738.0)	1.12 (3,225.0)	1 (3,239.4)	1.11 (1,634.2)	1 (1,631.1)
155	1.01 (3,273.0)	1 (3,270.2)	1.13 (2,823.2)	1 (2,841.6)	1.18 (1,527.8)	1 (1,528.1)
130	1.02 (2,788.5)	1 (2,786.7)	1.14 (2,400.4)	1 (2,418.9)	1.20 (1,462.1)	1 (1,463.5)
105	1.02 (2,285.4)	1 (2,282.6)	1.13 (1,993.2)	1 (2,004.8)	1.23 (1,280.2)	1 (1,283.3)
80	1.02 (1,775.6)	1 (1,772.1)	1.11 (1,542.0)	1 (1,543.2)	1.26 (1,084.0)	1 (1,089.9)
55	1.04 (1,258.0)	1 (1,263.2)	1.16 (1,095.9)	1 (1,103.6)	1.32 (798.9)	1 (812.8)
30	1.04 (702.3)	1 (710.8)	1.15 (628.9)	1 (632.3)	1.33 (487.6)	1 (498.4)
10	1.04 (240.9)	1 (240.6)	1.14 (224.4)	1 (228.5)	1.23 (178.2)	1 (184.3)

Supplementary material 6 | Hurdle and Poisson exponents (with BIC values) for LFDI flows

Definition of the urban system (N top cities)	Hurdle model with Poisson noise		Hurdle model with Negative binomial noise		Poisson model
	β_{count} (BIC)	β_{zero} (BIC)	β_{count} (BIC)	β_{zero} (BIC)	β (BIC)
355	1.21 (58,730.4)	1.71	0.81 (2,254.4)	1.71	1.28 (76,117.5)
330	1.21 (57,999.1)	1.77	0.84 (2,212.2)	1.77	1.28 (74,868.2)
305	1.21 (57,382.1)	1.86	0.85 (2,161.6)	1.86	1.28 (73,714.0)
280	1.21 (57,364.7)	1.82	0.84 (2,144.1)	1.82	1.28 (73,123.4)
255	1.21 (57,061.2)	1.86	0.84 (2,095.6)	1.86	1.28 (72,099.9)
230	1.22 (55,727.4)	2.01	0.86 (2,009.2)	2.01	1.28 (69,711.9)
205	1.22 (55,091.5)	2.17	0.86 (1,927.9)	2.17	1.28 (67,958.8)
180	1.23 (50,475.2)	2.33	0.93 (1,830.0)	2.33	1.28 (61,550.3)
155	1.25 (46,404.8)	2.90	1.03 (1,688.2)	2.90	1.29 (55,580.0)
130	1.25 (46,203.2)	2.53	1.00 (1,594.9)	2.53	1.28 (53,100.4)
105	1.26 (42,049.6)	3.73	1.02 (1,375.9)	3.73	1.28 (47,005.7)
80	1.29 (26,862.9)	3.50	1.19 (1,124.8)	3.50	1.30 (29,031.2)
55	1.31 (22,283.9)	6.30	1.43 (822.9)	6.30	1.31 (23,219.8)
30	-	-	-	-	1.30 (12,002.8)
10	-	-	-	-	1.22 (3,097.6)